Fuzzy Model-Based S-curve Regression for Working Capital Construction Management with Statistical Method

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ABSTRACT

The regression curve, obtained for project control of large-scale or small-scale engineering, is smoothly connected by a Takagi-Sugeno fuzzy model. A sample of data taken from Department of Rapid Transit Systems, Taipei City Government is used to demonstrate the concept of the proposed regression model using statistical method. This developed S-curve equation could be used in a variety of applications related to project control in the working capital management of construction firms.

Keywords: fuzzy model, fuzzy S-curve regression, working capital management

模糊模型建構的 S 曲線回歸與統計方法探討營建管理營運資金問題

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摘　　要

本文以最佳化方法找出的 S 曲線上下界，可用來取代以往統計學信賴區間的方式。文中，本研究收集捷運標案之估驗計價資料，進行 S-curve 實證分析與模型建構，以瞭解專案執行過程之短期財務需求，增進對現金管理問題的了解。此研究方法可應用於建設公司在營運資金管理的計畫控制相關問題。

關鍵詞：模糊理論，模糊 S 曲線回歸，營運資金管理

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I. INTRODUCTION

The regression analysis is one of the most widely used statistical methods for analyzing multifactor data and provides a way of empirically identifying how a variable is affected by other variables. It is also employed for making predictions and judging the strength of relationships. Therefore, it is essential a wide range of fields, including the social science, engineering analysis, medical research, business management and so on.

Some literature surveys indicated S-curves are helpful to project management in reporting current status and predicting the progress of project. Hence, they are widely used in industry and management for project control (see [1-3] and the references therein). For examples, Miskawi (1989) developed an S-curve equation capable of producing an S-curve envelope and it was used in the application to project control of petro-chemical industry. Consequently, the S-type distribution is believed to be suitable in regression on construction management, social economy and so on.

In the contracting business, construction firms are generally more concerned with short-term financial strategies than the long-term ones. Working capital management is the central issue of all short-term financial concerns. Most importantly, cash management is the ultimate goal for achieving high liquidity and profitability.

However, the present-day systems, such as capital management, engineering technology, environment and societal economy, become large in dimension and complexity so that the numerical data concerning the information of systems can not be obtained exactly. To solve the problems arising from complex systems may become very inefficient or even impossible if using the traditional mathematical tools that are not constructed for dealing with high dimensionality models. That is to say, the traditional least square regression may not be enough when dealing with curve fitting problems.

In recent two decades, some interesting approaches containing the regression model by fuzzy theory have been attracting increasingly attention, as proposed in the literature [4-8]. However, a literature survey indicates that the working capital management for large public constructions by using fuzzy S-curve regression has not been discussed. Therefore, the aim of this paper is to develop a practical model for construction firms to rationalize the amount of cash and current assets, which should be possessed in any point of time. Then, we will develop an S-curve regression model to achieve goals of project control in construction management.

This study is organized as follows. First, the data analysis by multiple regression model and the classic S-curve theory are reviewed. Then, based on fuzzy set theory and fuzzy inference engine as well as center of gravity defuzzification, a fuzzy S-curve was proposed for curve fitting problems. The fuzzy regression model is constructed from some linear fuzzy rules which are so-called Takagi-Sugeno fuzzy models. Finally, an example of two sets of data with simulations is given to demonstrate the methodology via the proposed method, and the conclusions are drawn.

II. MULTIPLE AND S-CURVE REGRESSION MODEL

The data consist of n observations on a dependent (response) variable y and p independent (explanatory) variables, $x_0, x_1, x_2, \Lambda, x_p$. The observations are usually represented as follows:

Observation Number	y	x_1	x_2	x_3	x_p
1	y_1	x_{11}	x_{21}	x_{31}	x_{p1}
2	y_2	x_{12}	x_{22}	x_{32}	x_{p2}
3	y_3	x_{13}	x_{23}	x_{33}	x_{p3}
M	M	M	M	M	M
n	y_n	x_{1n}	x_{2n}	x_{3n}	x_{pn}

The relationship between y and $x_0, x_1, x_2, \Lambda, x_p$ is formulated as a linear model.

The linear regression model is displayed as follows:

$$y_i = \beta_0 + \beta_1 x_{1i} + \beta_2 x_{2i} + \Lambda + \beta_p x_{pi} + u_i ,$$
for $i = 1,2,\Lambda\ ,n$, (1)

where $\beta_0, \beta_1, \beta_2, \Lambda\ , \beta_p$ are constants referred to as the model regression coefficients and u_i is a random disturbance. For any set of fixed values, $x_0, x_1, x_2, \Lambda\ , x_p$, which fall within the range of the data, the linear equation (1) could provide an acceptable approximation of the true relationship between y and the x's. That is to say, y is approximately a linear function of the x's, and u_i measures the discrepancy in that approximation for the ith observation. It is assumed that u's are random quantities, independently distributed with zero means and constant variance σ^2 .

Let

$$X = \begin{bmatrix} 1 & x_{11} & \Lambda & x_{p1} \\ 1 & x_{12} & \Lambda & x_{p2} \\ M & M & O & \\ 1 & x_{1n} & & x_{pn} \end{bmatrix}, \quad Y = \begin{bmatrix} y_1 \\ y_2 \\ M \\ y_n \end{bmatrix}, \quad u = \begin{bmatrix} u_1 \\ u_2 \\ M \\ u_n \end{bmatrix}$$

and $\beta = \begin{bmatrix} \beta_1 \\ \beta_2 \\ M \\ \beta_p \end{bmatrix}$.

