中正嶺學報 第三十四卷 第二期 民國 95.05.
JOURNAL OF C.C.I.T., VOL.34, NO.2, MAY, 2006

On Gross Error Detection Methods for Control Points in Image Registration

Chia-Sheng Hsieh and Tian-Yuan Shih

Department of Civil Engineering, National Chiao Tung University

ABSTRACT

The control point selection is a necessary step of image registration, which has an influence on the global efficiency of registration. Therefore, to ensure the quality of control points is an important issue. By using three error detection methods, such as data snooping, Tau test, and Danish robust estimation, this paper intends to explore the quality control issue of control points for image registration. Because the measurement error of control points has different levels of influences due to their variable geometric positions, a suitable method is required for the gross error removal of points. In the case study of the control point in image registration, Tau test is a more efficient method for detecting gross errors than data snooping and Danish robust estimation.

Keywords: blunder removal, statistical testing.

影像套合控制點偵錯方法之研究

謝嘉聲 史天元

國立交通大學土木工程學系

摘 要

在影像套合處理時，控制點選取為必要之步驟，且控制點選取的好壞，會影響整體套合的結果，因此確保控制點的品質，為套合處理時重要的要求。本研究以 data snooping、Tau test 及 Danish robust estimation 等三種誤差偵錯的方法，來進行研究，以固定的格子點及數位影像的控制點來分別探討。經研究後證實，控制點的量測誤差會因幾何位置的不同產生不同程度的影響，因此必須有適合的方法來進行點位的除錯。在格子點及數位影像控制點的例證中，Tau test 的偵錯方法比 data snooping 及 robust estimation 的方法更有效的偵測出錯誤點。

關鍵字：粗差移除、統計檢驗

文稿收件日期 94.09.09; 文稿修正後接受日期 95.02.07.
Manuscript received September 9, 2005; revised February 7, 2006.

I. INTRODUCTION

Image registration is an important step in photogrametric mapping, image vision, medical imaging and machine vision [1-5]. During the process of registration, some characteristic points located in both images are measured separately and considered as the control points. Then, the image coordinates of these points are substituted into the selected transformation model to obtain the transformation coefficients between these two images. For a higher level of precision, control points more than minimum requirement are measured for redundancies, and these control points are used to obtain the transformation parameters via the least squares method.

Selecting control points is a necessary step, and the quality of selected points will influence the calculation of transformation parameters and global registration efficiency. Thus, it is an important issue for quality assurance of control points. However, large errors due to homogeneity of image or other reasons are hard to avoid in both the manual measurement and automatic matching methods.

This research aims to explore the error detection capability of three schemes, data snooping, Tau test and Danish robust estimation. In order to compare the characteristics of the three gross error detection methods, simulation is performed. The differences between these gross error detection methods are evaluated by using different control point patterns in coordination with different error levels.

II. Error Detection Methods

The quality of the image registration is determined by two factors. The first is the transformation model that is used for describing the relation between two images. The second factor is the observation itself. Both factors may influence the residual behavior after the adjustment. The transformation model may be formulated from knowledge of the images, and may also be derived by regression analysis of observations. The coordinate observations may be contaminated by gross errors because of various kinds of probable factors through the measurement process. It is necessary to detect the gross error, in order to improve the precision of the transformation [6].

With the least square method, when there are gross errors in the observations, the magnitude of corresponding residuals may not always be larger than other residuals. This makes the gross errors difficult to be found. Therefore, it’s not reliable if the outlier is detected by simply examining the magnitude of the residual alone.

The basic concept of error detection is that the observations comprise only random errors, which are under certain observation (measurement) condition, and are distributed in normal state. Thus, statistical test may be applied to detect large errors or mistakes, namely, to remove the gross errors.

The gross error detection methods applied in this paper comprise data snooping, Tau test and Danish robust estimation as detailed below.

2.1 Data Snooping

Data snooping is a statistical test method developed by Baarda [7]. Assuming that the residuals indicate the linear function of observations, the normalized residuals are used for evaluation.

This method first utilizes a test of global model, e.g., using the statistic T as described in equation (1).

$$T = \frac{v^t P v}{\sigma_0^2}, \quad (1)$$

where

T : the statistic

v : residual

P : weight

σ_0^2 : prior variance

v^t : transpose matrix of v

When T is less than the threshold, the global model is considered correct, i.e. no major errors exist in the observations; in other words, mistakes exist in the observations, which would be further identified [8]. The threshold value is obtained from χ^2 distribution with the commonly applied significance level α of 0.15.

For detecting each individual observation, residuals can be standardized to obtain a standardized residual as W_i

$$W_i = \frac{v_i}{\sigma_{V_i}}, \quad (2)$$

where

W : statistic to be tested

σ_{V_i} : posterior standard deviation

Threshold K_α is obtained by a selected significance level α. In the case when $|W_i| > K_\alpha$, v_i is considered to be questionable.

The gross error detection procedures of data snooping include:

1. Calculate global model by all observed points;
2. Standardize the residual;
3. Check if the maximum standardized residual reflects an outlier. If not, stop error detection;
4. Remove the observation with gross error and calculate for the transformation parameters. Then, repeat step 2 and 3.

2.2 Tau Test

Since prior variance factor σ_0 is practically unknown, the residual will be subjected to standardization $\frac{v_i}{S_{v_i}}$, where v_i represents the residual of observations and S_{V_i} is the standard deviation of these residuals other than global model test.

$$W_i = \frac{|v_i|}{s_{V_i}}. \quad (3)$$

Unfortunately, since v_i and s_{V_i} are estimated from the same source, both of them are statistically related. Therefore, this value cannot be tested by t -distribution. According to Pope [9], a threshold of τ_α, calculated from the formula of τ distribution, is chosen. When the value of $|W|$ is larger than τ_α, the v_i is considered to be gross error.

The formula of τ distribution, relying on the F-distribution is given by:

$$\tau_\alpha = \left[\frac{rF_{1-\frac{\alpha}{n},1,r-1}}{r-1+F_{1-\frac{\alpha}{n},1,r-1}} \right]^{1/2} , \qquad (4)$$

where

γ : number of redundancies

α : the significance level

n : number of observations

Only a single gross error can be detected each time for this scheme. In the presence of several gross errors in the observation, error detection is performed sequentially until all gross errors are removed.

