中正嶺學報 第三十六卷 第一期 民國 96.11
JOURNAL OF C.C.I.T., VOL. 36, NO. 1, Nov, 2007

A Novel Closed-form Solution of Pursuit Trajectory for Non-maneuvering Target

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ABSTRACT

The method for deriving the analytic solution in the pure pursuit course is presented. The novel closed form solution for graphing the trajectory of pursuit guidance are validated and compared with the classical Howe's method. The Howe's solution used polar coordinate and was applied in the tail chase scenario only. Our proposed pursuit guidance law uses rectangular coordinate and it can be applied in the head-on scenario. Besides, it can further predict the hitting position and arc length of the missile trajectory. Thus the proposed method is also suitable to many other conditions.

Keywords: Pursuit guidance, Analytical solution, Head-on, Guidance law, Closed-form solution.

追逐軌跡於目標無閃躲時之新解

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摘要

本文係研究純追逐導引軌跡之解析解法，經推導可得出一組新的封閉解，經與舊有導引律常用之 Howe 解法比對檢驗，Howe 解法之特點為使用極座標解析，僅能適用於尾追目標場景。本文解法則適用於直角座標解析，並增加了迎頭追逐場景，此外並可計算出撞擊點與軌跡弧長，本文之解析方法可更廣泛運用於各導引追逐場景。

關鍵詞：追逐導引、解析解、迎頭、導引率、封閉解。

文稿收件日期 96.03.28；文稿修正後接受日期 96. 11.27
Manuscript received March 28, 2007; revised November 27, 2007

I. INTRODUCTION

In the missile guidance law design phase, one of the most straightforward means to assure an intercept is to keep the missile, which must have velocity superiority, pointed at the target. This is the principle of pursuit guidance[1] law which is an alternative terminal guidance law . The derivation of the pure pursuit course is provided in the textbooks by Locke[2] and Howe[3]. In 1955, Locke presented a closed form solution of pursuer range versus pursuer angular position. In addition, an advanced closed-form solution of pure pursuit trajectory for non-maneuvering targets was derived by Howe in 1965. His analytical procedure of the pursuit guidance law was frequently cited in the textbook and people. Since the Howe's solution is fundamentally a non-linear, anomalous behavior (e.g. head-on or singular) associated with polar coordinate. It is difficult to plot trajectories and compute the length of arc along missile trajectory from the Howe's solution without having an explicit relationship between instantaneous slant range, inclination angle of trajectory and the time. Due to this anomalous behavior, this paper has centered on the effect of the Cartesian coordinate system derivation upon trajectory computation convergence and simplicity. In this paper, a novel closed-form solution of the differential equations describing the pursuit trajectory of the pursuer for non-maneuvering target is discussed. This study also shows that the scattered published results in the field of pursuit guidance theory can be derived directly from a general differential equation derived from classical geometry.

II. Howe's closed-form solution

From Fig. 1, we have a fundamental equation of guidance as:

$$\dot{R} = V_T \cos(\beta - \theta_T) - V_M \cos(\beta - \theta) \tag{1}$$

$$\dot{\beta} = -\frac{V_T \sin(\beta - \theta_T) - V_M \sin(\beta - \theta)}{R} \tag{2}$$

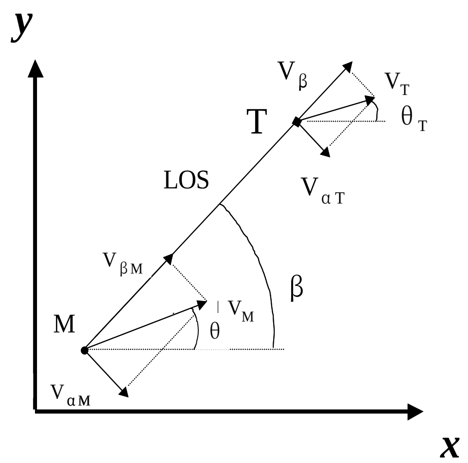

Fig. 1. Geometry of two-dimensional homing problem.

where θ_T is a function of time (forced function) and θ is dependent on a guidance law. For simplicity, we assume a non-maneuvering target with V_T = constant. Furthermore, let $\theta_T = 0$ since we can always reorient the (x , y) reference system of Fig. 1 [4]. For an ideal pursuit guidance $\theta = \beta$, and for $\theta_T = 0$, Eqs. (1) and (2) become

$$\dot{R} = V_T \cos\beta - V_M \tag{3}$$

$$\dot{\beta} = -\frac{V_T \sin\beta}{R} \tag{4}$$

The geometry required for deriving the pursuit course equations of motion is given in Fig. 2. Note that $\dot{\beta}$ is not zero unless $\beta = 0$ or π , i.e., unless the attack is a head-on or tail chase. Since $\dot{\theta} = \dot{\beta}$ in pursuit guidance, the missile will always have to turn during the attack unless it is a head-on or tail chase. By eliminating time from Eqs. (3) and (4), we can solve for β and hence the missile heading θ as a function of the range R .