The β's are estimated by minimizing the sum of squared residuals,

$$S(\beta) = \sum_{i=1}^{n} u_i^2$$
$$= \sum_{i=1}^{n} \left(y_i - \beta_0 - \beta_1 x_{i,1} - \beta_2 x_{i,2} \cdots - \beta_{p-1} x_{i,p-1} \right)^2 \tag{2}$$

which is known as the method of least squares. Minimization of $S(\beta)$ leads to the system of equations

$$(X^T X)\,\beta = X^T Y , \tag{3}$$

Assuming that $(X^T X)$ has an inverse and $\hat{\beta}$ is the least squares estimator of β . Then, $\hat{\beta}$ can be written explicitly as [9]

$$\hat{\beta} = (X^T X)^{-1} X^T Y , \tag{4}$$

where $\hat{\beta} = (\hat{\beta}_0, \hat{\beta}_1, \Lambda\ , \hat{\beta}_{p-1})$, for the linear model (1) for $i = 1,2,\Lambda\ ,n$. Meanwhile, the linear model (1) that represents the data will be

$$Y = X\,\beta + u , \tag{5}$$

where $x_{0i} = 1$ for all i. The assumptions made about u for least squares estimation are

$$\mathrm{E}(u) = 0 ; \quad \mathrm{Var}(u) = \mathrm{E}(uu^T) = \sigma^2 I_n , \tag{6}$$

that is, u_i 's are independent and have zero mean and constant variance. This implies

$$\mathrm{E}(Y) = X\,\beta \tag{7}$$

According to the above, the vector of predicted values $\hat{Y}$ corresponding to the observed Y is

$$\hat{Y} = X\,\hat{\beta} = V\,Y , \tag{8}$$

where $V = X(X^T X)^{-1} X^T$ denotes leverage.

Lemma 1: $V \in R^n$ is a projection if V is idempotent and symmetric.

Therefore, we can proof Eq. (7) from the viewpoint of the projection

$$\mathrm{E}(\hat{Y}) = \mathrm{E}(V\,Y) = V\,\mathrm{E}(Y) = V\,X\,\beta = X\,\beta \tag{9}$$

In similar viewpoint, we can readily obtain the expected value and the variance of residuals R

$$\mathrm{E}(R) = \mathrm{E}(Y) - \mathrm{E}(\hat{Y}) = X\,\beta - X\,\beta = 0 \tag{10}$$

$$\begin{aligned} &Var(R) = Var(Y - VY) = Var((I-V)Y) = \\ &(I-V)Var(Y)(I-V)^T = (I-V)\sigma^2 \end{aligned} \tag{11}$$

where $(I-V)$ is still a projection. Fig. 1 shows the relationship between the projection V and the projection $(I-V)$.

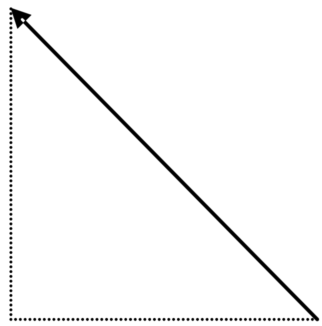

Fig. 1. The relationship between two projection vectors V and $(I-V)$.

The property of the least squares estimators is

$$Var(\hat{\beta}) = \mathrm{E}(\hat{\beta}-\beta)(\hat{\beta}-\beta)^T = \sigma^2 (X^T X)^{-1}, \tag{12}$$

Hereafter, in simple regression analysis, variation in Y can be regarded as the sum of what x_i can explain and the resultant of errors in the following for $i = 1,2,\cdots,n$

$$\sum_{i=1}^{n}(y_i - \bar{y})^2 = \hat{\beta}_1^2 \sum_{i=1}^{n}(x_i - \bar{x})^2 + \sum_{i=1}^{n}(y_i - \hat{y}_i)^2 \tag{13}$$

where $\bar{x}$ and $\bar{y}$ are the mean of x_i and y_i.

Similarly, in multiple regression model ($i = 1,2,\cdots,n$), Eq. (13) will be

$$\begin{aligned} &Y^T\left(I - \frac{1}{n}ee^T\right)Y = \\ &Y^T\left(V - \frac{1}{n}ee^T\right)Y + Y^T(I-V)Y \end{aligned}, \tag{14}$$

where $e = \begin{bmatrix} 1 & \cdots & 1 \\ \vdots & \ddots & \\ 1 & & 1 \end{bmatrix}_{n \times n}$.

Moreover, a typical S-curve figure is shown in Fig. 2. The x-axis and y-axis denote project duration and complete progress, respectively.

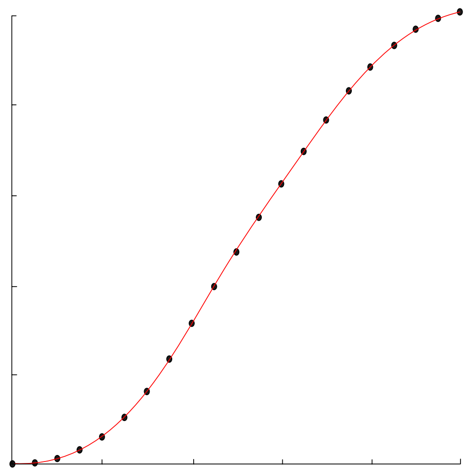

Fig.2. Typical S-curve figure.

Miskawi [6] proposed an S-curve equation which can be used in a variety of applications related to project control. The S-curve model is of the following form:

$$\begin{aligned} P = &\\ &\frac{3^T}{2}\sin\left[\frac{\pi(1-T)}{2}\right]\sin(\pi T)\log\left(\frac{T + (1.5 - T_p)}{T_p - T}\right) \\ &-2T^3 + 3T^2 \end{aligned} \tag{15}$$

where P denotes percentage completion of a project or an activity; T denotes time at any point of the duration of a project or an activity; T_P is shape factor.

Figure 3 is plotted with various values of T_P between T = 0 and T = 100% duration and the envelope of curves for T_P = 0 and T_P = 100% in Eq. (15).