2.3 Robust Estimation

The term of robust estimation means an estimation technique, which is robust with respect to the presence of gross errors in the data. In this context, gross errors are defined as observations, which do not fit into the stochastic model of parameter estimation. The least squares adjustment, as described in the previous section, is not a robust estimation technique due to the fact that the false observations e.g., point numbering errors in photogrammetric plotting, can lead to completely false results and may even prevent convergence of the adjustment, if it is processed iteratively due to the linearization of non-linear models.

There are varieties of such methods with a similar principle, which attempt to minimize the influence of gross errors in the observation (inclusive of possible mistakes) upon estimated parameters (unknowns from adjustment computation). The first step is to locate doubtful observations. And then, the influence of the identified observation is reduced. This is conducted by iteratively re-weight the observations in most schemes. In other words, the observation is more likely to be wrong if its weight is minimized [10].

Among all robust estimations, the so-called Danish method is successfully applied to the adjustments for geodetic and aerial surveying [10]. Still, the basic principle lies in that larger residual indicating a lower precision of observations. So, its weight is reduced if the residual of a certain observation exceeds the threshold. Equation (5) is used for re-assigning the weight.

$$P_{i+1}=P_i \cdot f(v_i), \qquad i=1,2,....$$

$$f(v_i) = \left[\begin{array}{ll} 1 \quad for & \frac{|v_i|\sqrt{P_1}}{s_0} < c \\ \exp(-\frac{|v_i|\sqrt{P_1}}{c \cdot s_0}) & else \end{array} \right. , \quad (5)$$

where

P : weight value

c : threshold value

v : residual

i : iteration number

It can thus be seen that the computing model of the least squares method hasn't yet been changed, apart from the re-weighting scheme. If there is an extremely large residual of a certain observation, the weight is reduced with a speed of an exponential function. The constant c is often set between 2 and 3 depending upon the number of redundancies and observations during adjustment computation [8]. Usually, c is equal to 3 for the test with smaller number of redundancies.

Danish method finds the residuals of every observation with the least squares adjustment, then utilizes them to calculate the weights of every observation again, and repeat least squares adjustment using the computed weights. The iteration is completed when the weights re-computed is not changed.

III. Tests and Discussion

In order to compare the capability of three gross error detection methods, simulations are performed. Two groups of data are simulated: one with five points and the other with twenty points. Regarding the gross error design, it includes single gross error in five point dataset, and both single and multiple gross errors in twenty point dataset.

The first order polynomial, Affine transformation, is a commonly used transformation model for image registration [11]. This model can be decomposed into six physical elements. These elements are, two translations, one rotation, scale and differential scale, and non-orthogonality of the axes, i.e., the skew factor [12]. The transformation equation is given by

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} a & b \\ d & e \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} c \\ f \end{bmatrix}. \qquad (6)$$

where

x,y : input point's coordinates

x',y' : output point's coordinates

a,b,c,d,e,f: transformation coefficients

3.1 Test of Five Points

Five points are selected from the 5×5 grid pattern, with the distribution shown in Fig. 1. These points are distributed around the periphery, except one at the center.

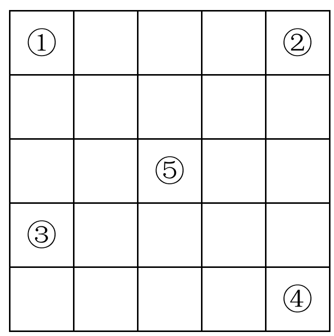

Fig. 1. Distribution of five points.

The method of testing is to give a large error for coordinate Y of one point among the five, to examine the residual pattern related to different error locations. The prior variance is selected as the identity matrix I. The degree of freedom, f, is 4.

Assuming that the magnitude of error is 4 pixels, the residuals on all five points is listed in Table 1.

Table 1.The Residuals for Different Gross Error Locations

Case 1			Case 2			Case 3			Case 4			Case 5		
Point	Residual		Point	Residual		Point	Residual		Point	Residual		Point	Residual	
	Vx	Vy		Vx	Vy		Vx	Vy		Vx	Vx		Vx	Vx
*1	0.00	-1.43	1	0.00	0.23	1	0.00	1.08	1	0.00	-1.16	1	0.00	0.70
2	0.00	0.81	*2	0.00	-0.60	2	0.00	-1.15	2	0.00	0.53	2	0.00	0.67
3	0.00	1.08	3	0.00	-0.80	*3	0.00	-1.53	3	0.00	0.70	3	0.00	0.89
4	0.00	-1.16	4	0.00	0.03	4	0.00	0.70	*4	0.00	-0.99	4	0.00	0.92
5	0.00	0.70	5	0.00	1.13	5	0.00	0.89	5	0.00	0.92	*5	0.00	-3.19
v^tv	5.74		v^tv	2.33		v^tv	6.12		v^tv	3.95		v^tv	12.75	

* indicates the point, on which the gross error of 4 pixel are introduced to Y coordinate

As can be seen in Table 1, the gross error of the same magnitude in different location has different levels of influence, in which the surrounding points, such as points 1, 2, 3 and 4 have less influence. If the gross error occurs at point 5 in the center, the global residual squares (v^tv) will be significantly increased from 2.33 to 12.75.

Moreover, when the gross error occur at point 2 and point 4, the corresponding residuals don't have maximum errors. This is because that the errors have been distributed by the transformation parameters based on the least squares method, causing other points' residuals to be larger and possibly lead to wrong results in the gross error detection.

The relation between residual and gross error is given by

$$V = Q_{vv}P \cdot \varepsilon. \tag{7}$$

The $Q_{vv}P$ matrix determines how the error in an observation propagates to the residual. In the five point case, the observations are [X_{p1}, Y_{p1}, X_{p2}, Y_{p2}, X_{p3}, Y_{p3}, X_{p4}, Y_{p4}, X_{p5}, Y_{p5}], and the matrix of $Q_{vv}P$ is obtained as

$$Q_{vv}P=\begin{bmatrix} 0.36 & 0 & -0.20 & 0 & -0.27 & 0 & 0.29 & 0 & -0.18 & 0 \\ & 0.36 & 0 & -0.2 & 0 & -0.27 & 0 & 0.29 & 0 & -0.18 \\ & & 0.22 & 0 & 0.29 & 0 & -0.13 & 0 & -0.17 & 0 \\ & & & 0.22 & 0 & 0.29 & 0 & -0.13 & 0 & -0.17 \\ & & & & 0.38 & 0 & -0.18 & 0 & -0.22 & 0 \\ & & & & & 0.38 & 0 & -0.18 & 0 & -0.22 \\ & & & & & & 0.25 & 0 & -0.23 & 0 \\ & & SYM & & & & & 0.25 & 0 & -0.23 \\ & & & & & & & & 0.80 & 0 \\ & & & & & & & & & 0.80 \end{bmatrix}.$$

