Optimum Process Mean for a One-Sided Specification Limit Product Considering Manufacturing Cost and Quadratic Quality Loss

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ABSTRACT

In this paper, we propose the mathematical models for determining the optimum process mean under the normal, log-normal, exponential, and Weibull quality characteristics. The quadratic quality loss function is used in measuring the quality loss of product. Assume that the linear manufacturing cost happens for a one-sided specification limit product. The manufacturing cost is positively and linearly proportional to deviation from the specification limit when the quality characteristic is within specification. The constant manufacturing cost happens when the quality characteristic exceeds specification. The objective is to find an optimal mean for the one-sided specification limit product with the minimum total loss of society including producer's manufacturing cost and consumer's quality loss.

Keywords: Quadratic Quality Loss Function, Specification Limit, Process Control, Process Mean

考量製造成本與二次品質損失的單邊規格界限產品最佳製程平均數設定

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摘 要

本文將提出產品具常態分配、對數常態分配、指數分配與韋伯分配品質特性的數學模式以決定最佳製程平均數。研究中採二次品質損失函數來量測產品的品質損失，假設產品具單邊規格界限且產品製造成本為線性函數，在規格界限內的製造成本與品質特性值偏離規格界限的距離成正比，而在規格界限外的製造成本則假設為常數。本研究的目標是要求出具單邊規格界限產品的最佳製程平均數，使得涵蓋生產者製造成本與消費者品質損失的總體社會損失為最小。

關鍵詞: 二次品質損失函數，規格界限，製程監控，製程平均數

文稿收件日期 94.3.22 ; 文稿修正後接受日期 95.2.7.
Manuscript received March 22, 2005; revised February 7, 2006.

I. INTRODUCTION

For a product with one-sided specification limit, one usually sets the upper specification limit (USL) for the smaller-the-better type quality characteristic and sets the lower specification limit (LSL) for the larger-the-better type quality characteristic. Taguchi [1] adopted quadratic quality loss function and redefined the product quality as the loss of society. For a larger-the-better type product, the quality loss approaches zero when the quality characteristic approaches infinity. However, the manufacturing cost is not possible without any bound. The trade-off problem is to find an optimal process mean for the larger-the-better quality characteristic with the highest quality level and the minimum manufacturing cost.

The optimum process level setting is used for selecting the manufacturing target. It has been a major topic of statistical process control. The selection of optimum process level/mean will directly affect the process defective rate, production cost, scrap cost, rework cost, and the cost of use. There are considerable attentions paid to the study of economic selection of process mean. Recently, Wu and Tang [2], Li [3-7], Li and Cherng [8, 9], Maghsoodloo and Li [10], Phillips and Cho [11], Li and Chou [12], Li and Wu [13, 14], Chen [15], and Chen and Chou [16] have addressed the different problems of unbalanced tolerance design with the asymmetric quadratic and linear quality loss functions.

The piecewise linear profit function of quality characteristic is usually applied in the filling/canning problem for determining the optimum manufacturing target and other important parameters, e.g., Hunter and Kartha [17], Carlsson [18], Bisgaard et al. [19], Golhar [20, 21], Golhar and Pollock [22, 23], Misiorek and Barnett [24], and Lee et al. [25, 26], and so forth.

The normal distribution is usually used in describing the characteristic of industrial product and the log-normal distribution has been used for random quantities of pollutants in water or air and for other phenomena with skewed distributions. The exponential and Weibull distributions have been used to describe the lifetime of certain electronic systems and they are the important distributions for studying the reliability and failure rates of systems.

In this paper, we propose the mathematical models for determining the optimum process mean under the normal, log-normal, exponential, and Weibull distributions. The quadratic quality loss function is used in measuring the quality loss of product. Assume that the linear manufacturing cost happens for a one-sided specification limit product. The manufacturing cost is positively and linearly proportional to deviation from the specification limit when the quality characteristic is within specification. The constant manufacturing cost happens when the quality characteristic exceeds specification. The objective is to find an optimal mean for the one-sided specification limit product with the minimum total loss of society including producer's manufacturing cost and consumer's quality loss.

II. QUADRATIC QUALITY LOSS FUNCTION FOR A PRODUCT WITH ONE-SIDED SPECIFICATION LIMIT

2.1 Smaller-The-Better Type Quality Characteristic

Assume that the quality characteristic X is the smaller-the-better type. Then the quadratic quality loss function is defined as

$$L(X)=\begin{cases} kX^2, & X \le T_U \\ D_U, & X > T_U \end{cases} \tag{1}$$

where T_U is the USL; D_U is the quality loss at the T_U; k is the quality loss coefficient.

When the loss function is kX^2, the constant k can be obtained as follows. Find the limit value Δ, above which an item performs unsatisfactorily and assess the corresponding consumer's penalty D_U. Substituting D_U and Δ in the loss function for an individual item, i.e., $L(X) = kX^2$, and solving for k, we obtain $k = \frac{D_U}{\Delta^2}$.

2.2 Larger-The-Better Type Quality Characteristic

Assume that the quality characteristic X is the larger-the-better type. Then the quadratic quality loss function is defined as

$$L(X) = \begin{cases} D_L, & X < T_L \\ \frac{k}{X^2}, & X \geq T_L \end{cases} \quad (2)$$

where T_L is the LSL; D_L is the quality loss at the T_L; k is the quality loss coefficient.