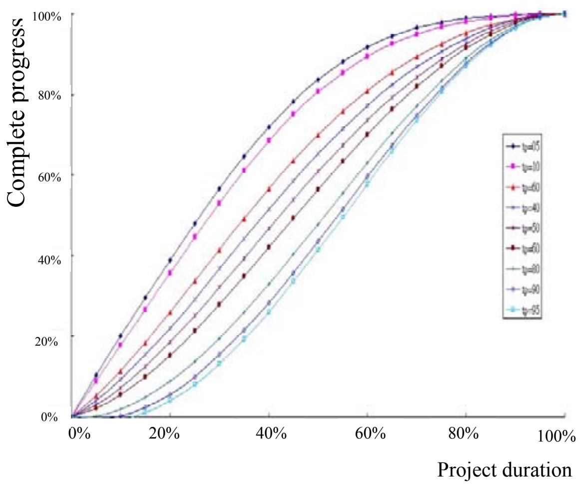

Fig.3. Miskawi S-curve figure.

Before introducing fuzzy S-curve regression, we give some relative definitions and operations about fuzzy numbers in the following.

III. FUZZY SET THEORY

Definition 1: Let R is a real number set. A fuzzy set $\tilde{A}$ on R is said to be a fuzzy number if the following conditions are satisfied:

(1) $\exists x_0 \in R$, such that $\mu_{\tilde{A}}(x_0)=1$; and membership function $\mu_{\tilde{A}}(x)$ is upper semi continuous; and

(2) $\forall \alpha \in (0,1]$, $A_\alpha \equiv \{x \mid \mu_{\tilde{A}}(x) \ge \alpha, x \in R\}$ is a convex set on R,

where x_0 is the mean value of $\tilde{A}$ and A_α is a crisp set. The convex set means that $\forall x \in [x_1, x_2]$, $f(x) \ge \min(f(x_1), f(x_2))$.

Definition 2: Extended Operations for LR-Representation of Fuzzy Sets

A triangular fuzzy number $\tilde{A}$ denoted by (m,β,γ) is defined as

$$\mu_{\tilde{A}}(x)=\begin{cases} 1-\dfrac{|m-x|}{\beta} & \text{if } m-\beta \le x \le m \\ 1-\dfrac{|x-m|}{\gamma} & \text{if } m+\gamma \ge x \ge m \\ 0 & \text{otherwise,} \end{cases} \tag{16}$$

where $m \in R$ is the center and $\beta>0$ is the left spread, $\gamma>0$ is the right spread of $\tilde{A}$.

If $\beta = \gamma$, then the triangular fuzzy number is called a symmetric triangular fuzzy number and denoted by (m,β).

Evidently for any $\forall \alpha \in (0,\alpha]$ the α-level set, $\tilde{A}_\alpha$, will be expressed as a closed interval $[p,q]$. Arithmetic operations about closed intervals can be obtained by use of the technique such as that in Zadeh [10]. Because they are proposed in several related references, it is not repeated here.

In the next section, the concept of a so-called Takagi-Sugeno fuzzy model is utilized in fuzzy inference engine to establish a fuzzy S-curve regression model. Based on this proposed regression methodology, data clusters are distributed into two overlapping clusters. Then, a T-S fuzzy model is then constructed based on the regression curves obtained for the two different data clusters.

IV. S-CURVE REGRESSION VIA TAKAGI-SUGENO FUZZY MODEL

For the curve fitting problems of the phenomena in civil construction management, we are used to choosing the following polynomial equation when k order curve fitting is adopted [8]:

$$y = a_k x^k, \tag{17}$$

by choosing the order k we can represent nonlinear relations. Parameters are determined so that the distance (or error) between an observed data point and its corresponding point on the polynomial will be minimal.

However, the S-curve fitting model by data of large-scale engineering must be different with that of small-scale engineering. In order to let an S-curve model be generally used in capital management for construction firms, Takagi-Sugeno fuzzy model is utilized to develop a practical model. That is to say, the fuzzy regression curve, obtained for project control of large-scale or small-scale engineering, is smoothly connected by a Takagi-Sugeno fuzzy model.

The Takagi-Sugeno fuzzy model was developed primarily from the pioneering work of Takagi and Sugeno [11], to represent the nonlinear relation of multiple input and output data, according to the format of fuzzy reasoning. In this paper, we distribute the data clusters of cost into two overlapping regions "low cost and high cost" such as shown by the membership functions of fuzzy sets *low* and *high* in Fig. 4.

We both apply polynomial regression curves for the respective cost clusters. Therefore, the *i*th rule of fuzzy inference is described by a set of fuzzy IF-THEN rules in the following form:

Rule 1: IF x is *low* THEN $y_1 = a_{1k} x_1^k$

Rule 2: IF x is *high* THEN $y_2 = a_{2k} x_2^k$

(18)

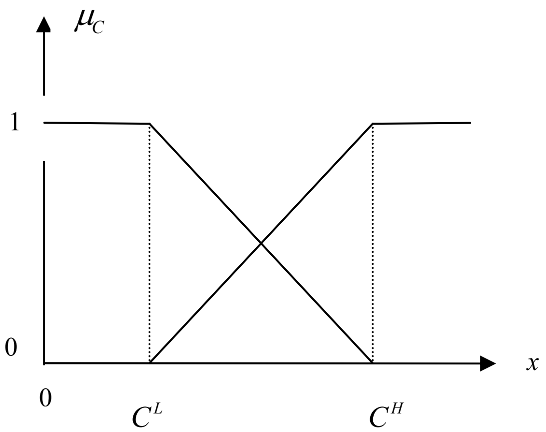

Fig.4. Fuzzy sets to represent low and high cost.

where in this case x, input, represents the cost and y_i (i = 1, 2), output, stands for progress of work. $i = 1, 2, \Lambda, r$; in which r is the number of IF-THEN rules and x is the premise variable. Using the center of gravity defuzzification, product inference, and single fuzzifier, the final output is inferred as follows:

$$y = \frac{\sum_{i=1}^{r} w_i y_i}{\sum_{i=1}^{r} w_i} = \sum_{i=1}^{r} h_i y_i \tag{19}$$

It is assumed that $w_i \geq 0$, $i = 1, 2, \Lambda, r$; $\sum_{i=1}^{r} w_i > 0$. Therefore, $h_i \geq 0$ and $\sum_{i=1}^{r} h_i = 1$.