For data snooping error detection method in case 5, there are 4 pixel errors at point 5. The number of observations is 10, and the redundancies are 4. Let the prior P value be 1, and the T value from equation (1) is 12.75 for the global model test. The threshold value $\chi^2_{0.15}(4)$ from the χ^2 distributation table is 6.582. Since T > $\chi^2_{\alpha}(f)$, the original null hypothesis is rejected, and all observations must be checked. After the standardization of all residuals, the values are listed in Table 2, wherein the calculated threshold K_α is 3.29. Thus, it is learned that the standardized residual of coordinate Y of point 5 exceeds a standard value, subject to the removal. Then, after readjustment and recalculation, $v^t v$ becomes zero. Therefore, the gross error in this case can be detected by data snooping successfully.

For the case 1 to 4, both the original null hypothesis and model transformation are acceptable from the global statistic analysis. Even if every observation is tested individually, every standardized residual is less than the threshold K_α = 3.29, so the gross error cannot be detected.

Table 2. Gross error detection of five points using data snooping

Case 1			Case 2			Case 3			Case 4			Case 5		
Point	Original Residual		Point	Original Residual		Point	Original Residual		Point	Original Residual		Point	Original Residual	
	Vx	Vy		Vx	Vy		Vx	Vy		Vx	Vy		Vx	Vy
*1	0.00	-1.43	1	0.00	0.23	1	0.00	1.08	1	0.00	-1.16	1	0.00	0.70
2	0.00	0.81	*2	0.00	-0.60	2	0.00	-1.15	2	0.00	0.53	2	0.00	0.67
3	0.00	1.08	3	0.00	-0.80	*3	0.00	-1.53	3	0.00	0.70	3	0.00	0.89
4	0.00	-1.16	4	0.00	0.03	4	0.00	0.70	*4	0.00	-0.99	4	0.00	0.92
5	0.00	0.70	5	0.00	1.13	5	0.00	0.89	5	0.00	0.92	*5	0.00	-3.19
$v^t v$	5.74		$v^t v$	2.33		$v^t v$	6.12		$v^t v$	3.95		$v^t v$	12.75	
Point	Local Redundancy		Point	Local Redundancy		Point	Local Redundancy		Point	Local Redundancy		Point	Local Redundancy	
	rix	riy		rix	riy		rix	riy		rix	riy		rix	riy
*1	0.36	0.36	1	0.36	0.36	1	0.36	0.36	1	0.36	0.36	1	0.36	0.36
2	0.22	0.22	*2	0.22	0.22	2	0.22	0.22	2	0.22	0.22	2	0.22	0.22
3	0.38	0.38	3	0.38	0.38	*3	0.38	0.38	3	0.38	0.38	3	0.38	0.38
4	0.25	0.25	4	0.25	0.25	4	0.25	0.25	*4	0.25	0.25	4	0.25	0.25
5	0.80	0.80	5	0.80	0.80	5	0.80	0.80	5	0.80	0.80	*5	0.80	0.80
Point	Standardized Residual		Point	Standardized Residual		Point	Standardized Residual		Point	Standardized Residual		Point	Standardized Residual	
	Uix	Uiy		Uix	Uiy		Uix	Uiy		Uix	Uiy		Uix	Uiy
*1	0.00	2.39	1	0.00	0.38	1	0.00	1.81	1	0.00	1.94	1	0.00	1.17
2	0.00	1.75	*2	0.00	1.28	2	0.00	2.47	2	0.00	1.13	2	0.00	1.44
3	0.00	1.75	3	0.00	1.28	*3	0.00	2.47	3	0.00	1.13	3	0.00	1.44
4	0.00	2.34	4	0.00	0.06	4	0.00	1.41	*4	0.00	1.98	4	0.00	1.86
5	0.00	0.78	5	0.00	1.26	5	0.00	1.00	5	0.00	1.03	*5	0.00	3.57

* indicates the point, on which the gross error of 4 pixel are introduced to Y coordinate

Table 3. Gross error detection of five points using Tau test

Case 1			Case 2			Case 3			Case 4			Case 5		
Point	Residual		Point	Residual		Point	Residual		Point	Residual		Point	Residual	
	Tix	Tiy		Tix	Tiy		Tix	Tiy		Tix	Tiy		Tix	Tiy
*1	0.00	2.00	1	0.00	0.50	1	0.00	1.46	1	0.00	1.95	1	0.00	0.65
2	0.00	1.46	*2	0.00	1.68	2	0.00	2.00	2	0.00	1.14	2	0.00	0.80
3	0.00	1.46	3	0.00	1.68	*3	0.00	2.00	3	0.00	1.14	3	0.00	0.80
4	0.00	1.95	4	0.00	0.08	4	0.00	1.14	*4	0.00	2.00	4	0.00	1.04
5	0.00	0.65	5	0.00	1.66	5	0.00	0.80	5	0.00	1.04	*5	0.00	2.00
v^tv	5.74		v^tv	2.33		v^tv	6.12		v^tv	3.95		v^tv	12.75	

* indicates the point, on which the gross error of 4 pixel are introduced to Y coordinate

Table 4. Gross error detection of five points using robust estimation

Case 1			Case 2			Case 3			Case 4			Case 5		
Point	Residual		Point	Residual		Point	Residual		Point	Residual		Point	Residual	
	Vx	Vy		Vx	Vy		Vy	Vx		Vx	Vy		Vx	Vy
*1	0.00	-1.43	1	0.00	0.23	1	0.00	1.08	1	0.00	-1.16	1	0.00	0.70
2	0.00	0.81	*2	0.00	-0.60	2	0.00	-1.15	2	0.00	0.53	2	0.00	0.67
3	0.00	1.08	3	0.00	-0.80	*3	0.00	-1.53	3	0.00	0.70	3	0.00	0.89
4	0.00	-1.16	4	0.00	0.03	4	0.00	0.70	*4	0.00	-0.99	4	0.00	0.92
5	0.00	0.70	5	0.00	1.13	5	0.00	0.89	5	0.00	0.92	*5	0.00	-3.19
v^tv	5.74		v^tv	2.33		v^tv	6.12		v^tv	3.95		v^tv	12.75	

* indicates the point, on which the gross error of 4 pixel are introduced to Y coordinate

If an error $\alpha = 5\%$ is selected in the Tau test, the threshold τ is 1.946 for the number of redundancies of 4, and all T_i values are calculated in Table 3.

