中正嶺學報 第三十三卷 第一期 民國 93.11.
JOURNAL OF C.C.I.T. VOL. 33, NO. 1, Nov. 2004

短文 Short Note

# All or None Inspection Policy under Quadratic Quality Loss Function

**Chung-Ho Chen**

*Department of Industrial Management, Southern Taiwan University of Technology*

## ABSTRACT

This article explores the problem of determining the all or none inspection policy under the out-of-control process. Assume that the variance of the process is known, but its mean is unknown. Over longer periods of time the mean may shift. In 1999, Baum has presented the inspection criterion applied to Taguchi and Clausing's SONY example. In this paper, we further propose the modified Baum's rule.

**Keywords**: Taguchi's Quality Loss Function, Uniform Distribution, Target Value, Out-Of-Control Process

# 考量二次品質損失函數的全數或全不檢驗策略

陳忠和

南台科技大學工業管理系

## 摘 要

本文探討擬訂失控製程的全數或全不檢驗策略的問題，假設製程變異數為已知，但是製程平均數卻是未知，最終製程平均數將會隨時間而改變。在1999年，Baum已提出應用於Taguchi與Clausing研究中SONY公司實例的檢驗法則，本篇論文將進一步提出修正的Baum檢驗法則。

**關鍵詞**: 田口品質損失函數，均等分配，目標值，失控製程

文稿收件日期 92.7.23; 文稿修正後以短文接受日期 93.4.27.
Manuscript received Jul. 23, 2003; revised as short paper April 27, 2004.

# I. INTRODUCTION

The traditional concept of conformance to specifications is that items meet the specification limits. In 1986, Taguchi [1] refined the quality of product and presented the quadratic quality loss function for reducing total losses to the society. Taguchi's [1] quadratic quality loss function has succeeded to be applied in the design of acceptance sampling plans and the method of statistical process control. Previous researchers (Kapur and Wang [2], Kapur [3], Kapur and Cho [4, 5]) have addressed the problem of setting the economic specification limits for the product when the process is not capable of specifications in the short term. They presented that the inspection in an on-line quality control system may be conducted as a short term approach to reduce variance of the items shipped to the customers. The economic specification limits must be determined on the basis of minimizing total loss to the customers as well as to the producers.

Taguchi and Clausing [6] have presented the paper entitled "Robust Quality" for addressing the expected loss from the customer. Baum [7] points out that "a major focus of Taguchi and Clausing's [6] article is to demonstrate that 100% inspection will not necessary reduce the expected loss suffered by the customer over a policy of 0% inspection". From Taguchi and Clausing's [6] example, Baum [7] further developed inspection decision rules for the San Diego plant of SONY company.

Suppose that the production process is stable, that is, in statistical control, we can apply Baum's [7] decision rule. However, the above stable process may change into out-of-control process. Over a certain period of time, the process is stable and the quality characteristic obeys the specified distribution. The variance of the characteristic, $\sigma^2$, is known but its mean, $\mu$, is unknown. However, over longer periods of time the mean may shift. Based on the above assumption, we further propose the modified Baum's [7] rule applied in the out-of-control process.

# II. BAUM'S DECISION RULE APPLIED TO THE SAN DIEGO PLANT

According to Baum [7], there are six assumptions in applying his rules as follows: (1) The production process is in statistical control. (2) The objective of plant is to select the alternative (100% inspection or 0% inspection) that minimizes the expected loss. (3) The plant has a perfect corrective action system, where the quality characteristic of any set that falls outside the specification limits is repaired to the target value. (4) The loss function represents the losses the producer imposes on the customer from shipping off-target units. (5) The distribution of quality characteristic, $X$, at the plant is uniformly distributed over the specification limits. (6) The distribution of $X$ is centered on the target value.

From Baum [7], we have the following inspection decision rule: 100% inspection should be performed if $C < \frac{B}{3}[\frac{1}{(1-p)^2} - 1 - 2p]$. The $C$ denotes the unit cost of inspection. $B$ denotes the unit repair cost or replacement cost. $P$ denotes the fraction of units repaired by the producer prior to shipment (= $\frac{b-a-2\lambda}{b-a}$). $a$ denotes the minimum value of quality characteristic $X$. $b$ denotes the maximum value of quality characteristic $X$. $\lambda$ denotes the producer's tolerance.