V. EXAMPLES

The observed data, normalized and represented by percentage, are given in Table 1, which is taken from Department of Rapid Transit Systems, Taipei City Government. These data were processed via above mentioned method. Data are divided into two parts including percentage completion of a project or an activity (Y) and duration time of a project or an activity (X). (X1, Y1) denotes the observed data of the first metro bid and (X2, Y2) means the observed data of the second metro bid. The data are normalized and transformed into the rate of percentage. In equation (1), if βi equals to zero, there will be no relationship between X and Y. Thus, we made the assumption for calculating every quantity of parameters, in which the values are shown in Table 2. Leverage and residue were calculated for the sake of verifying obtained data. The aim of examining these values of leverage and residue is to observe whether data distribution correspond to normality. Assuming that data were able to satisfactorily fit in with the normal distribution, the T value determined by residue and leverage was further being considered and summarized in Table 3. Plotting the graph in Fig. 5, the abscissa means *u* and the ordinate means t value, where *u* is data from the population. Each population is certainly conformed as a framework of normal distribution. From the residual plot, it is slightly as close to linear as might be desired. There seems to be a rough linear with an angular magnitude of 45 degrees, so the data set is considered to be normally distributed.

Cook's distance measures how much the unusual values effect when they are dropped. Similarly, Table 4 and Fig. 6 could be plotted by the data 2. These scatterplot clearly indicate that there is a positive association between X and Y. For this reason, the regression curve, obtained for project control of data 1 and data 2, would be smoothly connected by a Takagi-Sugeno fuzzy model by the following procedure.

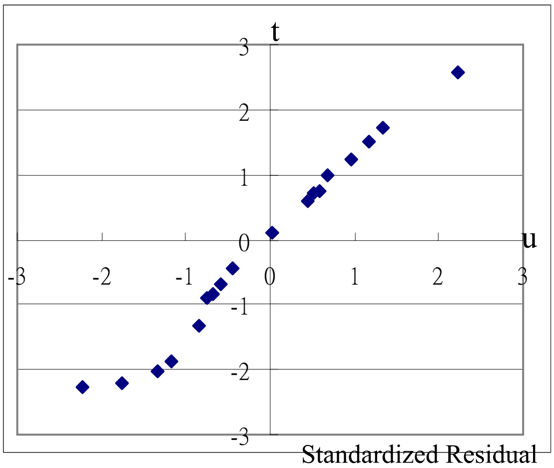

Fig. 5. Normal probability plot of t values versus standardized residual for various X1 and Y1.

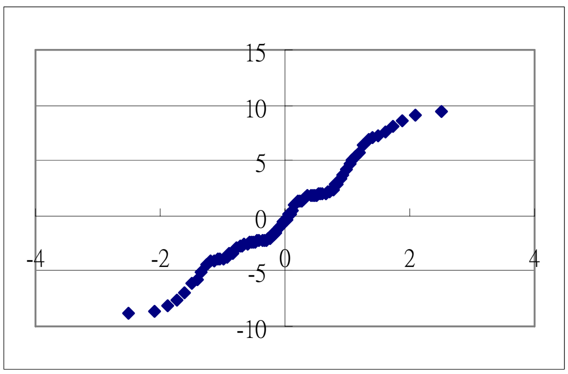

Fig. 6. Normal probability plot of t values versus standardized residual for various X2 and Y2.

The total cost of data 1 (X1, Y1) and data 2 (X2, Y2) are 1129589930 (= C^L) dollars and 5344703689 (= C^H) dollars, respectively. The polynomial regression models of rule 1 and rule 2 are given by data 1 and data 2 in Fig. 7 and these two models are foundation to establish another S-curve model. For example, if we have some new capital cost which is in the middle of these two engineering projects we will obtain the fuzzy inference with $w_1 = w_2 = 0.5$, and S-curve via T-S fuzzy model in Fig.7.

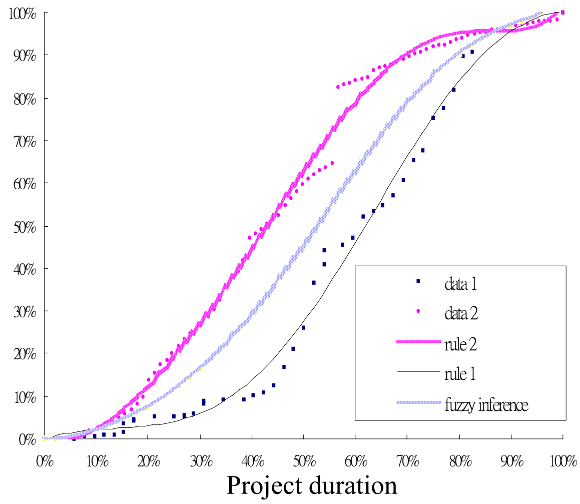

Fig.7. S-curve via T-S fuzzy model.

VI. CONCLUSIONS

In this paper, the data analysis by regression model with statistical method was used to deal with the project control problems in construction management. Furthermore, an S-curve regression method is proposed via T-S fuzzy models to curve fitting problems. The present paper, finally, attempts to develop a practical model for construction firms to rationalize the amount of cash and current assets, which should be possessed in any point of time. For this reason, this regression curve, obtained for project control of large-scale or small-scale engineering, is smoothly connected by a Takagi-Sugeno fuzzy model. The proposed method was demonstrated in a case study with statistical analysis.