When the loss function is $\frac{k}{X^2}$, the constant k can be obtained as follows. Find the limit value Δ, below which an item performs unsatisfactorily and assess the corresponding consumer's penalty D_L. Substituting D_L and Δ in the loss function for an individual item, i.e., $L(X) = \frac{k}{X^2}$, and solving for k, we obtain $k = D_L \Delta^2$.

III. LINEAR MANUFACTURING COST FOR A PRODUCT WITH ONE-SIDED SPECIFICATION LIMIT

3.1 Larger-The-Better Type Quality Characteristic

For a larger-the-better type product, the nominal value of quality characteristic approaches infinite. However, the more material inputs in the manufacturing process when the output value of product increases. Assume that the manufacturing cost is positively and linearly proportional to deviation from the LSL when the quality characteristic is within LSL and constant manufacturing cost happens when the quality characteristic is below the LSL. Then the manufacturing cost function is defined as

$$M(X) = \begin{cases} a, & X < T_L \\ a + b(X - T_L), & X \geq T_L \end{cases} \quad (3)$$

where a is the constant manufacturing cost; b is the variable manufacturing cost per unit.

3.2 Smaller-The-Better Type Quality Characteristic

For a smaller-the-better type product, the nominal value of quality characteristic approaches zero. However, the more unit machine cost inputs in the manufacturing process when the output value of product decreases. Assume that the manufacturing cost is positively and linearly proportional to deviation from the USL when the quality characteristic is within USL and constant manufacturing cost happens when the quality characteristic exceeds the USL. Then the manufacturing cost function is defined as

$$M(X) = \begin{cases} a + b(T_U - X), & X \leq T_U \\ a, & X > T_U \end{cases} \quad (4)$$

IV. NORMAL QUALITY CHARACTERISTIC

4.1 Smaller-The-Better Type Normal Quality Characteristic

Assume that the normal quality characteristic X follows an unknown mean μ and known variance σ^2. Let $\mu / \sigma > 5$, i.e., the probability of quality characteristic $X < 0$ is very small. From Appendix A, the total loss of society including the producer's manufacturing cost and the customer's quality loss for smaller-the-better type normal quality characteristic is

$$C_T = k[(\mu^2 + \sigma^2)\Phi(\frac{T_U - \mu}{\sigma}) - \sigma(T_U + \mu) \cdot \phi(\frac{T_U - \mu}{\sigma})] + D_U[1 - \Phi(\frac{T_U - \mu}{\sigma})] + [a + b(T_U - \mu)]\Phi(\frac{T_U - \mu}{\sigma}) + b\sigma\phi(\frac{T_U - \mu}{\sigma}) + a[1 - \Phi(\frac{T_U - \mu}{\sigma})] \quad (5)$$

where $\Phi(z)$ is the cumulative probability of a standard normal random variable with probability density function $\phi(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2}, -\infty < z < \infty.$

4.2 Larger-The-Better Type Normal Quality Characteristic

For the larger-the-better type normal quality characteristic, it is not easy to obtain the expected quality loss within specification. By adopting Taylor's series expansion and ignoring terms higher than the second order, we can obtain the approximate quality loss within specification. The total loss of society including the producer's manufacturing cost and the customer's quality loss is

$$C_T = \int_{T_L}^{\infty} k\frac{1}{x^2} f(x)dx + D_L \int_{-\infty}^{T_L} f(x)dx + \int_{T_L}^{\infty} [a + b(x - T_L)]f(x)dx + a\int_{-\infty}^{T_L} f(x)dx$$
$$\cong k[\frac{1}{\mu_x^2}(1 + \frac{3\sigma_x^2}{\mu_x^2})] + D_L\Phi(\frac{T_L - \mu}{\sigma}) + [a - b(T_L - \mu)][1 - \Phi(\frac{T_L - \mu}{\sigma})] + b\sigma\phi(\frac{T_L - \mu}{\sigma}) + a\Phi(\frac{T_L - \mu}{\sigma}) \quad (6)$$

where

$$\mu_x = \int_{T_L}^{\infty} xf(x)dx = \mu[1 - \Phi(\frac{T_L - \mu}{\sigma})] + \sigma\phi(\frac{T_L - \mu}{\sigma}) \quad (7)$$

$$\sigma_x^2 = \int_{T_L}^{\infty} x^2 f(x)dx - \mu_x^2 = (\mu^2 + \sigma^2)[1 - \Phi(\frac{T_L - \mu}{\sigma})] + \sigma(T_L + \mu)\phi(\frac{T_L - \mu}{\sigma}) - \mu^2[1 - \Phi(\frac{T_L - \mu}{\sigma})]^2 - 2\mu\sigma\phi(\frac{T_L - \mu}{\sigma})[1 - \Phi(\frac{T_L - \mu}{\sigma})] - \sigma^2[\phi(\frac{T_L - \mu}{\sigma})]^2 \quad (8)$$