As seen in Table 3, Tau test can also detect the error ($2.00 > \tau = 1.946$) in the case where gross error is located at point 5. In the case where gross error is located at point 2, it cannot detect any errors just like data snooping. In the case where gross error is located at points 1, 3, 4, Tau test can detect the error efficiently, unlike data snooping. So, Tau test shows better detection capability than data snooping in this simulation.

For the robust estimation test, the method utilizes Danish's formula, wherein the value of *c* is set as 3, and the results are listed in Table 4. It is shown that no gross error has been detected.

3.2 Test of Twenty Points

With the twenty points selected from a 7x7 pattern, the distribution is shown in Fig.2. In this case, there are 40 observations and 34 redundancies. Therefore, both the cases of a single gross error and cases of multiple gross errors can be studied.

The gross error of four pixels is introduced to the Y coordinate of point 3 in Fig. 2. The coordinates of the points and the corresponding residuals are listed in Table 5.

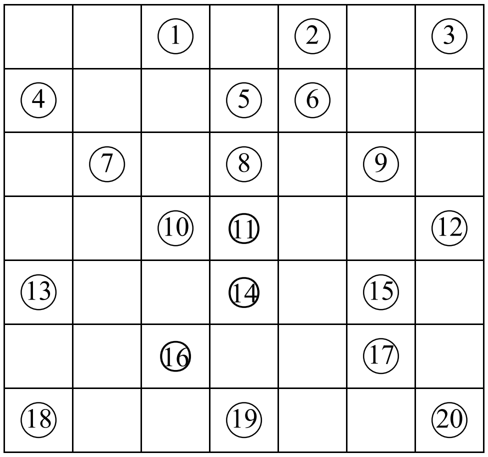

Fig. 2. Distribution of the twenty points.

As shown in Table 5, the residual of Y coordinate for point 3 is much larger than other residuals. This indicates that, if most of the observations of sufficient quantity are measured correctly and the geometric configuration is good, single gross error has little influence upon transformation parameter, and the residuals of the points will reflect the corresponding errors. In the adjustment theory, the strength of the configuration and redundancy is measured by the reliability of the data set.

For data snooping method, the global model test in this case, the T value is 11.927, $\chi^2_{0.15}(34)$ is 42.022. Since $T < \chi^2_\alpha(f)$, both original null hypothesis and transformation model are considered to be acceptable. So, no test is required for observation data. It can be seen from Table 6 that, the residual of Y coordinate for point 3 is significantly larger than other residuals.

Table 5. Coordinates of 20 points with one gross error and the corresponding residuals

Point	Coordinates in image 1		Coordinates in image 2		Adjusted Residual	
	X	Y	x	y	Vx	Vy
1	101	103	1.0	3.0	0.00	0.43
2	101	105	1.0	5.0	0.00	0.72
*3	101	107	1.0	11.0	0.00	2.98
4	102	101	2.0	1.0	0.00	0.00
5	102	104	2.0	4.0	0.00	0.44
6	102	105	2.0	5.0	0.00	0.58
7	103	102	3.0	2.0	0.00	0.01
8	103	104	3.0	4.0	0.00	0.30
9	103	106	3.0	6.0	0.00	0.59
10	104	103	4.0	3.0	0.00	0.01
11	104	104	4.0	4.0	0.00	0.16
12	104	107	4.0	7.0	0.00	0.60
13	105	101	5.0	1.0	0.00	0.41
14	105	104	5.0	4.0	0.00	0.02
15	105	106	5.0	6.0	0.00	0.31
16	106	103	6.0	3.0	0.00	0.25
17	106	106	6.0	6.0	0.00	0.17
18	107	101	7.0	1.0	0.00	0.68
19	107	104	7.0	4.0	0.00	0.25
20	107	107	7.0	7.0	0.00	0.18
v^tv					11.93	

* is the point, on which the gross error of 4 pixels is assigned to Y coordinates

Table 6. Gross error detection of 20 points with single gross error using data snooping

Point	Uix	Uiy
1	0.00	0.47
2	0.00	0.79
*3	0.00	*3.45
4	0.00	0.00
5	0.00	0.46
6	0.00	0.62
7	0.00	0.01
8	0.00	0.31
9	0.00	0.62
10	0.00	0.01
11	0.00	0.16
12	0.00	0.65
13	0.00	0.46
14	0.00	0.02
15	0.00	0.33
16	0.00	0.27
17	0.00	0.19
18	0.00	0.82
19	0.00	0.27
20	0.00	0.22

* is the point, on which the gross error of 4 pixels is assigned to Y coordinates

Although global model test is successfully passed in this case, some researchers suggested continuous checking of observations [6, 13]. The results are listed in Table 6.

It can be seen that, the standardized residual of coordinate Y of point 3 is 3.454, which is larger than the selected threshold of 3.29. This also indicates that when the reliability is high, the gross error has little influence upon unknown parameters.

If a significance level $\alpha = 5\%$ is selected in Tau test, the threshold $\tau = 3.047$, and all Ti values are calculated as in Table 7.

As seen in Table 7, the residuals of coordinate Y for point 3 are significantly larger than other residuals, and Tau test detected the gross error successfully.

For Danish's method, the value of *c* is 3, and the results are listed in Table 8. The result indicates that the gross error in each case has been efficiently detected.

3.3 Test of Multiple Errors

This test simulates 3 gross errors: an error of 4 pixels is incorporated in the Y component of point 3, 5 pixels in the X component of point 8, and -6 pixels in the X component of point 18. The comparative analysis is performed for cases of different gross locations and different magnitude of gross errors. The coordinates of points and the corresponding residuals are listed in Table 9. On points 3, 8, and 18, there are larger residuals.