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APPENDIX

Proof: A proof of Eq. (13) is given as follows: The following equality is satisfied in general

$$\sum_{i=1}^{n}\left(y_i-\bar{y}\right)^2=\sum_{i=1}^{n}\left(y_i-\hat{y}_i+\hat{y}_i-\bar{y}\right)^2 \qquad \text{(A1)}$$

$$\begin{aligned}&=\sum_{i=1}^{n}\left(y_i-\hat{y}_i\right)^2+\sum_{i=1}^{n}\left(\hat{y}_i-\bar{y}\right)^2\\&\quad+2\sum_{i=1}^{n}\left(y_i-\hat{y}_i\right)\left(\hat{y}_i-\bar{y}\right)\end{aligned} \qquad \text{(A2)}$$

$$\begin{aligned}&=\sum_{i=1}^{n}\left(y_i-\hat{y}_i\right)^2+\sum_{i=1}^{n}\left[\left(\hat{\beta}_0+\hat{\beta}_1x_i\right)-\bar{y}\right]^2\\&\quad+2\sum_{i=1}^{n}\left(y_i-\hat{y}_i\right)\left[\left(\hat{\beta}_0+\hat{\beta}_1x_i\right)-\bar{y}\right]\end{aligned} \qquad \text{(A3)}$$

$$\begin{aligned}&=\sum_{i=1}^{n}\left(y_i-\hat{y}_i\right)^2+\sum_{i=1}^{n}\left[\left(\bar{y}-\hat{\beta}_1\bar{x}\right)+\hat{\beta}_1x_i-\bar{y}\right]^2\\&\quad+2\hat{\beta}_1\sum_{i=1}^{n}\left(x_i-\bar{x}\right)\left(y_i-\hat{y}_i\right)\end{aligned} \qquad \text{(A4)}$$

$$\begin{aligned}&=\sum_{i=1}^{n}\left(y_i-\hat{y}_i\right)^2+\hat{\beta}_1^2\sum_{i=1}^{n}\left(x_i-\bar{x}\right)^2\\&\quad+2\hat{\beta}_1\sum_{i=1}^{n}x_i\left(y_i-\hat{y}_i\right)-2\hat{\beta}_1\sum_{i=1}^{n}\bar{x}\left(y_i-\hat{y}_i\right)\end{aligned} \qquad \text{(A5)}$$

Based on the following properties

$\sum_{i=1}^{n}\hat{y}_i \Big/ n=\bar{y}$ and

$$\frac{\partial S\left(\beta_0,\beta_1\right)}{\partial\beta_1}=\sum_{i=1}^{n}2\left(y_i-\beta_0-\beta_1x_i\right)\left(-x_i\right)^2=0. \qquad \text{(A6)}$$

Hence,

$$\hat{\beta}_1\sum_{i=1}^{n}\bar{x}\left(y_i-\hat{y}_i\right)=0,\ \hat{\beta}_1\sum_{i=1}^{n}x_i\left(y_i-\hat{y}_i\right)=0 \qquad \text{(A7)}$$

From Eq. (A1) and Eqs. (A5)-(A6), the proof of Eq. (13) is thereby completed.

Table 1a. The observed data of the first metro bid

Valuation times	Valuation month	Accumulative valuation total	X1	Y1
To begin construction	84/9	0	0.00%	0.00%
1	84/12	1,196,429	5.75%	0.11%
2	85/1	7,484,820	7.71%	0.66%
3	85/2	9,108,509	9.67%	0.81%
4	85/3	11,400,844	11.50%	1.01%
5	85/4	12,655,783	13.46%	1.12%
6	85/5	40,841,449	15.35%	3.62%
7	85/6	52,346,329	17.31%	4.63%
8	85/8	60,797,648	21.16%	5.38%
9	85/10	60,995,847	25.02%	5.40%
10	85/11	62,916,342	26.97%	5.57%
11	85/12	67,254,060	28.87%	5.95%
12	86/1	100,206,292	30.83%	8.87%
13	86/3	102,784,371	34.55%	9.10%
14	86/5	103,468,121	38.41%	9.16%
15	86/6	114,992,782	40.37%	10.18%
16	86/7	122,744,984	42.26%	10.87%
17	86/8	143,160,179	44.22%	12.67%
18	86/9	190,468,065	46.18%	16.86%
19	86/10	239,013,770	48.07%	21.16%
20	86/11	294,623,573	50.03%	26.08%
21	86/12	412,771,998	51.93%	36.54%
22	87/1	500,498,449	53.89%	44.31%
23	87/3	513,633,770	57.61%	45.47%
24	87/4	534,698,236	59.57%	47.34%
25	87/5	590,543,842	61.47%	52.28%
26	87/6	602,157,035	63.42%	53.31%
27	87/7	618,978,284	65.32%	54.80%
28	87/8	646,144,725	67.28%	57.20%
29	87/9	685,602,225	69.24%	60.69%
30	87/10	736,566,872	71.13%	65.21%
31	87/11	765,215,105	73.09%	67.74%
32	87/12	850,864,754	74.98%	75.33%
33	88/1	874,446,506	76.94%	77.41%
34	88/2	924,641,322	78.90%	81.86%
35	88/3	1,012,565,332	80.67%	89.64%
36	88/4	1,026,516,972	82.63%	90.88%
37	89/1	1,129,589,930	100.00%	100.00%

