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The Optimum Manufacturing Target under the Quality Loss Function

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ABSTRACT

In 1998, Wu and Tang presented the optimum manufacturing target based on Taguchi's quadratic quality loss function. However, they neglected to consider the linear quality loss function for a product in their model. In this paper, we further propose the modified Wu and Tang's model under the asymmetric linear quality loss function for determining the optimum manufacturing target.

Keywords: normal distribution, symmetric trapezoid distribution, quality loss function, manufacturing target

考量品質損失函數之最佳製造目標值設定

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摘 要

在1998年, Wu與Tang已提出基於田口二次品質損失函數來考量的最佳製造目標值設定問題; 然而, 他們卻未考慮產品具線性品質損失函數的情況於其模式中。本研究將進一步呈現考量非對稱線性品質損失函數的修正的Wu與Tang模式以決定最佳的製造目標值。

關鍵詞: 常態分配，對稱梯形分配，品質損失函數，製造目標值

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I. INTRODUCTION

Taguchi [1] redefined the product quality and adopts quadratic quality loss function for decreasing the total losses of society. The quadratic quality loss function has been successfully used in the economic design of control charts, sampling plans, specification limits, and tolerance zones. Recently, Wu and Tang [2], Li [3-7], Li and Cherng [8, 9], Maghsoodloo and Li [10], Li and Chou [11] and Li and Wu [12] addressed the different problems of unbalanced tolerance design with the asymmetric quality loss function.

Assume that the quality characteristic obeys a uniform or normal distribution. Wu and Tang [2] adopted quadratic asymmetric quality loss function for determining the optimum manufacturing target. They concluded that the average quality loss of a product can be decreased by shifting the manufacturing target from the design value. However, Wu and Tang [2] neglected to consider the linear quality loss function for a product in their model. The regular quadratic quality loss function is patently inappropriate in some situations. Trietsch [13] remarked that “One such case occurs when the expected cost of exceeding the tolerance limits is not equal to the right and to the left of the target. Missing by cutting too much, for instance, may imply scrap, while cutting too little only causes rework. When this is the case one possible response is fitting a loss function that is not symmetric, and not necessarily quadratic.”

The selection of optimum manufacturing target will directly affect the process defective rate, scrap or rework cost, and the loss to the customer. There are considerable attentions paid to the study of economic selection of manufacturing target. The linear quality loss/profit function is usually applied in the filling/canning problem for determining the optimum manufacturing target, e.g., Springer [14], Hunter and Kartha [15], Carlsson [16], Bisgaard et al. [17], Golhar [18, 19], Golhar and Pollock [20, 21], Cho and Leonard [22], Lee and Jang [23], Misiorek and Barnett [24], Phillips and Cho [25], Lee and Elsayed [26], Lee, et al. [27, 28], Duffuaa and Siddiqi [29] and so forth.

In this paper, we further propose the modified Wu and Tang’s [2] model with the asymmetric quality loss of a product within the tolerance zone for determining the optimum manufacturing target. The normal and symmetric trapezoid distributions are used in the modified model. The rest of the paper is organized as follows. The next section gives a description of the modified Wu and Tang’s [2] cost model with normal distribution. Section III concerns the modified Wu and Tang’s [2] cost model with symmetric trapezoid distribution. In Section IV, some numerical results and the sensitivity analyses of the parameters are presented. Finally, the conclusions and scope of further research are given in Section V.

II. MODIFIED WU AND TANG’ COST MODEL WITH NORMAL DISTRIBUTION

According to Wu and Tang [2], the average quality loss of a product per item is

$$
\begin{aligned}
L(x,T) =& \int_{m-T+x}^{m} f(y)k_1(y-m)^2\,dy + \\
& \int_{m}^{m+T+x} f(y)k_2(y-m)^2\,dy \\
=& \frac{3\lambda}{T\sqrt{2\pi}}\sum_{n=1}^{\infty}\frac{(\frac{-9}{2T^2})^{n-1}}{(n-1)!}\{k_1[\frac{T^{2n+1}-x^{2n+1}}{2n+1} \\
& +\frac{x}{n}(x^{2n}-T^{2n})+\frac{x^2}{2n-1}\cdot \\
& (T^{2n-1}-x^{2n-1})]+k_2[\frac{T^{2n+1}+x^{2n+1}}{2n+1} \\
& +\frac{x}{n}(x^{2n}-T^{2n})+\frac{x^2}{2n-1}\cdot \\
& (T^{2n-1}+x^{2n-1})]\} \qquad (1)
\end{aligned}
$$