Table 7. Gross error detection of 20 points with single gross error using Tau test

Point	Tix	Tiy
1	0.00	0.80
2	0.00	1.33
*3	0.00	*5.83
4	0.00	0.00
5	0.00	0.78
6	0.00	1.04
7	0.00	0.01
8	0.00	0.52
9	0.00	1.06
10	0.00	0.03
11	0.00	0.28
12	0.00	1.10
13	0.00	0.77
14	0.00	0.04
15	0.00	0.57
16	0.00	0.46
17	0.00	0.32
18	0.00	1.38
19	0.00	0.46
20	0.00	0.37

* is the point, on which the gross error of 4 pixels is assigned to Y coordinates

Table 8. Gross error detection of 20 points with single gross error using robust estimation

Point	Residual		Weight	
	Vx	Vy	Px	Py
1	0.00	0.00	1.00	1.00
2	0.00	0.00	1.00	1.00
*3	0.00	*4.00	1.00	*0.00
4	0.00	0.00	1.00	1.00
5	0.00	0.00	1.00	1.00
6	0.00	0.00	1.00	1.00
7	0.00	0.00	1.00	1.00
8	0.00	0.00	1.00	1.00
9	0.00	0.00	1.00	1.00
10	0.00	0.00	1.00	1.00
11	0.00	0.00	1.00	1.00
12	0.00	0.00	1.00	1.00
13	0.00	0.00	1.00	1.00
14	0.00	0.00	1.00	1.00
15	0.00	0.00	1.00	1.00
16	0.00	0.00	1.00	1.00
17	0.00	0.00	1.00	1.00
18	0.00	0.00	1.00	1.00
19	0.00	0.00	1.00	1.00
20	0.00	0.00	1.00	1.00

* is the point, on which the gross error of 4 pixels is assigned to Y coordinates

Table 9. Coordinates of 20 points with multiple gross errors and corresponding residual

Point	Coordinates in image 1		Coordinates in image 2		Adjusted Residual	
	X	Y	x	y	Vx	Vy
1	101	103	1.0	3.0	0.49	0.43
2	101	105	1.0	5.0	0.95	0.73
*3	101	107	1.0	11.0	1.41	-2.98
4	102	101	2.0	1.0	-0.24	0.00
5	102	104	2.0	4.0	0.45	0.44
6	102	105	2.0	5.0	0.67	0.59
7	103	102	3.0	2.0	-0.29	0.01
*8	103	104	8.0	4.0	-4.83	0.30
9	103	106	3.0	6.0	0.63	0.59
10	104	103	4.0	3.0	-0.34	0.02
11	104	104	4.0	4.0	-0.11	0.16
12	104	107	4.0	7.0	0.58	0.60
13	105	101	5.0	1.0	-1.08	-0.41
14	105	104	5.0	4.0	-0.39	0.03
15	105	106	5.0	6.0	0.07	0.32
16	106	103	6.0	3.0	-0.90	-0.26
17	106	106	6.0	6.0	-0.21	0.18
*18	107	101	1.0	1.0	4.36	-0.69
19	107	104	7.0	9.0	-0.95	-0.25
20	107	107	7.0	7.0	-0.26	0.19
vtpv					62.269	

* is the point, on which the gross error is assigned to Y coordinates

Table 10. Gross error detection of 20 points with multiple blunders using data snooping

Point	1st iteration		2nd iteration		3rd iteration		4 th iteration	
	Uix	Uiy	Uix	Uiy	Uix	Uiy	Uix	Uiy
1	0.54	0.47	0.47	0.49	0.00	0.52	0.00	0.00
2	1.04	0.79	0.42	0.89	0.00	0.92	0.00	0.00
*3	1.63	3.45	0.41	3.35	0.00	*3.34	-	-
4	0.27	0.00	0.48	0.11	0.00	0.08	0.00	0.00
5	0.46	0.46	0.37	0.47	0.00	0.50	0.00	0.00
6	0.71	0.62	0.35	0.67	0.00	0.70	0.00	0.00
7	0.31	0.01	0.37	0.09	0.00	0.07	0.00	0.00
*8	4.98	0.31	*4.8	0.29	-	-	-	-
9	0.66	0.62	0.28	0.69	0.00	0.70	0.00	0.00
10	0.35	0.01	0.28	0.08	0.00	0.06	0.00	0.00
11	0.11	0.16	0.26	0.10	0.00	0.12	0.00	0.00
12	0.62	0.65	0.21	0.72	0.00	0.73	0.00	0.00
13	1.20	0.46	0.30	0.72	0.00	0.71	0.00	0.00
14	0.40	0.02	0.21	0.07	0.00	0.05	0.00	0.00
15	0.07	0.33	0.17	0.32	0.00	0.33	0.00	0.00
16	0.95	0.27	0.18	0.46	0.00	0.45	0.00	0.00
17	0.22	0.19	0.12	0.13	0.00	0.14	0.00	0.00
*18	5.18	0.82	-	-	-	-	-	-
19	1.04	0.27	0.11	0.47	0.00	0.46	0.00	0.00
20	0.30	0.22	0.05	0.16	0.00	0.16	0.00	0.00
$v^t v$	62.269		34.721		11.172		0.0	

* is the point, on which the gross error is assigned to Y coordinates

Table 11. Gross error detection of 20 points with multiple blunders using Tau test

Point	1st iteration		2nd iteration		3rd iteration		4 th iteration	
	Tix	Tiy	Tix	Tiy	Tix	Tiy	Tix	Tiy
1	0.40	0.35	0.45	0.47	0.00	0.85	0.00	0.00
2	0.77	0.59	0.41	0.86	0.00	1.52	0.00	0.00
*3	1.21	2.55	0.40	3.22	0.00	5.48	-	-
4	0.21	0.00	0.46	0.11	0.00	0.15	0.00	0.00
5	0.35	0.34	0.36	0.46	0.00	0.82	0.00	0.00
6	0.53	0.46	0.34	0.65	0.00	1.15	0.00	0.00
7	0.23	0.01	0.36	0.09	0.00	0.12	0.00	0.00
*8	3.69	0.23	4.65	0.28	-	-	-	-
9	0.49	0.46	0.27	0.66	0.00	1.16	0.00	0.00
10	0.26	0.26	0.01	0.28	0.08	0.00	0.11	0.00
11	0.09	0.13	0.25	0.11	0.00	0.21	0.00	0.00
12	0.47	0.49	0.21	0.70	0.00	1.21	0.00	0.00
13	0.88	0.89	0.34	0.29	0.70	0.00	1.16	0.00
14	0.29	0.30	0.02	0.21	0.07	0.00	0.10	0.00
15	0.05	0.25	0.17	0.31	0.00	0.54	0.00	0.00
16	0.71	0.20	0.18	0.45	0.00	0.75	0.00	0.00
17	0.17	0.14	0.12	0.13	0.00	0.24	0.00	0.00
*18	3.83	0.61	-	-	-	-	-	-
19	0.77	0.20	0.11	0.45	0.00	0.76	0.00	0.00
20	0.23	0.16	0.05	0.16	0.00	0.28	0.00	0.00