Table 1b. The observed data of the second metro bid

Valuation times	Valuation month	Accumulative valuation total	X2	Y2
To begin construction	80/5	0	0.00%	0.00%
1	80/12	74,144,882	7.57%	1.39%
2	81/1	98,654,310	8.73%	1.85%
3	81/2	127,656,501	9.90%	2.39%
4	81/3	152,724,294	10.98%	2.86%
5	81/4	178,169,197	12.14%	3.33%
6	81/5	213,580,145	13.27%	4.00%
7	81/6	255,800,965	14.43%	4.79%
8	81/7	309,414,337	15.55%	5.79%
9	81/8	378,460,468	16.72%	7.08%
10	81/9	442,320,349	17.88%	8.28%
11	81/10	536,571,926	19.00%	10.04%
12	81/11	742,336,497	20.16%	13.89%
13	81/12	822,436,935	21.29%	15.39%
14	82/1	931,406,994	22.45%	17.43%
15	82/2	987,661,260	23.61%	18.48%
16	82/3	1,067,296,113	24.66%	19.97%
17	82/4	1,162,841,603	25.82%	21.76%
18	82/5	1,257,354,425	26.95%	23.53%
19	82/6	1,320,772,152	28.11%	24.71%
20	82/7	1,361,222,085	29.24%	25.47%
21	82/8	1,447,318,620	30.40%	27.08%
22	82/9	1,541,962,339	31.56%	28.85%
23	82/10	1,629,858,563	32.68%	30.49%
24	82/11	1,750,420,395	33.85%	32.75%
25	82/12	1,898,633,071	34.97%	35.52%
26	83/1	2,024,377,919	36.13%	37.88%
27	83/2	2,103,877,262	37.29%	39.36%
28	83/3	2,232,694,031	38.34%	41.77%
29	83/4	2,519,705,280	39.51%	47.14%
30	83/5	2,571,581,797	40.63%	48.11%
31	83/6	2,629,026,261	41.79%	49.19%
32	83/7	2,700,041,284	42.92%	50.52%
33	83/8	2,747,993,528	44.08%	51.42%
34	83/9	2,805,024,504	45.24%	52.48%
35	83/10	2,935,567,858	46.36%	54.92%
36	83/11	3,024,292,651	47.53%	56.58%
37	83/12	3,107,403,233	48.65%	58.14%
38	84/1	3,199,752,852	49.81%	59.87%
39	84/2	3,258,272,603	50.97%	60.96%
40	84/3	3,310,923,072	52.02%	61.95%
41	84/4	3,362,164,118	53.19%	62.91%
42	84/5	3,397,079,648	54.31%	63.56%
43	84/6	3,460,527,952	55.47%	64.75%
44	84/7	4,403,859,399	56.60%	82.40%
45	84/8	4,446,190,555	57.76%	83.19%
46	84/9	4,467,998,609	58.92%	83.60%
47	84/10	4,491,507,050	60.04%	84.04%
48	84/11	4,515,791,597	61.21%	84.49%
49	84/12	4,533,959,483	62.33%	84.83%
50	85/1	4,619,987,215	63.49%	86.44%
51	85/2	4,657,660,695	64.66%	87.15%
52	85/3	4,674,670,042	65.74%	87.46%
53	85/4	4,692,964,462	66.90%	87.81%
54	85/5	4,720,798,078	68.03%	88.33%
55	85/6	4,761,901,206	69.19%	89.10%
56	85/7	4,799,763,766	70.31%	89.80%

Table 1b. (Continued.)

Valuation times	Valuation month	Accumulative valuation total	X2	Y2
57	85/8	4,831,040,196	71.48%	90.39%
58	85/9	4,856,949,225	72.64%	90.87%
59	85/10	4,893,195,670	73.76%	91.55%
60	85/11	4,915,114,734	74.93%	91.96%
61	85/12	4,931,559,544	76.05%	92.27%
62	86/1	4,946,721,205	77.21%	92.55%
63	86/2	4,980,138,304	78.37%	93.18%
64	86/3	5,001,588,959	79.42%	93.58%
65	86/4	5,034,731,706	80.58%	94.20%
66	86/5	5,064,029,324	81.71%	94.75%
67	86/6	5,080,106,114	82.87%	95.05%
68	86/7	5,094,461,369	84.00%	95.32%
69	86/8	5,104,146,881	85.16%	95.50%
70	86/9	5,127,692,651	86.32%	95.94%
71	86/10	5,130,617,000	87.44%	95.99%
72	86/11	5,146,615,889	88.61%	96.29%
73	86/12	5,167,787,282	89.73%	96.69%
74	87/1	5,182,884,070	90.89%	96.97%
75	87/2	5,195,927,731	92.05%	97.22%
76	87/3	5,198,499,181	93.10%	97.26%
77	87/4	5,229,133,485	94.27%	97.84%
78	87/5	5,230,091,831	95.39%	97.86%
79	87/6	5,233,925,940	96.55%	97.93%
80	87/7	5,244,739,340	97.68%	98.13%
81	87/8	5,261,551,354	98.84%	98.44%
82	87/9	5,344,703,689	100.00%	100.00%

Table 2. The procedure for determining parameters on the relation between X1 and Y1

mean(x)	0.46
mean(y)	0.33
Sxx	2.411289275
Syy	3.614226574
Sxy	2.7955069
β1	
β0	-0.201239847
SSr	3.240946206
SSe	0.373280369
MSe	0.010368899
s	0.101827792
F	312.564156
F(1,36,0.05)	4.113161367
H0: β1 = 0	
H1: β1≠0	
∵ F＞F(1,36,0.05)	
∴ reject H0 ＝＞ Y1 is dependent upon X1	

