中正嶺學報 第三十三卷 第一期 民國 93.11.
JOURNAL OF C.C.I.T., VOL.33, NO.1, NOV., 2004.

Laminar Film Condensation on a Finite-size Horizontal Plate in a Porous Medium

Tong Bou Chang

Department of Mechanical Engineering
Nanya Institute of Technology

ABSTRACT

The problem of two-dimensional, steady state laminar film condensation on a finite-size isothermal horizontal plate in a porous medium is studied for the case in which a cold plate faces upwards in a stagnant saturation vapor. As in classical film condensation problems, it is assumed that; (a) the condensate and vapor are separated by a distinct boundary with no two-phase zone in between, and (b) the condensate has constant properties. A closed form solution has been obtained for the Nusselt number. A limiting case is also being considered here; the absence of any porous matrix.

***Keywords**:* horizontal plate condensation, laminar film condensation, porous medium

多孔隙介質中有限大小水平平板之層流膜狀凝結熱傳

張烔堡

南亞技術學院機械工程學系

摘　　要

本文旨在探討一有限大小之水平等溫平板於多孔隙介質中而低溫之平板上方具有飽和且靜止之蒸汽時之層流膜狀凝結問題。此問題作如同傳統膜狀凝結問題相同的假設：(1)凝結液與蒸汽分離，無兩相區。(2)凝結液具有等物理性質。本文獲得一無因次熱傳係數 Nusselt number 之預測公式。此外，無多孔隙介質存在時之特殊狀況亦在本文中探討

關鍵字：水平平板凝結，層流膜狀凝結，多孔隙介質

文稿收件日期 92.2.14 文稿修正後接受日期 93.3.23.
Manuscript received Feb. 14, 2003; revised Mar. 23, 2004.

I. INTRODUCTION

The problems of horizontal two-phase flow in a porous medium involving phase change have important applications in geothermal energy utilization [1] and thermal enhancement of oil recovery [2]. When two-phase flow exits in a porous medium, it is known that Darcy's law is applicable to both the liquid and the vapor phase.

The heat transfer rates of laminar film condensation on vertical or nearly vertical surfaces are adequately predicted by Nusselt's [3] analysis. Cheng [4] considered the problem of a downward condensate flowing along a cool inclined surface with porous medium that is filled with a dry saturated vapor. The earliest attempt to consider the condensation heat transfer rate on a horizontal surface was experimentally done by Popov [5] in 1951. Gerstmann and Griffith [6] then investigated the condensation on the underside of a horizontal plate both theoretically and experimentally. The case of the upper side condensation of a horizontal plate was first studied by Nimmo and Leppert [7,8]. However, they left one point in argument. The film condensation thickness at the plate edge is either assumed or specified by the particular boundary condition. Shigechi et. al. [9] obtained the condensate thickness and heat transfer results on a horizontal plate by adjusting the inclined angle of the vapor-liquid interface at the plate edge. Yang and Chen [10] used the concept of minimum mechanical energy [11] to obtain the boundary condition at the horizontal plate edge. In this study, the concept of minimum mechanical energy is also applied to obtain the minimum film thickness at the plate edge, and the dimensionless heat transfer coefficient can thus be properly solved.

II. ANALYSIS

Consider a finite-size horizontal plate with wall temperature T_w embedded in a porous medium filled with a dry vapor with temperature at T_{sat} as shown in Figure 1. If the wall temperature T_w is less than the saturated temperature T_{sat}, a film of condensate will be formed on the surface.

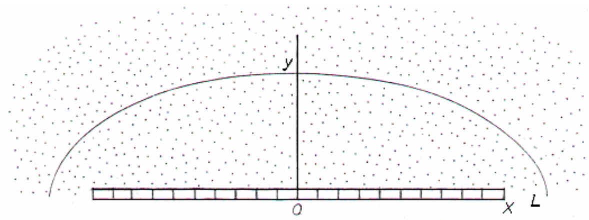

Figure 1. Condensate film flow on a finite-size horizontal plate with porous medium.