V. LOG-NORMAL QUALITY CHARACTERISTIC

5.1 Smaller-The-Better Type Log-Normal Quality Characteristic

Assume that the log-normal quality characteristic X follows an unknown mean μ and known variance σ^2. From Appendix B, the total loss of society including the producer's manufacturing cost and the customer's quality loss for smaller-the-better type log-normal quality characteristic is

$$C_T = ke^{2(\mu+\sigma^2)}\Phi(\frac{\log T_U - \mu - 2\sigma^2}{\sigma}) +$$
$$D_U[1-\Phi(\frac{\log T_U - \mu}{\sigma})] + (a + bT_U)\cdot$$
$$\Phi(\frac{\log T_U - \mu}{\sigma}) - be^{\mu+\frac{\sigma^2}{2}}\cdot$$
$$\Phi(\frac{\log T_U - \mu - \sigma^2}{\sigma}) +$$
$$a[1-\Phi(\frac{\log T_U - \mu}{\sigma})] \quad (9)$$

5.2 Larger-The-Better Type Log-Normal Quality Characteristic

From Appendix C, the total loss of society including the producer's manufacturing cost and the customer's quality loss for larger-the-better type log-normal quality characteristic is

$$C_T = ke^{2(\sigma^2-\mu)}[1-\Phi(\frac{\log T_L - \mu + 2\sigma^2}{\sigma})] +$$
$$D_L\Phi(\frac{\log T_L - \mu}{\sigma}) + be^{\mu+\frac{\sigma^2}{2}}\cdot$$
$$[1-\Phi(\frac{\log T_L - \mu - \sigma^2}{\sigma})] +$$
$$a\Phi(\frac{\log T_L - \mu}{\sigma}) \quad (10)$$

VI. EXPONENTIAL QUALITY CHARACTERISTIC

6.1 Smaller-The-Better Type Exponential Quality Characteristic

Assume that the exponential quality characteristic X follows an unknown scale parameter θ. From Appendix D, the total loss of society including the producer's manufacturing cost and the customer's quality loss for smaller-the-better type exponential quality characteristic is

$$C_T = k\{-e^{-\frac{T_U}{\theta}}[T_U^2 + 2\theta(T_U + \theta)] + 2\theta^2\} +$$
$$D_U e^{-\frac{T_U}{\theta}} + (a + bT_U)[1 - e^{-\frac{T_U}{\theta}}]$$
$$-b[\theta - (T_U + \theta)e^{-\frac{T_U}{\theta}}] + ae^{-\frac{T_U}{\theta}} \quad (11)$$

6.2 Larger-The-Better Type Exponential Quality Characteristic

For the larger-the-better type exponential quality characteristic, it is not easy to obtain the expected quality loss within specification. By adopting Taylor's series expansion and ignoring terms higher than the second order, we can obtain the approximate quality loss within specification. The total loss of society including the producer's manufacturing cost and the customer's quality loss is

$$C_T = \int_{T_L}^{\infty}\frac{k}{x^2}f(x)dx + D_L\int_0^{T_L}f(x)dx +$$
$$\int_{T_L}^{\infty}[a + b(x - T_L)]f(x)dx +$$
$$a\int_0^{T_L}f(x)dx$$
$$\cong k[\frac{1}{\mu_x^2}(1 + \frac{3\sigma_x^2}{\mu_x^2})] + D_L(1 - e^{-\frac{T_L}{\theta}}) +$$
$$(a - bT_L)e^{-\frac{T_L}{\theta}} + be^{-\frac{T_L}{\theta}}(T_L + \theta)$$
$$+ a(1 - e^{-\frac{T_L}{\theta}}) \quad (12)$$

where

$$\mu_x = \int_{T_L}^{\infty} xf(x)dx$$
$$= \int_{T_L}^{\infty} x\frac{1}{\theta}e^{-\frac{x}{\theta}}dx$$
$$= (\theta + T_L)e^{-\frac{T_L}{\theta}} \quad (13)$$

$$\sigma_x^2 = \int_{T_L}^{\infty} x^2 f(x)dx - \mu_x^2$$
$$= \int_{T_L}^{\infty} x^2 \frac{1}{\theta} e^{-\frac{x}{\theta}} dx - \mu_x^2$$
$$= T_L^2 e^{-\frac{T_L}{\theta}} + 2\theta[(\theta + T_L)e^{-\frac{T_L}{\theta}}]$$
$$- [(\theta + T_L)e^{-\frac{T_L}{\theta}}]^2 \quad (14)$$

VII. WEIBULL QUALITY CHARACTERISTIC

7.1 Smaller-The-Better Type Weibull Quality Characteristic

Assume that the Weibull quality characteristic X follows an unknown scale parameter θ and known shape parameter β. From Appendix E, the total loss of society including the producer's manufacturing cost and the customer's quality loss for smaller-the-better type Weibull quality characteristic is

$$C_T = k\{-T_U^2 e^{-(\frac{T_U}{\theta})^\beta} + \frac{2\theta^2}{\beta}\Gamma(\frac{2}{\beta}) \cdot$$
$$I[\frac{2}{\beta}, \ (\frac{T_U}{\theta})^\beta]\} + D_U e^{-(\frac{T_U}{\theta})^\beta} +$$
$$(a + bT_U)[1 - e^{-(\frac{T_U}{\theta})^\beta}] - b\{-T_U e^{-(\frac{T_U}{\theta})^\beta}$$
$$+ \frac{\theta}{\beta}\Gamma(\frac{1}{\beta}) I[\frac{1}{\beta}, \ (\frac{T_U}{\theta})^\beta]\} + ae^{-(\frac{T_U}{\theta})^\beta} \quad (15)$$

where

$$\Gamma(t) = \int_0^\infty u^{t-1} e^{-u} du, \quad t > 0 \quad (16)$$
$$I[t_1, \ t_2] = \frac{1}{\Gamma(t_1)} \int_0^{t_2} u^{t_1 - 1} e^{-u} du,$$
$$t_1 > 0, t_2 > 0 \quad (17)$$