# III. MODIFIED BAUM'S RULE

Assume that the quality characteristic $X$ obeys the uniform distribution, the variance of the characteristic, $\sigma^2$, is known but its mean, $\mu$, is unknown. However, over longer periods of time the mean may shift. Let $T$ denote the target value. The probability density function (p.d.f.) of $X$ is $f(x)=\frac{1}{b-a}, a \le x \le b$. Suppose that the process mean $\mu$ shifts from target by $\delta$ standard deviations, that is, $\mu = T + \delta\sigma$.

### 3.1 Case 1

We have the following condition: $a < T-\lambda$ and $T+\lambda < b$. The expected loss with 0% inspection is

$$E(L/N) = k[\sigma^2 + (\mu - T)^2] = k(1+\delta^2)\sigma^2 \tag{1}$$

The $k$ is the quality loss coefficient ($=\frac{B}{\lambda^2}$) and $\sigma^2 = \frac{(b-a)^2}{12}$.

The expected loss given 100% inspection is

$$E(L/I) = \int_{T-\lambda}^{T+\lambda}[C + k(x-T)^2] f(x)dx + \int_{a}^{T-\lambda}(C+B) f(x)dx + \int_{T+\lambda}^{b}(C+B) f(x)dx = C + Bp + k(1-p)\frac{\lambda^2}{3} \tag{2}$$

The fraction of units repaired by the producer prior to shipment $p = P(X < T-\lambda) + P(X > T+\lambda) = \frac{b-a-2\lambda}{b-a}$. Hence, 100% inspection should be performed if $E(L/N) > E(L/I)$, i.e.,

$$C < \frac{B}{3}[\frac{1+\delta^2}{(1-p)^2} - 1 - 2p] \tag{3}$$

### 3.2 Case 2

We have the following condition: $T-\lambda < a < T$ and $T+\lambda < b$. The expected loss with 0% inspection is

$$E(L/N) = k[\sigma^2 + (\mu - T)^2] = k(1+\delta^2)\sigma^2 = \frac{B}{\lambda^2}(1+\delta^2)\frac{(T-a+\lambda)^2}{12(1-p)^2} \tag{4}$$

The fraction of units repaired by the producer prior to shipment $p = \frac{b-T-\lambda}{b-a}$.

The expected loss given 100% inspection is

$$E(L/I) = \int_{a}^{T+\lambda}[C + k(x-T)^2] f(x)dx + \int_{T+\lambda}^{b}(C+B) f(x)dx + C + Bp + (1-p)\frac{B}{3\lambda^2}[\lambda^2 + \lambda(a-T) + (a-T)^2] \tag{5}$$

Hence, 100% inspection should be performed if $E(L/N) > E(L/I)$, i.e.,

$$C < \frac{B}{3}[\frac{(1+\delta^2)(T-a+\lambda)^2}{4\lambda^2(1-p)^2} - 3p - (1-p)\frac{\lambda^2 + \lambda(a-T) + (a-T)^2}{\lambda^2}] \tag{6}$$

### 3.3 Case 3

We have the following condition: $a < T-\lambda$ and $T < b < T+\lambda$. The expected loss with 0% inspection is

$$E(L/N) = k[\sigma^2 + (\mu - T)^2] = k(1+\delta^2)\sigma^2 = \frac{B}{\lambda^2}(1+\delta^2)\frac{(b+\lambda-T)^2}{12(1-p)^2} \tag{7}$$

The fraction of units repaired by the producer prior to shipment $p$ = $\frac{T-\lambda-a}{b-a}$.

The expected loss given 100% inspection is

$$E(L/I)=\int_{T-\lambda}^{b}[C+k(x-T)^2]f(x)dx+\int_{a}^{T-\lambda}(C+B)f(x)dx$$
$$=C+Bp+(1-p)\frac{B}{3\lambda^2}[\lambda^2-\lambda(b-T)+(b-T)^2] \quad (8)$$

Hence, 100% inspection should be performed if $E(L/N)>E(L/I)$, i.e.,

$$C<\frac{B}{3}[\frac{(1+\delta^2)(b+\lambda-T)^2}{4\lambda^2(1-p)^2}-3p-(1-p)\frac{\lambda^2-\lambda(b-T)+(b-T)^2}{\lambda^2}] \quad (9)$$

### 3.4 Case 4

We have the following condition: $T-\lambda<a<T$ and $T<b<T+\lambda$. The expected loss with 0% inspection is

$$E(L/N)=k[\sigma^2+(\mu-T)^2]$$
$$=k(1+\delta^2)\sigma^2 \quad (10)$$
$$=\frac{B}{\lambda^2}(1+\delta^2)\frac{(b-a)^2}{12}$$