* is the point, on which the gross error is assigned to Y coordinates

Table 12. Gross error detection of 20 points with multiple blunders using robust estimation

Point	Debugged Residual		Debugged Weight	
	Vx	Vy	Px	Py
1	0.00	0.00	1.00	1.00
2	0.00	0.00	1.00	1.00
*3	0.00	-4.00	1.00	0.00
4	0.00	0.00	1.00	1.00
5	0.00	0.00	1.00	1.00
6	0.00	0.00	1.00	1.00
7	0.00	0.00	1.00	1.00
*8	-5.00	0.00	0.00	1.00
9	0.00	0.00	1.00	1.00
10	0.00	0.00	1.00	1.00
11	0.00	0.00	1.00	1.00
12	0.00	0.00	1.00	1.00
13	0.00	0.00	1.00	1.00
14	0.00	0.00	1.00	1.00
15	0.00	0.00	1.00	1.00
16	0.00	0.00	1.00	1.00
17	0.00	0.00	1.00	1.00
*18	6.00	0.00	0.00	1.00
19	0.00	0.00	1.00	1.00
20	0.00	0.00	1.00	1.00

* is the point, on which the gross error is assigned to Y coordinates

For the data snooping method in the global model test, the T value is 62.269, $\chi^2_{0.15}(34)$ is 42.022. Since $T > \chi^2_\alpha(f)$, the original null hypothesis is rejected, and all observations must be checked. After standardization of all residuals, the values are listed in Table 10, wherein the calculated threshold K_α is 3.29.

Because the gross errors are removed one at a time, there are four iterations performed. Only in the first iteration, value T is larger than the threshold, and less than the threshold in the other iterations. As already mentioned in single gross error case, it is recommended that all observations should be checked for gross errors.

If an error α = 5% is selected in Tau test, the threshold τ = 3.047, and all Ti values are calculated as shown in Table 11. It can be seen from table 11 that Tau test can correctly locate all gross errors.

For the Danish method, the results are listed in Table 12. It can be seen that, the gross errors have been efficiently detected. The residuals of wrong coordinates for points 3, 8, and 18 have about 4, 5, and 6 pixels, respectively, and the corresponding weight is also approximately 0.

3.4 Test of Random Errors

This test simulates random errors from the observations. The mean of random errors generator is 0, and the standard deviation is 1. The X and Y components of points are independent. The random errors and the corresponding residuals are listed in Table 13.

Table 13. Coordinates of 20 point with random errors and corresponding residual

Point	Coordinate		Random		Coordinates		Adjusted	
	X	Y	x	y	x	y	Vx	Vy
1	101	103	0.60	-0.42	1.60	2.58	-0.88	0.34
2	101	105	-0.16	0.89	0.84	5.89	-0.18	-0.85
3	101	107	-0.73	0.50	0.27	7.50	0.33	-0.36
4	102	101	0.29	-0.24	2.29	0.76	-0.47	0.01
5	102	104	0.84	0.66	2.84	4.66	-1.11	-0.73
6	102	105	-0.27	0.04	1.73	5.04	-0.03	-0.05
7	103	102	-2.08	-0.64	0.92	1.36	1.90	0.42
8	103	104	-1.30	-1.64	1.70	2.36	1.06	1.53
9	103	106	-0.19	0.62	2.81	6.62	-0.10	-0.62
10	104	103	-0.50	-1.07	3.50	1.93	0.32	0.86
11	104	104	0.26	-0.25	4.26	3.75	-0.46	0.09
12	104	107	-1.54	-0.71	2.46	6.29	1.25	0.72
13	105	101	0.93	-0.86	5.93	0.14	-1.01	0.50
14	105	104	1.74	1.30	6.74	5.30	-1.90	-1.51
15	105	106	-1.56	-0.89	3.44	5.11	1.34	0.79
16	106	103	0.17	1.12	6.17	4.12	-0.27	-1.42
17	106	106	-1.23	-0.68	4.77	5.32	1.04	0.54
18	107	101	-1.06	0.24	5.94	1.24	1.05	-0.70
19	107	104	-0.65	-1.23	6.35	2.77	0.56	0.94
20	107	107	2.28	0.39	9.28	7.39	-2.46	-0.52

As shown in Table 13, the largest error is 2.48 at the X component of point 20. The magnitude of adjusted residuals ranges from 0.03 to 2.76.

For the data snooping method, the global model test, T value is 37.40 and $\chi^2_{0.15}(34)$ =42.022. Since T < $\chi^2_\alpha(f)$, both original null hypothesis and transformation model are considered to be acceptable. Although global model test is passed, the individual observations are still screened for gross errors. The results are shown in Table 14.

The calculated threshold K_α is 3.29. It can thus be seen that none of the residuals exceed the threshold value. The residual of the X component of point 20 is the largest.

If a significance level α = 5% is selected in Tau test, the threshold τ = 3.047, and all Ti values are calculated in Table 15. All observations are judged to be correct.

For the robust estimation method, the result is listed in Table 16. No gross error identified.

In the case of random errors test, data snooping, Tau test and robust methods accepted for all observations. These three methods performed correctly and not misled by the random error.

Table 14. Gross error detection of 20 points with random errors using data snooping

Point	Uix	Uiy	Point	Uix	Uiy
1	0.97	0.38	11	0.47	0.10
2	0.20	0.93	12	1.37	0.78
3	0.38	0.42	13	1.12	0.56
4	0.54	0.02	14	1.96	1.56
5	1.17	0.77	15	1.43	0.84
6	0.03	0.05	16	0.29	1.52
7	2.03	0.45	17	1.13	0.59
8	1.09	1.58	18	1.25	0.83
9	0.11	0.65	19	0.62	1.03
10	0.34	0.89	20	2.93	0.61

Table 15. Error detection of 20 points with random errors using Tau test

Point	Tix	Tiy	Point	Tix	Tiy
1	0.93	0.36	11	0.45	0.09
2	0.19	0.90	12	1.31	0.75
3	0.37	0.40	13	1.08	0.53
4	0.52	0.01	14	1.89	1.50
5	1.12	0.74	15	1.37	0.81
6	0.03	0.05	16	0.28	1.46
7	1.95	0.43	17	1.09	0.57
8	1.05	1.52	18	1.20	0.80
9	0.10	0.63	19	0.60	0.99
10	0.32	0.86	20	2.81	0.59