7.2 Larger-The-Better Type Weibull Quality Characteristic

For the larger-the-better type Weibull quality characteristic, it is not easy to obtain the expected quality loss within specification. By adopting Taylor's series expansion and ignoring terms higher than the second order, we can obtain the approximate quality loss within specification. The total loss of society including the producer's manufacturing cost and the customer's quality loss is

$$C_T = \int_{T_L}^{\infty} \frac{k}{x^2} f(x)dx + D_L \int_0^{T_L} f(x)dx +$$
$$\int_{T_L}^{\infty} [a + b(x - T_L)] f(x)dx + a\int_0^{T_L} f(x)dx$$
$$\cong k[\frac{1}{\mu_x^2}(1 + \frac{3\sigma_x^2}{\mu_x^2})] + D_L[1 - e^{-(\frac{T_L}{\theta})^\beta}] +$$
$$(a - bT_L)e^{-(\frac{T_L}{\theta})^\beta} + b\{T_L e^{-(\frac{T_L}{\theta})^\beta} +$$
$$\frac{\theta}{\beta}\Gamma(\frac{1}{\beta})\{1 - I[\frac{1}{\beta}, \ (\frac{T_L}{\theta})^\beta]\}\} +$$
$$a[1 - e^{-(\frac{T_L}{\theta})^\beta}] \quad (18)$$

where

$$\mu_x = \int_{T_L}^{\infty} xf(x)dx$$
$$= \int_{T_L}^{\infty} x \frac{\beta}{\theta}(\frac{x}{\theta})^{\beta - 1} e^{-(\frac{x}{\theta})^\beta} dx$$
$$= T_L e^{-(\frac{T_L}{\theta})^\beta} + \frac{\theta}{\beta}\Gamma(\frac{1}{\beta}) \cdot$$
$$\{1 - I[\frac{1}{\beta}, \ (\frac{T_L}{\theta})^\beta]\} \quad (19)$$

$$\sigma_x^2 = \int_{T_L}^{\infty} x^2 f(x)dx - \mu_x^2$$

$$= \int_{T_L}^{\infty} x^2 \frac{\beta}{\theta}(\frac{x}{\theta})^{\beta-1} e^{-(\frac{x}{\theta})^{\beta}} dx - \mu_x^2$$

$$= T_L^2 e^{-(\frac{T_L}{\theta})^{\beta}} + \frac{2\theta^2}{\beta}\Gamma(\frac{2}{\beta}) \cdot$$

$$\{1 - I[\frac{2}{\beta},\ (\frac{T_L}{\theta})^{\beta}]\}$$

$$-\{\frac{\theta}{\beta}\Gamma(\frac{1}{\beta})\{1 - I[\frac{1}{\beta},\ (\frac{T_L}{\theta})^{\beta}]\}\}^2 \quad (20)$$

The optimum process mean μ^* for Eqs. (5), (6), (9), (10), (11), (12), (15), and (18) can been obtained by using linear search techniques for single variable, such as, the golden section search method or the Fibonacci search method.

VIII. NUMERICAL EXAMPLES

8.1 Normal Quality Characteristic

Assume that the quality characteristic X is normally distributed with known standard deviation $\sigma = 0.5$ and unknown mean μ.

8.1.1 Smaller-the-better type normal quality characteristic

Suppose that the USL = 9.5, the quality loss coefficient k = 0.5, and quality loss at the USL is $D_U = 45.125$. The constant manufacturing cost a = 5 and the variable manufacturing cost per unit b = 9. The objective is to find an optimal process mean such that it will be at the optimum level possible, while the same time minimizing the manufacturing cost and quality loss. By solving Eq. (5), the optimum process mean for the smaller-the-better type normal quality characteristic is μ^* = 9.49307 with $C_T^* = 48.2923$. Figure 1 shows the curve of the total loss of society.

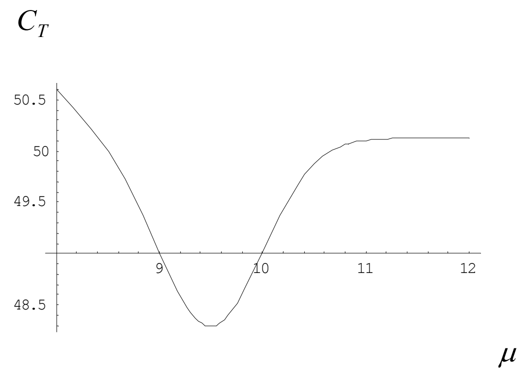

Fig. 1. The total loss of society for the smaller-the-better type normal quality characteristic.