The k_1 and k_2 denote the coefficients of quality loss to the left and right of the design target, $k_1 > k_2$. m denotes the design target. x denotes a positive distance of manufacturing target that is

away from the design target. $2T$ denotes the tolerance zone (it is defined by six times of the standard deviation covering 99.73% of the cumulated probability). λ denotes the modification factor used to ensure that all the products fall
inside the tolerance zone even though the original probability density function covers an infinite range ($\lambda = \frac{1}{0.9973}$).

$$f(y) = \frac{\lambda}{\frac{T}{3}\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{y-m}{\frac{T}{3}}\right)^2},$$

$$m - T + x \le y \le m + T + x \qquad (2)$$

Now, we further adopt the linear asymmetric quality loss for a product within tolerance zone. The modified Wu and Tang's [2] model is as follows:

$$L(x,T) = \int_{m-T+x}^{m} f(y)k_1(m-y)dy + \int_{m}^{m+T+x} f(y)k_2(y-m)dy$$

$$= \frac{3\lambda}{T\sqrt{2\pi}} \sum_{n=1}^{\infty} \frac{\left(\frac{-9}{2T^2}\right)^{n-1}}{(n-1)!} \{(k_1+k_2)\cdot \frac{T^{2n}-x^{2n}}{2n} + \frac{x}{2n-1}[x^{2n-1}(k_1+k_2) + T^{2n-1}(k_2-k_1)]\} \qquad (3)$$

III. MODIFIED WU AND TANG'S COST MODEL WITH SYMMETRIC TRAPEZOID DISTRIBUTION

The linear combination of independent and identical random variables often exists in the real world. That is, the product with quality characteristic Y is composed of n parts each with quality characteristic X_i and $Y = X_1 + ... + X_n$. Killmann and Collani [30] pointed out that 'generally the only feature of X_i which is known with certainty is its bounded support, which follows from technical conditions. In such a case the normal approximation which is based on an unbounded support of each X_i may lead to a distribution of Y which does not reflect reality sufficiently well'. In this section, we shall consider the particular case of $Y = X_1 + X_2$, where X_1 and X_2 are two independent and uniformly distributed random variable with $X_1 \sim U[\frac{a}{2}, b-c+\frac{a}{2}], X_2 \sim U[\frac{a}{2}, c-\frac{a}{2}]$, which leads to that Y follows a symmetric trapezoid distribution.

A random variable Y is said to have a symmetric trapezoid distribution, if its probability density function is given by

$$f(y) = \begin{cases} \frac{1}{b-c}\frac{y-a}{c-a}, & a \le y < c \\ \frac{1}{b-c}, & c \le y < b-(c-a) \\ \frac{1}{b-c}\frac{b-y}{c-a}, & b-(c-a) \le y < b \end{cases} \qquad (4)$$

The expected value and variance of the random variable Y are $E(Y) = \frac{a+b}{2}$ and $Var(Y) = \frac{(b-c)^2 + (c-a)^2}{12}$, respectively. The value of c has the following constraint: $a < c < \frac{a+b}{2}$. If $c = \frac{a+b}{2}$, then the random variable Y obeys a symmetric triangular distribution. Figure 1 shows the probability density function of a symmetric trapezoid distribution. By setting $a = m - T + x$,

$b = m + T + x$ and $c^{'} = c + x$, we can obtain the modified Wu and Tang's [2] model.