Table 16. Error detection of 20 points with random error using robust estimation

Point	Residual		Weight		Point	Residual		Weight	
	Vx	Vy	Px	Py		Vx	Vy	Px	Py
1	-0.88	0.34	1.00	1.00	11	-0.46	0.09	1.00	1.00
2	-0.18	-0.85	1.00	1.00	12	1.25	0.72	1.00	1.00
3	0.33	-0.36	1.00	1.00	13	-1.01	0.50	1.00	1.00
4	-0.47	0.01	1.00	1.00	14	-1.90	-1.51	1.00	1.00
5	-1.11	-0.73	1.00	1.00	15	1.34	0.79	1.00	1.00
6	-0.03	-0.05	1.00	1.00	16	-0.27	-1.43	1.00	1.00
7	1.90	0.42	1.00	1.00	17	1.04	0.54	1.00	1.00
8	1.06	1.53	1.00	1.00	18	1.05	-0.70	1.00	1.00
9	-0.10	-0.62	1.00	1.00	19	0.56	0.94	1.00	1.00
10	0.32	0.86	1.00	1.00	20	-2.46	-0.52	1.00	1.00

3.5 Test of Digital Images

To fully understand how these three error detection methods are applied to digital images, Kodak DC210 digital cameras are used for shooting digital images of the façade to one of the building in the university campus of NCTU. The control points are generated by image matching with interest operator, and their distribution on the images is shown in Fig. 3. The coordinates of control points are listed in Table 17.

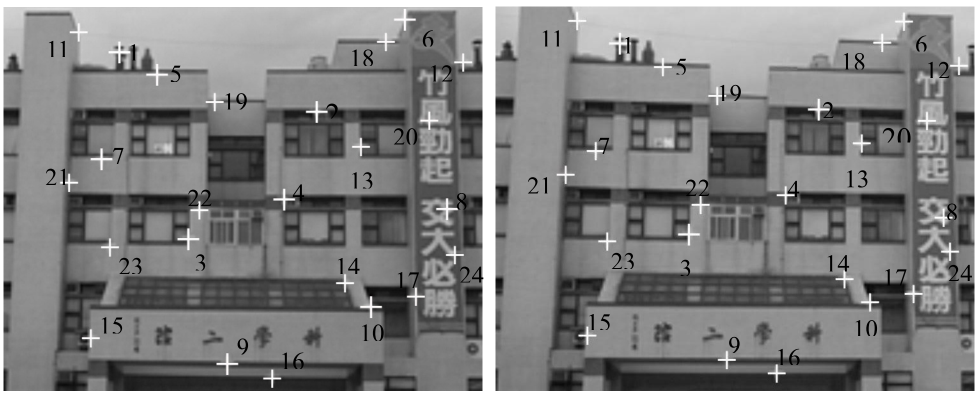

Fig.3. Test image for a building

Table 17. Image coordinates of control points

Point	X	Y	x	y	Point	X	Y	x	y	Point	X	Y	x	y	Point	X	Y	x	y
1	517	83	194	95	7	489	250	156	264	13	895	230	571	252	19	666	160	346	178
2	826	176	504	198	*8	1030	327	700	368	14	870	444	545	463	20	1002	190	674	216
3	625	375	302	393	9	686	570	361	590	15	471	530	142	551	21	438	287	109	301
4	775	313	453	333	10	911	481	585	500	16	756	592	439	611	22	642	330	320	348
5	576	118	261	133	11	453	52	127	61	17	981	464	652	486	23	501	387	174	404
6	964	32	638	62	12	1055	99	724	130	18	934	67	603	94	24	1041	399	709	420

* is the point with gross error

As seen in Fig. 4, the position of point 8 has a remarkable error as it is not located at the same point in the two images, while other points are located at the corresponding positions with visual inspection.

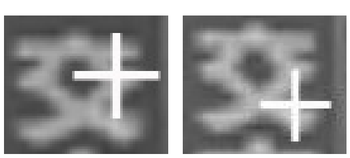

Fig. 4. A pair of positions of point 8.

Error detections are now performed with data snooping, Tau test, and Danish robust estimation.

For the data snooping method, the global model test, T value is 670.074. Since T> $\chi^2_\alpha(f)$, all observations must be checked. After standardization of all residuals, the values are listed as in Table 18, wherein the calculated threshold K_α is 3.29.

It can thus be seen that many residuals exceed the threshold value. Point 8 shall be removed, as its standardized residual of coordinate Y is the maximum one. Then, we readjust and recalculate until the 13th iteration. During this process, 12 points identified as gross errors are removed, and the remaining 12 points meet the test standard.

For the Tau test, all Ti values are calculated as in Table 19. Tau test can detect the gross error in an efficient manner, while the other observations are all judged to be correct.

For the Danish method, the results are listed in Table 20. No gross error was identified.

During gross error detection in digital images, both Tau test and the date snooping method can correctly identified the gross error, whereas the data snooping also identified other 12 correct observation as gross error. And, the Danish method failed to identify the gross error. This indicates that Danish method can only identify gross error with error magnitude. Thus, Tau test method is best suited for selecting control points for digital images.

Table 18. Error detection of image control points using data snooping

Point	1st iteration		2nd iteration		…	13th iteration	
	Uix	Uiy	Uix	Uiy	…	Uix	Uiy
1	1.45	3.32	1.57	3.90	…	-	-
2	4.19	0.55	4.02	0.31	…	2.51	2.14
3	1.95	0.69	1.91	0.89	…	3.12	1.89
4	3.82	1.13	3.67	0.36	…	1.58	0.66
5	10.28	1.54	10.33	1.81	…	-	-
6	0.94	4.29	0.65	5.71	…	0.63	1.17
7	9.40	0.15	9.30	0.66	…	-	-
8	3.02	14.50	-	-	…	-	-
9	0.21	1.54	0.07	2.23	…	-	-
10	0.50	5.59	0.18	4.05	…	1.38	0.04
11	2.29	5.08	2.10	6.03	…	-	-
12	4.04	3.01	4.47	4.93	…	2.78	2.06
13	2.51	2.24	2.27	1.04	…	1.68	1.58
14	1.31	4.49	1.04	3.17	…	1.70	0.07
15	5.60	8.57	5.54	8.24	…	-	-
16	9.46	1.42	9.27	0.38	…	-	-
17	2.30	4.30	2.71	2.44	…	0.06	2.31
18	4.68	1.80	4.96	3.08	…	-	-
19	5.36	0.64	5.32	0.47	…	-	-
20	1.03	0.91	1.39	0.77	…	0.42	0.12
21	5.55	1.26	5.41	0.54	…	-	-
22	3.08	0.20	3.03	0.42	…	1.73	0.04
23	2.99	2.92	2.93	2.59	…	-	-
24	5.16	7.06	5.65	5.04	…	2.34	2.32