8.1.2 Larger-the-better type normal quality characteristic

Suppose that the LSL = 9.2, the quality loss coefficient k = 1058, and quality loss at the LSL is $D_L = 12.5$. The constant manufacturing cost a = 5 and the variable manufacturing cost per unit b = 2. The objective is to find an optimal process mean such that it will be at the optimum level possible, while the same time minimizing the manufacturing cost and quality loss. By solving Eq. (6), the optimum process mean for the smaller-the-better type normal quality characteristic is μ^* = 10.7869 with $C_T^* = 17.3664$. Figure 2 shows the curve of the total loss of society.

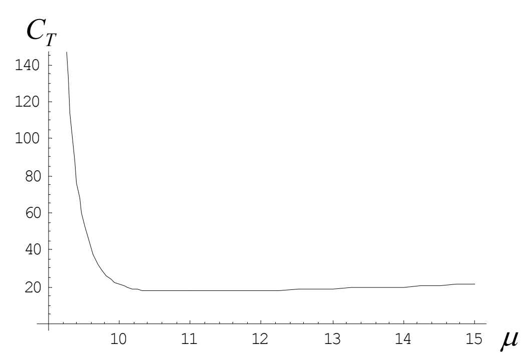

Fig. 2. The total loss of society for the larger-the-better type normal quality characteristic.

8.2 Log-Normal Quality Characteristic

Assume that the quality characteristic X follows log-normal distribution with known standard deviation $\sigma = 0.01$ and unknown mean μ.

8.2.1 Smaller-the-better type log-normal quality characteristic

Suppose that the USL = 9.5, the quality loss coefficient k = 0.5, and quality loss at the USL is $D_U = 2.5341$. The constant manufacturing cost a = 5 and the variable manufacturing cost per unit b = 30. The objective is to find an optimal process mean such that it will be at the optimum level possible, while the same time minimizing the manufacturing cost and quality loss. By solving Eq. (9), the optimum process mean for the smaller-the-better type log-normal quality characteristic is μ^* = 2.339 (the optimum mean value $e^{2.339}$ = 10.3709 before the variable transformation) with C_T^* = 7.5341. Figure 3 shows the curve of the total loss of society.

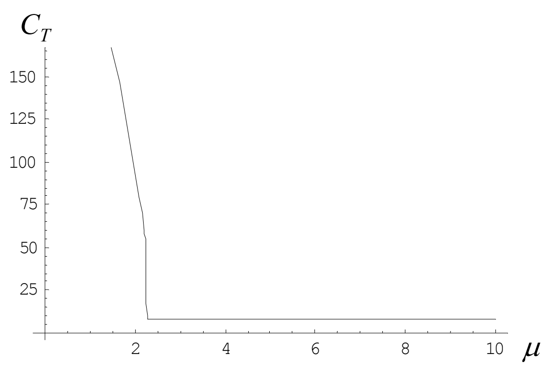

Fig. 3. The total loss of society for the smaller-the-better type log-normal quality characteristic.

8.2.2 Larger-the-better type log-normal quality characteristic

Suppose that the LSL = 9.2, the quality loss coefficient k = 61.5608, and quality loss at the LSL is $D_L = 12.5$. The constant manufacturing cost a = 5 and the variable manufacturing cost per unit b = 2. The objective is to find an optimal process mean such that it will be at the optimum level possible, while the same time minimizing the manufacturing cost and quality loss. By solving Eq. (10), the optimum process mean for the smaller-the-better type normal quality characteristic is μ^* = 2.2467 (the optimum mean value $e^{2.2467}$ = 9.4565 before the variable transformation) with C_T^* = 6.26009. Figure 4 shows the curve of the total loss of society.

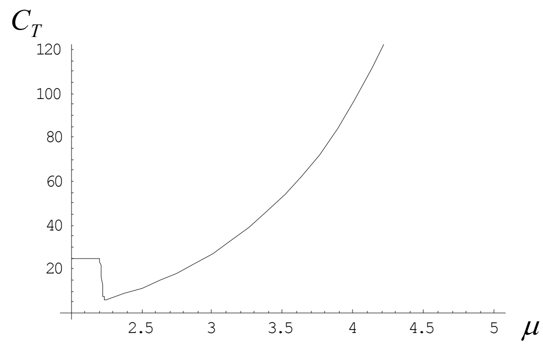

Fig. 4. The total loss of society for the larger-the-better type log-normal quality characteristic.

8.3 Exponential Quality Characteristic

Assume that the quality characteristic X follows exponential distribution with unknown scale parameter θ.

8.3.1 Smaller-the-better type exponential quality characteristic

Suppose that the USL = 9.5, the quality loss coefficient k = 0.5, and quality loss at the USL is $D_U = 45.125$. The constant manufacturing cost a = 5 and the variable manufacturing cost per unit b = 5.8. The objective is to find an optimal process mean such that it will be at the optimum level possible, while the same time minimizing the manufacturing cost and quality loss. By solving Eq. (11), the optimum process mean for the smaller-the-better type normal quality characteristic is $\mu^* =$ 9.9467 with $C_T^* = 49.1165$. Figure 5 shows the curve of the total loss of society.