Under steady-state conditions, a laminar film condensation boundary layer will be established with maximum depth existing at the center of plate and gradually decreasing to minimum depth at the plate edge. The condensate flow rate within the boundary layer increases from zero at x=0 to a maximum value at the edge of the plate. To analyze this problem, the following assumptions are made：

1. The flow is steady and laminar.
2. The inertia within the film is negligible（a creeping film flow is assumed）.
3. The wall temperature and vapor temperature are uniform and are kept constant.
4. The kinetic energy of the film flow is negligible.

5. The properties of porous medium, dry vapor, and condensate are constant.

Under the aforementioned assumptions, the governing equations for the condensate film are：

continuity：

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad (1)$$

momentum：

$$0=-\frac{\partial P}{\partial x}-\frac{\mu}{K}u+\mu\frac{\partial^2 u}{\partial y^2} \qquad (2)$$

energy：

$$u\frac{\partial T}{\partial x}=\alpha_e\frac{\partial^2 T}{\partial y^2} \qquad (3)$$

pressure：

$$P=P_{sat}+\rho g(\delta-y) \qquad (4)$$

Cheng [4] reduced the momentum equation (Eq. (2)) to Darcy's velocity U_D, and allowed for a slip in velocity at the wall. Using U_D as the velocity, Eq. (2) was solved as $U_D=\frac{-K}{\mu}\frac{\partial P}{\partial x}=\frac{-\rho g K}{\mu}\frac{d\delta}{dx}$, and the temperature profile was almost linear for small values of the Jakob number (Ja $<<1$). However, the slip assumption of velocity at the wall will over predict the heat transfer to the wall and a higher condensate flux will be obtained.

Substituting Eq. (4) into Eq. (2) yields

$$\mu\frac{\partial^2 u}{\partial y^2}-\frac{\mu}{K}u-\rho g\frac{d\delta}{dx}=0 \qquad (5)$$

In this equation a no-shear boundary condition, $\frac{\partial u}{\partial y}=0$, is used for the momentum equation at $y=\delta$, and, $u=0$ at $y=0$, is the same as the assumption used in the classical Nusselt analysis. This model would give a more accurate prediction than the model used in [4] whenever the porous medium matrix plays a minor role in the condensation process.

Using this assumption, Eq. (5) can be solved to obtain the velocity profile of

$$u=\frac{\rho g K}{\mu}\frac{d\delta}{dx}\left(-1+\cosh\frac{y}{\sqrt{K}}-\sinh\frac{y}{\sqrt{K}}\tanh\frac{\delta}{\sqrt{K}}\right) \qquad (6).$$

At the boundary $y=\delta$, the velocity is $u\big|_{y=\delta}=\frac{\rho g K}{\mu}\frac{d\delta}{dx}\left(-1+\mathrm{sech}\frac{\delta}{\sqrt{K}}\right)$. To show the nature of this solution, two limiting cases are considered here. First, as $K\to\infty$, (in the absence of any porous medium), Eq. (5) is reduced to

$$u=\frac{\rho g}{\mu}\frac{d\delta}{dx}\left(\frac{y^2}{2}-y\delta\right) \qquad (7)$$

which is the same as Nusselt's solution for the velocity profile. Second, when δ is small, the velocity at the interface will be less than U_D.