Table 3. Summary of various statistic indexes of data 1

predicted	residual	vii	ti	rank	fi(inverse)-mu(i)	Cook's distance
-0.20124	0.20	11.384491086332700%	2.579054	38	2.221518	0.427262
-0.13458	0.14	9.330570761130920%	1.719014	35	1.327902	0.152046
-0.11185	0.12	8.693122040707480%	1.495552	34	1.182917	0.106474
-0.08913	0.10	8.087536772249210%	1.223549	32	0.950014	0.065865
-0.06792	0.08	7.550881324256510%	0.978885	29	0.67449	0.039132
-0.04519	0.06	7.006909567506390%	0.705501	27	0.516847	0.018752
-0.02328	0.06	6.512542175900590%	0.742169	28	0.593821	0.019185
-0.00056	0.05	6.031159342653480%	0.583166	26	0.442822	0.010914
0.044077	0.01	5.178351646110030%	0.120465	20	0.032988	0.000396
0.088827	-0.03	4.446750804529420%	-0.42983	13	-0.44282	0.004299
0.111434	-0.06	4.124144943600130%	-0.68671	11	-0.59382	0.010142
0.133462	-0.07	3.840147495847290%	-0.90994	9	-0.75981	0.016533
0.156185	-0.07	3.578557453706840%	-0.82913	10	-0.67449	0.012757
0.199313	-0.11	3.169698041500240%	-1.32793	8	-0.85106	0.028862
0.244063	-0.15	2.866792342620700%	-1.8663	5	-1.18292	0.0514
0.266786	-0.16	2.760292775861720%	-2.01849	4	-1.3279	0.057828
0.288698	-0.18	2.687773567461660%	-2.20133	2	-1.75683	0.066921
0.311421	-0.18	2.643862924205690%	-2.25858	1	-2.22152	0.069265
0.334144	-0.17	2.631815732914890%	-2.02398	3	-1.5079	0.055363
0.356055	-0.14	2.650375600859130%	-1.76631	6	-1.05928	0.04247
0.378779	-0.12	2.700917333071330%	-1.44295	7	-0.95001	0.028898
0.400806	-0.04	2.780327133444750%	-0.43321	12	-0.51685	0.002684
0.423529	0.02	2.893620357792430%	0.239601	22	0.165664	0.000855
0.466657	-0.01	3.196274084022500%	-0.14661	15	-0.30134	0.000355
0.48938	-0.02	3.401906291616170%	-0.19615	14	-0.37115	0.000677
0.511407	0.01	3.631658899757340%	0.140012	21	0.099108	0.000369
0.534014	0.00	3.898592425295710%	-0.01125	18	-0.09911	2.57E-06
0.556042	-0.01	4.189017969485710%	-0.09912	16	-0.23296	0.000215
0.578765	-0.01	4.519990592717870%	-0.08352	17	-0.16566	0.000165
0.601488	0.01	4.882826667915200%	0.066947	19	-0.03299	0.000115
0.6234	0.03	5.262881114258720%	0.355738	23	0.232964	0.003515
0.646123	0.03	5.688306112959060%	0.388552	24	0.301337	0.004553
0.668034	0.09	6.128714164109050%	1.06172	31	0.851058	0.036798
0.690757	0.08	6.616728086312390%	1.040482	30	0.759809	0.038354
0.71348	0.11	7.136605460480900%	1.316022	33	1.059277	0.066549
0.734001	0.16	7.633466516961280%	2.038582	37	1.756826	0.171725
0.756724	0.15	8.213981991043090%	1.915025	36	1.507905	0.164095
0.958101	0.04	14.751111101504100%	0.547466	25	0.371149	0.025931

Table 4. The procedure for determining parameters on the relation between X2 and Y2

mean(x)	0.53
mean(y)	0.60
Sxx	6.188857
Syy	9.827913
Sxy	7.574913
β1	1.22396
β0	-0.0546
SSr	9.27139
SSe	0.556522
MSe	0.006871
s	0.082889
F	1349.421
F(1,81,0.05)	3.95886
H0: β1 = 0	
H1: β1≠0	
∵ F＞F(1,81,0.05)	
∴ reject H0 ＝＞ Y2 is dependent upon X2	