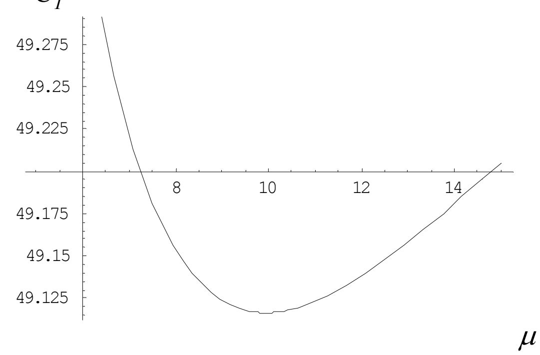

Fig. 5. The total loss of society for the smaller-the-better type exponential quality characteristic.

8.3.2 Larger-the-better type exponential quality characteristic

Suppose that the LSL = 9.2, the quality loss coefficient k = 1058, and quality loss at the LSL is $D_L = 12.5$. The constant manufacturing cost a = 5 and the variable manufacturing cost per unit b = 60. The objective is to find an optimal process mean such that it will be at the optimum level possible, while the same time minimizing the manufacturing cost and quality loss. By solving Eq. (12), the optimum process mean for the smaller-the-better type normal quality characteristic is $\mu^* =$ 10.2263 with $C_T^* = 387.727$. Figure 6 shows the curve of the total loss of society.

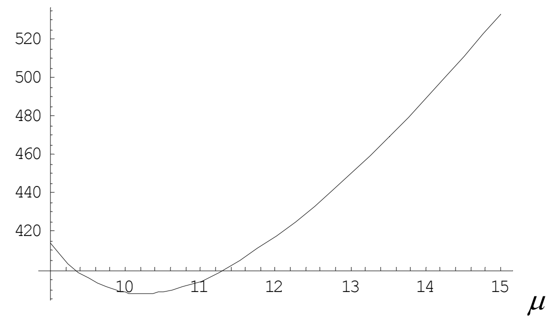

Fig. 6. The total loss of society for the larger-the-better type exponential quality characteristic.

8.4 Weibull Quality Characteristic

Assume that the quality characteristic X follows Weibull distribution with shape parameter $\beta = 0.9$ and unknown scale parameter θ .

8.4.1 Smaller-the-better type weibull quality characteristic

Suppose that the USL = 9.5, the quality loss coefficient k = 0.5, and quality loss at the USL is $D_U = 45.125$. The constant manufacturing cost a = 5 and the variable manufacturing cost per unit b = 5.8. The objective is to find an optimal process mean such that it will be at the optimum level possible, while the same time minimizing the manufacturing cost and quality loss. By solving Eq. (15), the optimum process mean for the smaller-the-better type normal quality characteristic is $\mu^* =$ 7.4700 with $C_T^* = 42.649$. Figure 7 shows the curve of the total loss of society.

C_T

C_T

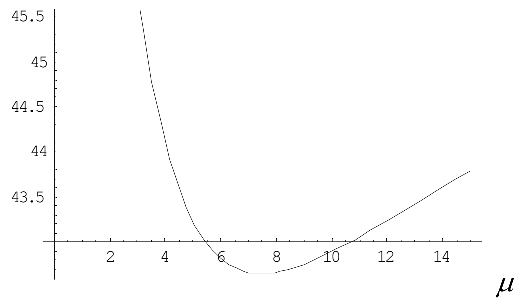

Fig. 7. The total loss of society for the smaller-the-better type Weibull quality characteristic.

8.4.2 Larger-the-better type weibull quality characteristic

Suppose that the LSL = 9.2, the quality loss coefficient k = 1058, and quality loss at the LSL is $D_L = 12.5$. The constant manufacturing cost a = 5 and the variable manufacturing cost per unit b = 60. The objective is to find an optimal process mean such that it will be at the optimum level possible, while the same time minimizing the manufacturing cost and quality loss. By solving Eq. (18), the optimum process mean for the smaller-the-better type normal quality characteristic is $\mu^* =$ 9.7837 with $C_T^* = 418.415$. Figure 8 shows the curve of the total loss of society.

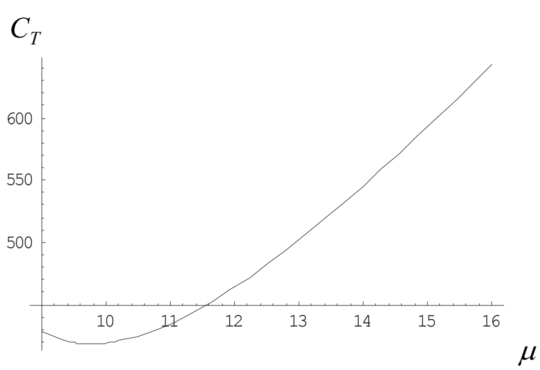

Fig. 8. The total loss of society for the larger-the-better type Weibull quality characteristic

IX. CONCLUSIONS

In this paper, we have presented the mathematical models for determining the optimum process mean under the normal, log-normal, exponential, and Weibull quality characteristics. By solving the optimization model, we can obtain the minimum total loss of society including the producer's manufacturing cost and the customer's quality loss. The modified model considering different manufacturing cost and linear quality loss of product may be significant for further study.