Using Nusselt's classical analysis method, the overall energy balance in the liquid film can be written as

$$\left(k_e\frac{\partial T}{\partial y}\bigg|_{y=0}\right)dx=\frac{d}{dx}\left\{\int_0^{\delta}\rho u\left[h_{fg}+Cp(T_{sat}-T)\right]dy\right\}dx \qquad (8)$$

with the left side as the energy transfer from liquid film to solid plate, and the right hand side as the net energy flux across the liquid film (from x to x+dx). With linear temperature profile assumption, and $h_{fg}>>Cp\Delta T$, Eq. (8) can be simplified to

$$k_e\frac{\Delta T}{\delta}=\frac{d}{dx}\left\{\int_0^{\delta}\rho u h_{fg}dy\right\} \qquad (9)$$

Substituting Eq. (6) into Eq. (9) yields

$$\delta\frac{d}{dx}\left\{\frac{d\delta}{dx}\left(\sqrt{K}\tanh\frac{\delta}{\sqrt{K}}-\delta\right)\right\}=\frac{k_e\Delta T\mu}{\rho^2 gKh_{fg}} \qquad (10)$$

and by defining the following dimensionless parameters as:

$$Ja=\frac{Cp\Delta T}{h_{fg}} \qquad (11a)$$

and

$$Ra_e=\frac{\rho^2 g\Pr_e L^3}{\mu^2} \qquad (11b)$$

$$\Pr_e=\frac{\mu Cp}{k_e} \qquad (11c)$$

$$Da=\frac{K}{L^2} \qquad (11d)$$

we may rewrite the above energy equation as

$$\delta\frac{d}{dx}\left\{\frac{d\delta}{dx}\left(\sqrt{K}\tanh\frac{\delta}{\sqrt{K}}-\delta\right)\right\}=\frac{Ja}{Ra_e}\frac{L}{Da} \qquad (12)$$

with the following boundary conditions:

$$\frac{d\delta}{dx}=0 \quad \text{at} \quad x=0 \qquad (13a)$$

$$\delta=\delta_{\min} \quad \text{at} \quad x=L \qquad (13b)$$

With these boundary conditions, we still cannot solve Eq. (12), since $\delta_{\min}$ is unknown.

However, the film thickness at the plate edge cannot be zero, in fact, it should be established by the application of a minimum mechanical energy principle [10,11], just like that used in open-channel hydraulics. The principle states that a fluid flowing across a hydrostatic pressure gradient and off the plate will adjust itself so that the rate of mechanical energy within the fluid will be minimal with respect to the boundary layer at the plate edge. The minimum (critical) thickness can thus be calculated by setting the derivative of mechanical energy with respect to δ equal to zero for the steady flow rate;

$$\left[\frac{\partial}{\partial\delta}\int_0^{\delta}\left(\frac{u^2}{2}+gy+\frac{P}{\rho}\right)\rho u dy\right]_{\dot{m}_c}=0 \qquad (14)$$

where $\dot{m}_c$ is the critical value of mass flowing out of the plate edge.

Substituting Eq. (4) and Eq. (6) into Eq. (14) and setting $\tanh\frac{\delta}{\sqrt{K}}=1$ (because the permeability of many practically porous materials is at the order of $10^{-10}\sim10^{-16}$ m^2 ; see Table 1.1 in reference [12] or Table 2.3 in reference [13]; and then δ is much greater than $\sqrt{K}$), yields:

$$\left.\frac{d\delta}{dx}\right|_L=-\left(\frac{\mu^2}{\rho^2 gK^2}\delta_{\min}\right)^{1/2} \qquad (15)$$

By introducing the normalized variables of $x^*=x/\sqrt{K}$, $\delta^*=\delta/\sqrt{K}$, for Eq. (12), which can be written as

$$\delta^*\frac{d}{dx^*}\left\{\frac{d\delta^*}{dx^*}\left(\tanh\delta^*-\delta^*\right)\right\}=\frac{Ja}{Ra_e Da^{3/2}} \qquad (16)$$

Eq. (13a) becomes

$$\frac{d\delta^*}{dx^*}=0 \quad \text{at} \quad x^*=0 \qquad (16a)$$

and Eq. (15) becomes

$$\left.\frac{d\delta^*}{dx^*}\right|_{x_L^*}=-\left(\frac{\Pr_e\delta^*_{\min}}{Ra_e Da^{3/2}}\right)^{1/2} \qquad (16b)$$

From Eq. (16) and the corresponding boundary conditions Eq. (16a) and Eq. (16b), we can obtain the dimensionless film thickness δ^* in terms of Ja, Ra_e, $\Pr_e$, and Da.