Table 5. Summary of various statistic indexes of data 2

predicted	residual	vii	ti	rank	fi(inverse)-mu(i)	Cook's distance
-0.0546	0.05	5.767156503401010%	2.82532	66	0.803498	0.244267
0.038058	-0.02	4.559836975644180%	-1.24227	38	-0.1211	0.036866
0.052256	-0.03	4.391194563307480%	-1.73429	35	-0.213	0.069071
0.066577	-0.04	4.225503189478150%	-2.19069	30	-0.37072	0.105866
0.079796	-0.05	4.076483717794940%	-2.62593	22	-0.64632	0.14652
0.093993	-0.06	3.920624280536490%	-3.11057	18	-0.8035	0.197413
0.107824	-0.07	3.772976915463430%	-3.47336	16	-0.88995	0.236514
0.122022	-0.07	3.625701939474530%	-3.79298	15	-0.93576	0.270622
0.13573	-0.08	3.487631536573090%	-3.9799	12	-1.08684	0.286195
0.150051	-0.08	3.347726509022280%	-4.04959	11	-1.14306	0.284008
0.164249	-0.08	3.213384455033370%	-4.15901	10	-1.20316	0.287143
0.177957	-0.08	3.087801011304330%	-3.95773	13	-1.03385	0.249536
0.192155	-0.05	2.962005931854530%	-2.71584	20	-0.72267	0.11257
0.205986	-0.05	2.843645398095070%	-2.65459	21	-0.684	0.103126
0.220184	-0.05	2.726434779914820%	-2.33709	27	-0.46972	0.076546
0.234382	-0.05	2.613572622465010%	-2.52398	23	-0.60954	0.085482
0.247233	-0.05	2.515162391927400%	-2.41848	25	-0.53831	0.075454
0.261431	-0.04	2.410584801903660%	-2.22892	28	-0.43626	0.061359
0.275262	-0.04	2.312893029395440%	-2.03115	34	-0.244	0.048839
0.28946	-0.04	2.216899900641260%	-2.15197	32	-0.30675	0.052496
0.303291	-0.05	2.127570577473210%	-2.46738	24	-0.57356	0.066171
0.317489	-0.05	2.040161909988570%	-2.36974	26	-0.50372	0.058477
0.331686	-0.04	1.957101703234370%	-2.19106	29	-0.40327	0.047915
0.345395	-0.04	1.881031767180230%	-2.0537	33	-0.27524	0.040428
0.359715	-0.03	1.805895084460960%	-1.63317	37	-0.15158	0.024527
0.373424	-0.02	1.738113593770520%	-0.92353	40	-0.06044	0.007543
0.387621	-0.01	1.672184822824990%	-0.44691	42	5.47E-10	0.001698
0.401819	-0.01	1.610604512609880%	-0.41627	43	0.030205	0.001418
0.414671	0.00	1.558613195517490%	0.153366	45	0.090725	0.000186
0.428991	0.04	1.504876911615890%	2.14665	63	0.683999	0.035203
0.4427	0.04	1.457581272785100%	1.943288	59	0.538312	0.027929
0.456898	0.04	1.412869991265620%	1.77093	55	0.403274	0.022473
0.470728	0.03	1.373496260662420%	1.743729	54	0.370724	0.021172
0.484926	0.03	1.337369440412500%	1.480523	52	0.306747	0.014856
0.499124	0.03	1.305591080893000%	1.298348	50	0.244003	0.01115
0.512833	0.04	1.279034652513750%	1.838745	57	0.469721	0.021902
0.527153	0.04	1.255621901919150%	1.953774	60	0.573561	0.02427
0.540861	0.04	1.237353918903610%	2.049216	62	0.64632	0.026305
0.555059	0.04	1.222706995192790%	2.20586	64	0.722675	0.030116
0.569257	0.04	1.212408532212390%	2.039057	61	0.609537	0.025514
0.582109	0.04	1.206836128565220%	1.889821	58	0.50372	0.021814
0.596429	0.03	1.204823776788280%	1.651231	53	0.338562	0.016625
0.610137	0.03	1.207041645632380%	1.286926	49	0.213002	0.010118
0.624335	0.02	1.213612211347610%	1.170823	48	0.182205	0.00842
0.638166	0.19	1.224194073309260%	9.393195	83	2.510731	0.54676
0.652364	0.18	1.239349100294050%	9.075554	82	2.095296	0.516804
0.666562	0.17	1.258852588009270%	8.565949	81	1.879025	0.467733
0.68027	0.16	1.281809667304910%	8.096306	80	1.726056	0.425569
0.694591	0.15	1.310120848834980%	7.600871	79	1.605273	0.383474
0.708299	0.14	1.341366373494330%	7.080718	77	1.416509	0.340829
0.722497	0.14	1.378001297018220%	7.17825	78	1.504199	0.359984
0.736817	0.13	1.419356891255160%	6.814434	76	1.338533	0.334295
0.750036	0.12	1.461457697786990%	6.303809	75	1.267941	0.294684
0.764234	0.11	1.510875596389120%	5.763864	74	1.203157	0.254823
0.778065	0.11	1.563196670559560%	5.328394	73	1.143064	0.225434
0.792263	0.10	1.621199030431240%	5.000857	72	1.086838	0.20606
0.805971	0.09	1.681327434567610%	4.662517	71	1.033853	0.185878

Table 5. (Continued.)

predicted	residual	vii	ti	rank	fi(inverse)-mu(i)	Cook's distance
0.820291	0.08	1.748469375083080%	4.23736	70	0.983622	0.159764
0.834489	0.07	1.819404656954750%	3.762423	69	0.935758	0.131162
0.848198	0.07	1.892020020263520%	3.413438	68	0.889948	0.112351
0.862518	0.06	1.972206373485870%	2.896267	67	0.845932	0.084382
0.876226	0.05	2.053110182158350%	2.358988	65	0.762464	0.058323
0.890424	0.04	2.141176899838690%	1.781234	56	0.436257	0.034711
0.904622	0.03	2.233592078249450%	1.380809	51	0.275239	0.02178
0.917474	0.02	2.320993229309790%	0.931509	47	0.15158	0.010309
0.931672	0.01	2.421692975146640%	0.525251	46	0.121097	0.003423
0.945502	0.00	2.523969641881920%	0.101641	44	0.060437	0.000134
0.9597	-0.01	2.633253848988320%	-0.4684	41	-0.0302	0.002967
0.973531	-0.02	2.743892965063770%	-1.03566	39	-0.09073	0.01513
0.987729	-0.03	2.861761633439720%	-1.66821	36	-0.18221	0.040994
1.001927	-0.04	2.983978762546100%	-2.16898	31	-0.33856	0.072349
1.015635	-0.06	3.106107633529760%	-2.84443	19	-0.76246	0.129683
1.029956	-0.07	3.238017918876780%	-3.4245	17	-0.84593	0.196218
1.043664	-0.08	3.368435235224150%	-3.92294	14	-0.98362	0.268227
1.057862	-0.09	3.507783800139200%	-4.50867	9	-1.26794	0.369494
1.07206	-0.10	3.651480825784680%	-5.11072	8	-1.33853	0.494947
1.084911	-0.11	3.785300890290230%	-5.75197	7	-1.41651	0.650823
1.099232	-0.12	3.938611574454910%	-6.19328	6	-1.5042	0.78633
1.11294	-0.13	4.089514742661920%	-6.89107	5	-1.60527	1.012391
1.127138	-0.15	4.250080797003030%	-7.58981	4	-1.72606	1.278469
1.140969	-0.16	4.410675505643330%	-8.20406	3	-1.87902	1.55283
1.155167	-0.17	4.579826021253990%	-8.78207	1	-2.51073	1.850855
1.169365	-0.17	4.753324997595080%	-8.71789	2	-2.0953	1.896448