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APPENDIX

A. Total Loss of Society For Smaller-The-Better Type Normal Quality Characteristic

$$C_T = \int_{-\infty}^{T_U} kx^2 f(x)dx + D_U \int_{T_U}^{\infty} f(x)dx + \int_{-\infty}^{T_U} [a+b(T_U - x)]f(x)dx + a\int_{T_U}^{\infty} f(x)dx$$

$$= k\int_{-\infty}^{\frac{T_U-\mu}{\sigma}} (\mu + z\sigma)^2 \phi(z)dz + D_U \cdot \int_{\frac{T_U-\mu}{\sigma}}^{\infty} \phi(z)dz + \int_{-\infty}^{\frac{T_U-\mu}{\sigma}} [a+b(T_U - z\sigma - \mu)]\phi(z)dz + a\int_{\frac{T_U-\mu}{\sigma}}^{\infty} \phi(z)dz$$

$$= k[(\mu^2+\sigma^2)\Phi(\frac{T_U-\mu}{\sigma}) - \sigma(T_U+\mu)\cdot \phi(\frac{T_U-\mu}{\sigma})] + D_U[1-\Phi(\frac{T_U-\mu}{\sigma})] + [a+b(T_U-\mu)]\Phi(\frac{T_U-\mu}{\sigma}) + b\sigma\phi(\frac{T_U-\mu}{\sigma}) + a[1-\Phi(\frac{T_U-\mu}{\sigma})]$$

where $f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}(\frac{x-\mu}{\sigma})^2}, -\infty < x < \infty.$

$\Phi(z)$ is the cumulative probability of a standard normal random variable with probability density function $\phi(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2}, -\infty < z < \infty.$

B. Total Loss of Society for Smaller-The-Better Type Log-Normal Quality Characteristic

$$C_T = \int_0^{T_U} kx^2 f(x)dx + D_U \int_{T_U}^{\infty} f(x)dx + \int_0^{T_U} [a+b(T_U - x)]f(x)dx + a\int_{T_U}^{\infty} f(x)dx$$

$$= \int_{-\infty}^{\frac{\log T_U-\mu}{\sigma}} k\frac{1}{\sqrt{2\pi}} e^{2(z\sigma+\mu)-\frac{1}{2}z^2} dz + D_U \int_{\frac{\log T_U-\mu}{\sigma}}^{\infty} \phi(z)dz + \int_{-\infty}^{\frac{\log T_U-\mu}{\sigma}} [a+b(T_U - e^{z\sigma+\mu})]\frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2} dz + a\int_{\frac{\log T_U-\mu}{\sigma}}^{\infty} \phi(z)dz$$

$$= ke^{2(\mu+\sigma^2)} \int_{-\infty}^{\frac{\log T_U-\mu}{\sigma}-2\sigma} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}y^2} dy + D_U[1-\Phi(\frac{\log T_U-\mu}{\sigma})] + (a+bT_U)\cdot \Phi(\frac{\log T_U-\mu}{\sigma}) - be^{\mu+\frac{\sigma^2}{2}} \cdot \int_{-\infty}^{\frac{\log T_U-\mu}{\sigma}-\sigma} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}y^2} dy + a[1-\Phi(\frac{\log T_U-\mu}{\sigma})]$$

$$= ke^{2(\mu+\sigma^2)}\int_{-\infty}^{\frac{\log T_U-\mu}{\sigma}-2\sigma}\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}y^2}dy +$$
$$D_U[1-\Phi(\frac{\log T_U-\mu}{\sigma})]+(a+bT_U)\cdot$$
$$\Phi(\frac{\log T_U-\mu}{\sigma})-be^{\mu+\frac{\sigma^2}{2}}\cdot$$
$$\int_{-\infty}^{\frac{\log T_U-\mu}{\sigma}-\sigma}\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}y^2}dy + a[1-$$
$$\Phi(\frac{\log T_U-\mu}{\sigma})]$$

where

$$f(x)=\frac{1}{\sqrt{2\pi}\sigma x}e^{-\frac{1}{2}(\frac{\log x-\mu}{\sigma})^2}, 0<x<\infty.$$