The numerical procedure of solving this problem is to guess an initial thickness at plate center（$\delta^*\big|_{x^*=0}$）and substitute $\delta^*\big|_{x^*=0}$ into Eq.（16）. With the aid of Eq.（16a）, the δ^* value along the x^* direction can be calculated. After all δ^* values（at every arid point）have been calculated, check the resulting thickness gradient at the plate edge with Eq.（16b）and obtain a modified initial value for $\delta^*\big|_{x^*=0}$. The process is repeated until the condition of Eq.（16b）is satisfied.

Since the slope of film thickness is not flat at the plate edge, the progressively finer grid size toward the plate edge（with 1200 total grid points）is used in the calculations.

The mean Nusselt number is also calculated as

$$\overline{Nu} = \frac{\bar{h}L}{k_e} \qquad (17)$$

where

$$\bar{h} = \frac{1}{L}\int_0^L h(x)dx = \frac{1}{L}\int_0^L \frac{k_e}{\delta}dx = \frac{1}{L}\int_0^{x_L^*} \frac{k_e}{\delta^*}dx^*$$

III. RESULTS AND DISCUSSION

At first, the porous medium is assumed absent: i.e. the permeability of liquid approaches infinity in the present work. With $K \to \infty$, substituting Eq.（7）into Eqs.（8）and（14）yields

$$\delta \frac{d}{dx}\left(\delta^3 \frac{d\delta}{dx}\right) = -\frac{3Ja}{Ra}L^3 \qquad (18)$$

$$\left.\frac{d\delta}{dx}\right|_L = -\left(\frac{35}{6}\frac{\Pr}{Ra}\frac{L^3}{\delta_{\min}^3}\right)^{1/2} \qquad (18a)$$

and

$$\frac{d\delta}{dx} = 0 \quad \text{at} \quad x = 0 \qquad (18b)$$

The above equations can be solved by the same numerical procedures mentioned in the previous section, and the resulting δ value along the x-direction can be obtained.

Figure.2 shows the calculated Nusselt number decreases when Ja/Ra increases. In fact, the relationship is almost linear and can be expressed as：

$$\overline{Nu}(Ja/Ra)^{0.2} = 0.824 - 0.14(Ja/\Pr) \qquad (19)$$

for $Ja/\Pr < 0.1$.

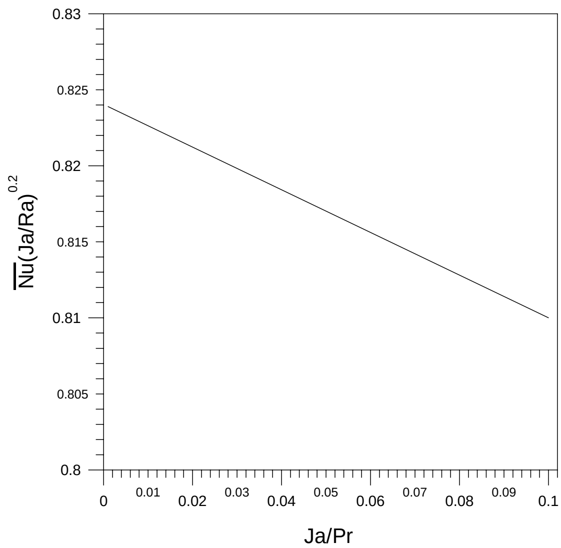

Figure 2. Dimensionless heat transfer coefficient without porous medium.