The expected loss given 100% inspection is

$$E(L/I)=\int_{a}^{b}[C+k(x-T)^2]f(x)dx$$
$$=C+\frac{B}{3\lambda^2}[(b-T)^2+(a-T)(b-T)+(a-T)^2] \quad (11)$$

Hence, 100% inspection should be performed if $E(L/N)>E(L/I)$, i.e.,

$$C<\frac{B}{\lambda^2}\{\frac{(1+\delta^2)(b-a)^2}{12}-\frac{1}{3}[(b-T)^2+(a-T)(b-T)+(a-T)^2]\} \quad (12)$$

## IV. NUMERICAL EXAMPLES

### 4.1 Example 1

Assume that we are in the condition of case 1, i.e., $a<T-\lambda$ and $T+\lambda<b$. Consider that the uniform quality characteristic $X$ with known variance $\sigma^2$ and unknown mean $\mu$. The p.d.f. of $X$ is $f(x)=\frac{1}{b-a}, a\le x\le b$. Let $b$-$a$ = 10. Assume that the unit cost of inspection $C$ = 1, the unit repair cost $B$ = 2, and the producer's tolerance $\lambda$ = 3. Hence, we have the process variance $\sigma^2=\frac{25}{3}$, the quality loss coefficient $k=\frac{2}{9}$, and the fraction of units repaired by the producer prior to shipment $p$ = 0.4. Suppose that the process mean $\mu$ shifts from target $T$ by $\delta$ standard deviations, that is, $\mu=T+\delta\sigma$. From Eq. (3), we obtain the conclusion that the 100% inspection is better than 0% inspection when $\delta<-0.434$ or $\delta>0.434$. Table 1 lists the expected losses with 0% and 100% inspection when $\delta=0(0.1)0.5$.

### 4.2 Example 2

Assume that we are in the condition of case 2, i.e., $T-\lambda<a<T$ and $T+\lambda<b$. Consider the uniform quality characteristic $X$ with known variance $\sigma^2$ and unknown mean $\mu$. The p.d.f. of $X$ is $f(x)$ = $\frac{1}{b-a}, a\le x\le b$. Let $b$-$a$ = 10 and $a$-$T$ = -2. Assume that the unit cost of inspection $C$ = 1.5, the unit repair cost $B$ = 2, and the producer's tolerance $\lambda$ = 3.

Hence, we have the process variance $\sigma^2 = \frac{25}{3}$, the quality loss coefficient $k = \frac{2}{9}$, and the fraction of units repaired by the producer prior to shipment $p$ = 0.5. Suppose that the process mean $\mu$ shifts from target $T$ by $\delta$ standard deviations, that is, $\mu = T + \delta\sigma$. From Eq. (6), we obtain the conclusion that the 100% inspection is better than 0% inspection when $\delta < -0.3$ or $\delta > 0.3$. Table 2 lists the expected losses with 0% and 100% inspection when $\delta = 0(0.1)0.5$.

Table 1. The expected losses of the 0% and 100% inspection for example 1.

| $\delta$ | $E(L/N)$ | $E(L/I)$ |
|---|---|---|
| 0 | 1.852 | 2.200 |
| 0.1 | 1.870 | 2.200 |
| 0.2 | 1.926 | 2.200 |
| 0.3 | 2.019 | 2.200 |
| 0.4 | 2.148 | 2.200 |
| 0.5 | 2.315 | 2.200 |

Table 2. The expected losses of the 0% and 100% inspection for example 2.

| $\delta$ | $E(L/N)$ | $E(L/I)$ |
|---|---|---|
| 0 | 1.852 | 2.0185 |
| 0.1 | 1.870 | 2.0185 |
| 0.2 | 1.926 | 2.0185 |
| 0.3 | 2.019 | 2.0185 |
| 0.4 | 2.148 | 2.0185 |
| 0.5 | 2.315 | 2.0185 |

### 4.3 Example 3

Assume that we are in the condition of case 3, i.e., $a < T - \lambda$ and $T < b < T + \lambda$. Consider the uniform quality characteristic $X$ with known variance $\sigma^2$ and unknown mean $\mu$. The p.d.f. of $X$ is $f(x) = \frac{1}{b-a}, a \le x \le b$. Let $b$-$a$ = 10 and $b$-$T$ = 2. Assume that the unit cost of inspection $C$ = 1, the unit repair cost $B$ = 2, and the producer's tolerance $\lambda$ = 3. Hence, we have the process variance $\sigma^2 = \frac{25}{3}$, the quality loss coefficient $k = \frac{2}{9}$, and the fraction of units repaired by the producer prior to shipment $p$ = 0.5. Suppose that the process mean $\mu$ shifts from target $T$ by $\delta$ standard deviations, that is, $\mu = T + \delta\sigma$. From Eq. (9), we obtain the conclusion that the 100% inspection is better than 0% inspection when $\delta < -0.469$ or $\delta > 0.469$. Table 3 lists the expected losses with 0% and 100% inspection when $\delta = 0(0.1)0.5$.