3.1 Quadratic Asymmetric Quality Loss Function

The modified Wu and Tang's [2] model is as follows:

$$
\begin{aligned}
&L(x,T) \\
&= \int_{m-T+x}^{c+x} \frac{1}{m+T-c}\,\frac{y-m+T-x}{c-m+T}\,k_1(y-m)^2\,dy \\
&\quad + \int_{c+x}^{m} \frac{1}{m+T-c}\,k_1(y-m)^2\,dy + \\
&\quad \int_{m}^{2m+x-c} \frac{1}{m+T-c}\,k_2(y-m)^2\,dy + \\
&\quad \int_{2m+x-c}^{m+T+x} \frac{1}{m+T-c}\,\frac{m+T+x-y}{c-m+T}\,k_2(y-m)^2\,dy \\
&= \frac{k_1}{(c-m+T)(m+T-c)}\{\frac{1}{4}[(c+x-m)^4 - \\
&\quad (T-x)^4] + (T-x)\frac{1}{3}[(c+x-m)^3 + \\
&\quad (T-x)^3]\} - \frac{k_1}{3(m+T-c)}(c+x-m)^3 + \\
&\quad \frac{k_2}{3(m+T-c)}(m+x-c)^3 + \\
&\quad \frac{k_2}{(c-m+T)(m+T-c)}\{\frac{1}{3}(T+x)[(T+x)^3 \\
&\quad -(m+x-c)^3] - \frac{1}{4}[(T+x)^4 - \\
&\quad (m+x-c)^4]\} \qquad (5)
\end{aligned}
$$

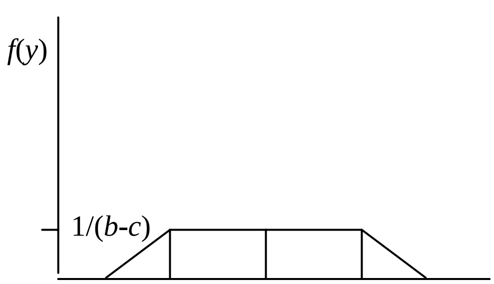

Fig.1. The probability density function of a symmetric trapezoid distribution.

3.2 Linear Asymmetric Quality Loss Function

Now, we further adopt the linear asymmetric quality loss within tolerance zone. The modified Wu and Tang's [2] model is as follows:

$$
\begin{aligned}
&L(x,T) \\
&= \int_{m-T+x}^{c+x} \frac{1}{m+T-c}\,\frac{y-m+T-x}{c-m+T}\,k_1(m-y)\,dy \\
&\quad + \int_{c+x}^{m} \frac{1}{m+T-c}\,k_1(m-y)\,dy + \\
&\quad \int_{m}^{2m+x-c} \frac{1}{m+T-c}\,k_2(y-m)\,dy + \\
&\quad \int_{2m+x-c}^{m+T+x} \frac{1}{m+T-c}\,\frac{m+T+x-y}{c-m+T}\,k_2(y-m)\,dy \\
&= \frac{-k_1}{(c-m+T)(m+T-c)}\{\frac{1}{3}[(c+x-m)^3 + \\
&\quad (T-x)^3] + (T-x)\frac{1}{2}[(c+x-m)^2 - \\
&\quad (T-x)^2]\} + \frac{k_1}{2(m+T-c)}(c+x-m)^2 + \\
&\quad \frac{k_2}{2(m+T-c)}(m+x-c)^2 + \\
&\quad \frac{k_2}{(c-m+T)(m+T-c)}\{\frac{1}{2}(T+x)[(T+x)^2 \\
&\quad -(m+x-c)^2] - \frac{1}{3}[(T+x)^3 - \\
&\quad (m+x-c)^3]\} \qquad (6)
\end{aligned}
$$

For given parameters, we can adopt direct search method for finding the optimal x value of the above models (1), (3), (5), and (6).

IV. NUMERICAL EXAMPLE AND SENSITIVITY ANALYSIS

4.1 Example 1

Suppose that a rod is put together in sections. One considers the length, y, of the assembled rod is normally distributed with unknown mean and standard deviation 4/3. The different costs occur for the assembled rod within the specification limits and out-of-specification. Let the design target $m = 10$ and the bilateral tolerance $T = 4$. The positive distance of manufacturing target that is away from the design target be x. The truncated probability density function is

$$f(y) = \frac{1/0.9973}{\frac{4}{3}} e^{-\frac{1}{2}\left(\frac{y-10}{4/3}\right)^2}, 6 + x \le y \le 14 + x.$$

We would like to find the optimum manufacturing target based on minimizing the average quality loss of a product per item.