C. Total Loss of Society for Larger-The-Better Type Log-Normal Quality Characteristic

$$C_T=\int_{T_L}^{\infty}k\frac{1}{x^2}f(x)dx+D_L\int_0^{T_L}f(x)dx+$$
$$\int_{T_L}^{\infty}[a+(x-T_L)]f(x)dx+a\int_0^{T_L}f(x)dx$$
$$=\int_{\frac{\log T_L-\mu}{\sigma}}^{\infty}k\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}z^2-2z\sigma}dz+$$
$$D_L\int_0^{\frac{\log T_L-\mu}{\sigma}}\phi(z)dz$$
$$+\int_{\frac{\log T_L-\mu}{\sigma}}^{\infty}[a+b(e^{z\sigma+\mu})]\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}z^2}dz$$
$$+a\int_0^{\frac{\log T_L-\mu}{\sigma}}\phi(z)dz$$
$$=ke^{2(\sigma^2-\mu)}\int_{\frac{\log T_L-\mu}{\sigma}+2\sigma}^{\infty}\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}y^2}dy+$$
$$D_L\Phi(\frac{\log T_L-\mu}{\sigma})+(a-bT_L)[1-$$
$$\Phi(\frac{\log T_L-\mu}{\sigma})]+be^{\mu+\frac{\sigma^2}{2}}\cdot$$
$$\int_{\frac{\log T_L-\mu}{\sigma}-\sigma}^{\infty}\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}y^2}dy$$
$$+a\Phi(\frac{\log T_L-\mu}{\sigma})$$
$$=ke^{2(\sigma^2-\mu)}[1-\Phi(\frac{\log T_L-\mu+2\sigma^2}{\sigma})]+$$
$$D_L\Phi(\frac{\log T_L-\mu}{\sigma})+be^{\mu+\frac{\sigma^2}{2}}[1-$$
$$\Phi(\frac{\log T_L-\mu-\sigma^2}{\sigma})]+a\Phi(\frac{\log T_L-\mu}{\sigma})$$

where

$$f(x)=\frac{1}{\sqrt{2\pi}\sigma x}e^{-\frac{1}{2}(\frac{\log x-\mu}{\sigma})^2}, 0<x<\infty.$$

D. Total Loss of Society for Smaller-The-Better Type Exponential Quality Characteristic

$$C_T = \int_0^{T_U} kx^2 f(x)dx + D_U \int_{T_U}^{\infty} f(x)dx + \int_0^{T_U} [a + b(T_U - x)]f(x)dx + a\int_{T_U}^{\infty} f(x)dx$$

$$= k[-x^2 e^{-\frac{x}{\theta}}\Big|_0^{T_U} + 2\theta\int_0^{T_U} x\frac{1}{\theta}e^{-\frac{x}{\theta}}dx] + D_U e^{-\frac{T_U}{\theta}} + (a + bT_U)[-e^{-\frac{x}{\theta}}\Big|_0^{T_U}] - b[-xe^{-\frac{x}{\theta}}\Big|_0^{T_U} + \int_0^{T_U} e^{-\frac{x}{\theta}}dx] + ae^{-\frac{T_U}{\theta}}$$

$$= k\{-e^{-\frac{T_U}{\theta}}[T_U^2 + 2\theta(T_U + \theta)] + 2\theta^2\} + D_U e^{-\frac{T_U}{\theta}} + (a + bT_U)[1 - e^{-\frac{T_U}{\theta}}] - b[\theta - (T_U + \theta)e^{-\frac{T_U}{\theta}}] + ae^{-\frac{T_U}{\theta}}$$

where

$$f(x) = \frac{1}{\theta}e^{-\frac{x}{\theta}}, x \ge 0, \theta > 0.$$

E. Total Loss of Society for Smaller-The-Better Type Weibull Quality Characteristic

$$C_T = \int_0^{T_U} kx^2 f(x)dx + D_U \int_{T_U}^{\infty} f(x)dx + \int_0^{T_U} [a + b(T_U - x)]f(x)dx + a\int_{T_U}^{\infty} f(x)dx$$

$$= k[-x^2 e^{-(\frac{x}{\theta})^\beta}\Big|_0^{T_U} + 2\int_0^{T_U} xe^{-(\frac{x}{\theta})^\beta}dx] + D_U \int_{(\frac{T_U}{\theta})^\beta}^{\infty} e^{-y}dx + (a + bT_U)[-e^{-z}\Big|_0^{(\frac{T_U}{\theta})^\beta}] - b[-xe^{-(\frac{x}{\theta})^\beta}\Big|_0^{T_U} + \int_0^{T_U} e^{-(\frac{x}{\theta})^\beta}dx] + a\int_{(\frac{T_U}{\theta})^\beta}^{\infty} e^{-y}dx$$

$$= k\{-T_U^2 e^{-(\frac{T_U}{\theta})^\beta} + \frac{2\theta^2}{\beta}\Gamma(\frac{2}{\beta})I[\frac{2}{\beta},\ (\frac{T_U}{\theta})^\beta]\} + D_U e^{-(\frac{T_U}{\theta})^\beta} + (a + bT_U)[1 - e^{-(\frac{T_U}{\theta})^\beta}] - b\{-T_U e^{-(\frac{T_U}{\theta})^\beta} + \frac{\theta}{\beta}\Gamma(\frac{1}{\beta})I[\frac{1}{\beta},\ (\frac{T_U}{\theta})^\beta]\} + ae^{-(\frac{T_U}{\theta})^\beta}$$

where

$$f(x) = \frac{\beta}{\theta}(\frac{x}{\theta})^{\beta-1}e^{-(\frac{x}{\theta})^\beta}, x \ge 0, \beta > 0, \theta > 0.$$

$$\Gamma(t) = \int_0^{\infty} u^{t-1}e^{-u}du,\quad t > 0.$$

$$I[t_1,\ t_2] = \frac{1}{\Gamma(t_1)}\int_0^{t_2} u^{t_1-1}e^{-u}du,\quad t_1 > 0,\ t_2 > 0.$$