Table 3. The expected losses of the 0% and 100% inspection for example 3.

| $\delta$ | $E(L/N)$ | $E(L/I)$ |
|---|---|---|
| 0 | 1.852 | 2.259 |
| 0.1 | 1.870 | 2.259 |
| 0.2 | 1.926 | 2.259 |
| 0.3 | 2.019 | 2.259 |
| 0.4 | 2.148 | 2.259 |
| 0.5 | 2.315 | 2.259 |

### 4.4 Example 4

Assume that we are in the condition of case 4, i.e., $T - \lambda < a < T$ and $T < b < T + \lambda$. Consider the uniform quality characteristic $X$ with known variance $\sigma^2$ and unknown mean $\mu$. The p.d.f. of $X$ is $f(x) = \frac{1}{b-a}, a \le x \le b$. Let $b$-$a$ = 10 and $b$-$T$ = 4. Assume that the unit cost of inspection $C$ = 0.1, the unit repair cost $B$ = 2, and the producer's tolerance $\lambda$ = 3. Hence, we have the process variance $\sigma^2 = \frac{25}{3}$, $a$-$T$ = -6, the quality loss coefficient $k = \frac{2}{9}$, and the fraction of units repaired by the producer prior to shipment $p$ = 0. Suppose that the process mean $\mu$ shifts from target $T$ by $\delta$ standard deviations, that is, $\mu = T + \delta\sigma$. From Eq. (12), we obtain the conclusion that the 100% inspection is better than 0% inspection when $\delta < -0.417$ or $\delta > 0.417$. Table 4 lists the expected losses with 0% and 100% inspection when $\delta = 0(0.1)0.5$.

Table 4. The expected losses of the 0% and 100% inspection for example 4.

| $\delta$ | $E(L/N)$ | $E(L/I)$ |
|---|---|---|
| 0 | 1.852 | 2.174 |
| 0.1 | 1.870 | 2.174 |
| 0.2 | 1.926 | 2.174 |
| 0.3 | 2.019 | 2.174 |
| 0.4 | 2.148 | 2.174 |
| 0.5 | 2.315 | 2.174 |

## V. CONCLUDING REMARKS

If the process is under control but not capable of specifications, then the inspection in an on-line quality control system may be conducted as short term approach to reduce variance of the items shipped to the customers. Kapur and Wang [2] pointed out that if we can’t improve the present process, then a short term approach to decrease variance of the units shipped to the customer is to put specification limits on the process and truncate the distribution by inspection. The 0% or 100% inspection in Kapur and Wang [2] be adopted according to minimizing the total loss of society. For the out-of-control process, further improvements in quality must come from improving the process capability by statistical process control in order to reduce the process variance and adjust the process mean. In this paper, we have presented the modified Baum’s [7] rule under the uniform quality characteristic. Further direction of study can extend this method to other quality characteristic and consider the asymmetric quality loss function of product for determining the all or none inspection policy.

## REFERENCES

[1] Taguchi, G., Introduction to Quality Engineering, Tokyo, Asian Productivity Organization, 1986.

[2] Kapur, K. C. and Wang, C. J., “Economic design of specifications based on Taguchi's concept of quality loss function,” in Quality: Design, Planning, and Control, DeVor, R. E. and Kapoor, S. G., Ed., Boston, The Winter Annual Meeting of the American Society of Mechanical Engineers, pp. 23-36, 1987.

[3] Kapur, K. C., “An approach for development of specifications for quality improvement,” Quality Engineering, Vol. 1, No. 1, pp. 63-77, 1988.

[4] Kapur, K. C. and Cho, B.-R., “Economic design and development of Specifications,” Quality Engineering, Vol. 6, No. 3, pp. 401-417, 1994.

[5] Kapur, K. C. and Cho, B.-R., “Economic design of the specification region for multiple quality characteristics,” IIE Transactions, Vol. 28, No.3, pp. 237-248, 1996.

[6] Taguchi, G. and Clausing, D., “Robust quality,” Harvard Business Review, January-February, pp. 65-75, 1990.

[7] Baum, P., “Inspection decision rules applied to Taguchi’s Sony example,” Proceedings of the Section on Quality and Productivity, American Statistical Association, Maryland, pp. 1-4, 1999.