For water vapor condensing on a smooth plate, the value of $Ja/\Pr$ is about 0.0025 at one atmosphere. The resulting Nusselt number, Eq.（19）, is about

$$\overline{Nu} = 0.8237(Ra/Ja)^{0.2} \qquad (20)$$

Toru et al. [9] specified the inclination angle for the boundary condition at the plate edge, and obtained the mean Nusselt number of

$$\overline{Nu} = 0.821(Ra / Ja)^{0.2} \qquad (21a)$$

By using a new transformation method and the concept of minimum mechanical energy, Yang and Chen [10] obtained the mean Nusselt number of

$$\overline{Nu} = (0.81 \sim 0.82)(Ra / Ja)^{0.2} \qquad (21b)$$

Both are very close to our result (Eq. (20)) .

The experimental data on a finite-size horizontal plate by Nimmo and Leppert [8] is fitted into the following form:

$$\overline{Nu} = 0.64(Ra / Ja)^{0.2} \qquad (22)$$

Compare Eq. (20) and Eq. (22) . The present value is about 28% higher than that of Nimmo's [8], which may be caused by neglecting the effect of surface tension and impurities existing between the water and the plate. If the surface tension effect is included, the boundary layer thickness will become larger, $\overline{Nu}$ will become smaller, and thus closer to the experimental data obtained by Nimmo and Leppert.

Figure 3 illustrates the resulting Nusselt number when the effect of porous medium is included in the analysis. The relationship can be expressed as:

$$\overline{Nu}\left(\frac{Ja}{Ra_e Da}\right)^{1/3} = 1.448 - 0.037(Ja / \Pr_e) \qquad (23)$$

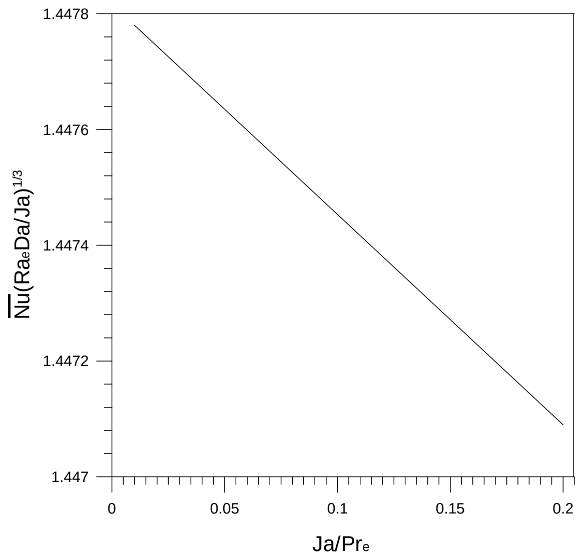

Figure 3. Dimensionless heat transfer coefficient with porous medium.

Because $Ja / \Pr_e$ is very small, Eq. (23) can be rewritten as

$$\overline{Nu} = 1.447\left(\frac{Ra_e Da}{Ja}\right)^{1/3} \quad \text{for } Da < 10^{-2} \qquad (24)$$

The constraint of $Da = K / L^2 < 10^{-2}$ in the above equation is usually satisfied for common porous materials, because the permeability of many practically porous materials is at the order of $10^{-10} \sim 10^{-16}$ m^2 (see Table 1.1 in reference [12] or Table 2.3 in reference [13]). Cheng [4] considered the problem of film condensation along a cool inclined surface in a porous medium. He obtained the local Nusselt number but the result is not valid for horizontal or nearly horizontal cases. Therefore, the result obtained in this study should be able to expand the data bank regarding film condensation in a porous medium.

Compare the Nusselt number of Eq. (24) to that of Eq. (22) , and we find the former is about

1 ~ 2 orders smaller than the latter. Two reasons are given for this difference. First, the effective conductivity of porous material is higher than that of liquid. The heat transfer rate of porous medium is thus higher, and the liquid condensate will be thicker. Second, the drag of condensate flowing in the porous medium is higher than in the non-porous medium. The developed condensate thickness will thus be thicker, and the resulting Nusselt number will be smaller.