4.1.1 Quadratic Asymmetric Quality Loss Function

Assume that the monetary loss per item of below lower specification limit is $A_1 = 1600$ and the monetary loss per item of exceeding upper specification limit is $A_2 = 800$. Hence, we have the coefficient of quality loss to the left of the design target is $k_1 = \frac{A_1}{T^2}$ =100 and the coefficient of quality loss to the right of the design target is $k_2 = \frac{A_2}{T^2}$ = 50. From Eq. (1), we have the optimal solution of Wu and Tang's [2] model at x^* = 2.51 and $L(x^*, T) = 578.2743$.

4.1.2 Linear Asymmetric Quality Loss Function

Assume that the linear asymmetric quality loss at the specification limits is defined as $A_1 = 400$ and $A_2 = 200$. Hence, we have $k_1 = \frac{A_1}{T}$ =100 and $k_2 = \frac{A_2}{T}$ = 50. From Eq. (3), we have the optimal solution of modified Wu and Tang's [2] model with the normal distribution under the linear asymmetric quality loss function which is at $x^* = 2.74$ and $L(x^*, T) = 550.4416$.

4.2 Example 2

The numerical data is the same that in Example 1, except that the length, y, of the assembled rod is a symmetric trapezoid distribution. Let a = 6+x, b = 14+x, and c' = 8+x. The probability density function of y is

$$f(y) = \begin{cases} \frac{y-6-x}{12}, & 4+x \le y < 8+x \\ \frac{1}{6}, & 8+x \le y < 12+x \\ \frac{14+x-y}{12}, & 12+x \le y < 14+x \end{cases}$$

We would like to find the optimum manufacturing target based on minimizing the average quality loss of a product per item.

4.2.1 Quadratic Asymmetric Quality Loss Function

Assume that the quadratic asymmetric quality loss at the specification limits is defined as A_1 = 1600 and A_2 = 800. Hence, we have $k_1 = \frac{A_1}{T^2}$ =100 and $k_2 = \frac{A_2}{T^2}$ = 50.From Eq. (5), we have the optimal solution of modified Wu and Tang's [2] model with the symmetric trapezoid distribution under the quadratic asymmetric quality loss function which is at $x^* = 0.53$ and $L(x^*, T) = 229.4317$.

4.2.2 Linear Asymmetric Quality Loss Function

Assume that the linear asymmetric quality loss at the specification limits is defined as $A_1 = 400$ and $A_2 = 200$. Hence, we have $k_1 = \frac{A_1}{T}$ =100 and $k_2 = \frac{A_2}{T}$ = 50. From Eq. (6), we have the optimal solution of modified Wu and Tang's [2] model with the symmetric trapezoid distribution under the linear asymmetric quality loss function which is at $x^* = 1$ and $L(x^*, T) = 104.1667$.

4.2.3 Sensitivity Analysis

In this section, the sensitivity analysis of parameters is conducted to study the effects of different parameters on the optimal solution. From Tables 1 through 7, we have the following results:

1. As the quality loss coefficient k_1 of normal distribution increases, the x increases slowly and $L(x,T)$ increases slowly.
2. As the quality loss coefficient k_2 of normal distribution increases, the x decreases slowly and $L(x,T)$ increases vastly.
3. As the bilateral tolerance T of normal distribution increases, the x increases vastly and $L(x,T)$ increases vastly.
4. As the quality loss coefficient k_1 of symmetric trapezoid distribution increases, the x increases slowly and $L(x,T)$ increases slowly.
5. As the quality loss coefficient k_2 of symmetric trapezoid distribution increases, the x decreases slowly and $L(x,T)$ increases slowly.
6. As the design target m of symmetric trapezoid distribution increases, the x increases vastly and $L(x,T)$ increases vastly.
7. As the bilateral tolerance T of symmetric trapezoid distribution increases, the x increases vastly and $L(x,T)$ increases vastly.
8. The $L(x,T)$ of modified Wu and Tang's [2] model with quadratic quality loss function is larger than that of modified model with linear quality loss function.

Table 1. Effect of k_1 for normal quality characteristic.