Table 19. Gross error detection of image control points using Tau test

Point	1st iteration		2nd iteration	
	Tix	Tiy	Tix	Tiy
1	0.32	0.72	0.39	0.96
2	0.91	0.12	0.99	0.08
3	0.43	0.15	0.47	0.22
4	0.83	0.25	0.90	0.09
5	2.24	0.34	2.54	0.45
6	0.20	0.94	0.16	1.40
7	2.05	0.03	2.28	0.16
8	0.66	*3.16	-	-
9	0.05	0.34	0.02	0.55
10	0.11	1.22	0.04	0.99
11	0.50	1.11	0.52	1.48
12	0.88	0.66	1.10	1.21
13	0.55	0.49	0.56	0.26
14	0.29	0.98	0.25	0.78
15	1.22	1.87	1.36	2.02
16	2.06	0.31	2.28	0.09
17	0.50	0.94	0.67	0.60
18	1.02	0.39	1.22	0.76
19	1.17	0.14	1.31	0.12
20	0.22	0.20	0.34	0.19
21	1.21	0.27	1.33	0.13
22	0.67	0.04	0.74	0.10
23	0.65	0.64	0.72	0.64
24	1.13	1.54	1.39	1.24

Table 20. Gross error detection of image control points using robust estimation

Point .	Debugged Residual		Debugged Weight	
	Vx	Vy	Px	Py
1	-1.33	3.04	1.00	1.00
2	-4.05	0.54	1.00	1.00
3	-1.88	-0.67	1.00	1.00
4	-3.74	1.11	1.00	1.00
5	-9.66	1.45	1.00	1.00
6	-0.85	-3.88	1.00	1.00
7	8.86	0.14	1.00	1.00
8	2.84	-13.63	1.00	1.00
9	-0.19	-1.41	1.00	1.00
10	-0.47	5.24	1.00	1.00
11	2.02	4.50	1.00	1.00
12	3.65	-2.73	1.00	1.00
13	-2.43	2.17	1.00	1.00
14	-1.25	4.28	1.00	1.00
15	5.01	-7.66	1.00	1.00
16	-8.58	1.28	1.00	1.00
17	2.13	3.99	1.00	1.00
18	4.33	-1.67	1.00	1.00
19	-5.15	0.61	1.00	1.00
20	0.97	0.85	1.00	1.00
21	5.15	-1.17	1.00	1.00
22	-2.99	-0.19	1.00	1.00
23	2.81	-2.74	1.00	1.00
24	4.79	6.55	1.00	1.00

IV. Conclusions

Based on the simulation, it is shown that, although the magnitude of gross error is the same, the residual patterns may vary with the location of the gross error. It is also shown that even if the global model test of data snooping method is passed, it may still be able to identify the gross error if the individual residuals are tested.

In the real image test, data snooping identified the gross error correctly, but also mis-identified other 11. This indicates that data snooping is very sensitive to model and measurement errors, and will possibly make misjudgment. For the Danish method, there are cases shown that it is insensitive to the gross errors.

Tau test can efficiently detect the gross errors almost in all tests. It shows stronger detection capability than data snooping and Danish robust estimation methods in the experiments conducted in this study.

Acknowledgements

The authors are benefited from fruitful discussions with Dr. Gee-Wen, Lee, CCIT. The test image pairs were supported from Mr. Wei-Chia Hong, ITRI. We specially thank reviewers for their constructive suggestions.

References

[1] Dare, P., and Dowman, I., “An Improved Model for Automatic Feature-Based Registration of SAR and Spot Images,” ISPRS Journal of Photogrammetry and Remote Sensing, Vol. 56, Issue: 1, pp. 13-28, 2001.

[2] Barbara, Z., and Flusser, J., “Image Registration Methods: a Survey,” Image and Vision Computing, Vol. 21, Issue: 11, pp. 977-1000, 2005.

[3] Lee, C., Tublin, M. E., and Chapman, B. E., “Registration of MR and CT Images of the Liver: Comparison of Voxel Similarity and Surface Based Registration Algorithms,” Computer Methods and Programs in Biomed, Vol.78, Issue: 2, pp. 101-114, 2005.

[4] Jongen, C., Pluim, P. W., Nederkoorn, P. J., Viergever, M. A., and Niessen, W. J.,

"Construction and Evaluation of an Average CT Brain Image for Inter-Subject Registration," Computers in Biology and Medicine, Vol.34, Issue: 8, pp. 647-662, 2004.

[5] Liu, Y., and Wei, B., "Developing Structural Constraints for Accurate Registration of Overlapping Range Images," Robotics and Autonomous Systems, Vol. 47, Issue: 1, pp. 11-30, 2004.

[6] Kavouras, M., On the Detection of Outliers and the Determination of Reliability in Geodetic Networks, Technical Report No 87. Department of Surveying Engineering, University of New Brunswick, Fredericton, N.B. Canada, 1982.

[7] Kraus, K., Jansa, J., and Kager, H., Photogrammetry, Vol.2, Advanced Methods and Applications, Dümmler Verlag, Bonn, Germany, 1997.

[8] Caspary, W. F., Concepts of Network and Deformation Analysis, School of Surveying, The University of New South Wales, Kensington, N.S.W., Australia, 1988.

[9] Pope, A. J., The Statistics of Residuals and the Detection of Outliers, NOAA Technical Report NOS 65 NGS 1, U.S. Department of Commerce, National Geodetic Survey, Rockville, Md., 1976.

[10] Wang, S. J., and Qiu, C. M., A Research on Reliability Analysis of Large-Scale Network and Feasible Error Detection Methods, Research Report of National Science Commission, National Cheng Kung University, Tainan, 1983.

[11] Jensen, J. R., Introductory Digital Image Processing – A remote Sensing Perspective. Prentice-Hall, New Jersey, 1986.

[12] Mikhail, E. M., Bethel, J. S., and McGlone, J. C., Introducation to Modern Photogrammetry, John Wiley & Sons Inc, New York, 2001.

[13] Kuang, S., Geodetic Network Analysis and Optimal Design: Concepts and Applications. Ann Arbor press, Chelsea, Michigan, 1996.