IV. CONCLUSION

A closed form solution has been obtained in for the Nusselt number of steady state laminar film condensation on a finite-size isothermal horizontal plate in a porous medium. The correlation $\overline{Nu} = 1.447\left(\frac{Ra_e Da}{Ja}\right)^{1/3}$ is suitable for most of the engineering application. The limiting case, of absence of any porous matrix is also solved in this study.

V. REFERENCE

[1] Cheng, P., "Heat transfer in geothermal system,"Adv. Heat Transfer, Vol. 14, No.1, pp. 1-105 , 1978.

[2] Weinstein, H. G., Wheeler, J. A., and Woods, E. G., "Numerical model for thermal process," Soc. Petrol. Engrs. Jl 00, No.1, pp. 65-77, 1977.

[3] Nusselt, W., "Die oberflachen Kondensation des Wasserdampes," Zeitsehrift des Vereines Deutscher Ingenieure, Vol. 60, No.2, pp. 541-546, 1916.

[4] Cheng, P., "Film Condensation along An Inclined Surface in a Porous Medium," Int. J. Heat and Mass Transfer, Vol. 24, No.6, pp. 983~990, 1981.

[5] Popov, V. D., "Heat Transfer during Vapor Condensation on a Horizontal Surfaces," Trudy Kiev. Teknol. Inst. Pishch, Prom. Vol. 11, No.1, pp. 87~97, 1951.

[6] Gerstmann, J. and Griffith, P., "Laminar Film Condensation on the Underside of Horizontal and Inclined Surfaces," Int. J. Heat and Mass Transfer, Vol. 10, No.2, pp. 567~580, 1976.

[7] Leppert, G. and Nimmo, B., "Laminar Film Condensation on Surface Normal to Body or Inertial Forces," J. Heat Transfer, Vol. 80, No.1, pp. 178~179, 1968.

[8] Nimmo, B. and Leppert, G., "Laminar Film Condensation on a Finite Horizontal Surface," 4th Int. Heat Transfer Conference, pp. 402~403, 1970.

[9] Shigechi, T., Kawae, N., Tokita, Y., and Yamada, T., "Film Condensation Heat Transfer on a Finite-Size Horizontal Plate Facing Upward," JSME series B, Vol.56, No.1, pp. 205~210, 1990.

[10] Yang, S. A. and Chen, C. K., "Laminar Film Condensation on A Finite-Size Horizontal Plate with Suction at Wall," Apply Math. Modeling, Vol.16, No.1, pp. 325-329, 1992.

[11] Bakhmeteff, B. K. , Hydraulics of Open Channel, McGraw-Hill, New York, pp.39~41, 1966.

[12] Nield, D. A. and Bejan, A., Convection in Porous Media, Springer-Verlag, New York, pp. 4, 1992.

[13] Kaviany, M, Principles of Heat Transfer in Porous Media, Springer-Verlag, New York, pp.

26, 1991.

VI. NOMENCLATURE

Cp	Specific heat at constant pressure
Da	Darcy number defined in Eq.（11d）
g	acceleration of gravity
h	heat transfer coefficient
h_{fg}	heat of vaporization
Ja	Jakob number defined in Eq.（11a）
k	thermal conductivity
K	permeability of the porous medium
L	half of plate width
$\dot{m}$	condensate mass flux
Nu	Nusselt number defined in Eq.（17）
P	pressure
Ra	Rayleigh number
T	temperature
ΔT	saturation temperature minus wall temperature
u, v	horizontal and vertical velocity component
U_D	Darcy velocity $= -\frac{\rho g K}{\mu}\frac{d\delta}{dx}$

Geek symbols

δ	condensate film thickness
δ_0	condensate film thickness at plate center
μ	liquid viscosity
ρ	liquid density
α	thermal diffusivity

Superscripts

—	indicates average quantity
*	indicates dimensionless variable

Subscripts

o	quantity ay plate center
c	critical quantity
min	minimum quantity or quantity at plate edge
sat	saturation property
w	quantity at wall
e	effective property