	Quadratic Model		Linear Model	
k_1	x	$L(x,T)$	x	$L(x,T)$
96	2.49	572.5666	2.45	546.2045
97	2.50	574.0073	2.53	547.3808
98	2.50	575.4378	2.62	548.4746
99	2.51	576.8678	2.68	549.4912
100	2.51	578.2743	2.74	550.4416
101	2.51	579.6818	2.80	551.3327
102	2.52	581.0790	2.84	552.1720
103	2.52	582.4629	2.89	552.9654

Table 2. Effect of k_2 for normal quality characteristic.

	Quadratic Model		Linear Model	
k_2	x	$L(x,T)$	x	$L(x,T)$
46	2.54	542.9921	3.07	512.2743
47	2.53	551.8720	3.01	522.0488
48	2.52	560.7150	2.94	531.6860
49	2.52	569.5123	2.85	541.1612
50	2.51	578.2743	2.74	550.4416
51	2.50	587.0044	2.61	559.4854
52	2.49	595.7019	2.46	568.2466
53	2.49	604.3618	2.30	576.6985

Table 3. Effect of T for normal quality characteristic.

	Quadratic Model		Linear Model	
T	x	$L(x,T)$	x	$L(x,T)$
3.7	2.32	457.6769	2.54	470.9717
3.8	2.38	495.8014	2.60	496.7739
3.9	2.45	535.9832	2.67	523.2635
4.0	2.51	578.2743	2.74	550.4416

4.1	2.57	622.7388	2.81	578.3078
4.2	2.63	669.4293	2.88	606.8616
4.3	2.70	718.3914	2.95	636.1041
4.4	2.76	769.6841	3.02	666.0348

Table 4. Effect of k_1 for symmetric trapezoid quality characteristic.

	Quadratic Model		Linear Model	
k_1	x	$L(x,T)$	x	$L(x,T)$
95	0.50	224.4792	0.93	102.3037
96	0.50	225.4861	0.95	102.6860
97	0.51	226.4844	0.96	103.0629
98	0.52	227.4747	0.97	103.4355
99	0.53	228.4573	0.99	103.8034
100	0.53	229.4317	1.00	104.1667
101	0.54	230.3964	1.01	104.5257
102	0.55	231.3539	1.03	104.8803
103	0.56	232.3045	1.04	105.2304

Table 5. Effect of k_2 for symmetric trapezoid quality characteristic.

	Quadratic Model		Linear Model	
k_2	x	$L(x,T)$	x	$L(x,T)$
46	0.60	218.5653	1.11	98.5761
47	0.58	221.3332	1.08	100.0017
48	0.57	224.0676	1.05	101.4086
49	0.55	226.7649	1.03	102.7967
50	0.53	229.4317	1.00	104.1667
51	0.52	232.0637	0.97	105.5191
52	0.50	234.6667	0.95	106.8539
53	0.49	237.2359	0.92	108.1716

Table 6. Effect of m for symmetric trapezoid quality characteristic.

	Quadratic Model		Linear Model	
m	x	$L(x,T)$	x	$L(x,T)$
8.5	0.47	187.5395	0.75	92.0139
9.0	0.48	195.8113	0.83	94.5835
9.5	0.51	209.7532	0.92	98.7691
10.0	0.53	229.4317	1.00	104.1667
10.5	0.57	254.8831	1.08	110.4969
11.0	0.60	286.1286	1.17	117.5597
11.5	0.64	323.1735	1.25	125.2083

Table 7. Effect of T for symmetric trapezoid quality characteristic.

	Quadratic Model		Linear Model	
T	x	$L(x,T)$	x	$L(x,T)$
2.5	0.39	117.2850	0.75	75.3472
3.0	0.43	148.8691	0.83	84.5835
3.5	0.48	186.2517	0.92	94.2236
4.0	0.53	229.4317	1.00	104.1667
4.5	0.59	278.4046	1.08	114.3431
5.0	0.64	333.1672	1.17	124.7025
5.5	0.69	393.7192	1.25	135.2083
6.0	0.75	460.0587	1.33	145.8334

V. CONCLUSIONS

In this paper, we have presented the modified Wu and Tang's [2] model with asymmetric quality loss of a product within the tolerance zone for determining the optimum manufacturing target. The normal and symmetric trapezoid distributions are used in the modified model. The further study can extend this method to the smaller-the-better, the larger-the-better or the multiple quality characteristics for determining the optimum manufacturing target.

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