Dividing Eq. (3) by (4), we obtain

$$\frac{dR}{R} = (-\cot\beta + \gamma \csc\beta) d\beta \tag{5}$$

where $\gamma = V_M / V_T$, the ratio of missile to target velocity. After integration, Eq. (5)

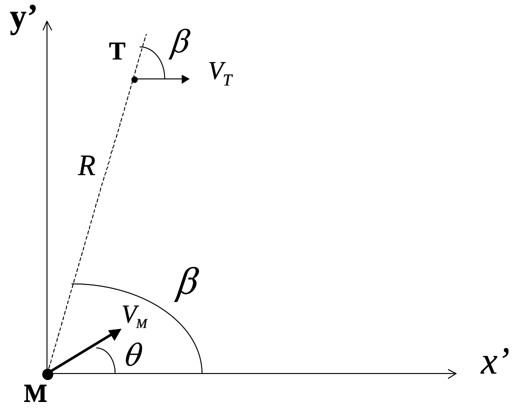

Fig. 2. Typical geometry of two-dimensional pursuit problem.

Becomes

$$\ln R = -\ln\left|\sin\beta\right| + \gamma \ln\left|\tan\frac{\beta}{2}\right| + const$$

Assuming that $0 \le \beta \le \pi$, we can drop the absolute-value signs and write

$$\frac{R\sin\beta}{(\tan\frac{\beta}{2})^{\gamma}} = \frac{R_0\sin\beta_0}{(\tan\frac{\beta_0}{2})^{\gamma}} = K = const \tag{6}$$

where R_0 and β_0 are initial values of range and line-of-sight angle, respectively. As the missile approaches the target, R approaches zero. Also from Eq. (6), it is evident that β must also approach zero for the left-hand side of Eq. (6) to keep R be constant. Thus we obtain an important conclusion that in ideal pursuit guidance, the trajectory always terminates in a tail chase with $\theta = \beta = 0$.

Substituting R from Eq. (6) into (4), we obtain

$$\dot{\beta} = -\frac{V_T}{K}\frac{\sin^2\beta}{(\tan\frac{\beta}{2})^{\gamma}} \tag{7}$$

Near the trajectory termination, when $\beta << 1$ and $\sin\beta \cong \beta$, $\tan\frac{\beta}{2} \cong \frac{\beta}{2}$, Eq. (7) becomes

$$\dot{\beta} \cong -\frac{V_T 2^{\gamma}}{K}\beta^{2-\gamma} \tag{8}$$

III. Novel closed-form solution

In the same geometry of two-dimensional homing problem as Fig. 2, the missile path is considered to be under continuous control throughout the trajectory. To simplify the discussion, let us restrict the problem to Cartesian coordinate, as shown in Fig. 3. Here we designate target and missile positions with respect to a two-dimensional inertial frame with coordinates, (x', y'). There are two techniques for deriving the differential equation of pursuit trajectory. Both techniques lead to a functional relation between the two variables x and y, not the variables R and β. This is the major difference between the Howe's solution and our proposed one.

Suppose that the curve of the pursuit trajectory whose length we wish to find is the graph of $x = x(y)$ from $y = 0$ to $y = y$. We calculate the arc length with the following equation [5,6]:

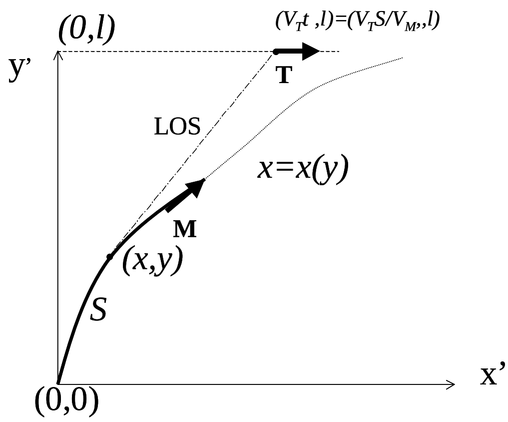

Fig. 3. Typical geometry of pursuit in Cartesian coordinate.

Fig. 3 shows the path of the missile that moved from point $(0,0)$ to point (x, y) and the path of the target that moved from point $(0,l)$ to point $(\frac{V_T}{V_M}S, l)$. In an ideal pursuit course, the tangent line of $x = x(y)$ passing through point $(x,y)\sqrt{a^2+b^2}$ is line-of-sight (LOS), and thus we have

$$\frac{dx}{dy} = \frac{\frac{V_T}{V_M}S - x}{l-y} \tag{10}$$

From this, we see

$$(l-y)\frac{dx}{dy} = \frac{S}{\gamma} - x \tag{11}$$

Differentiating Eqs. (9) and (11) by variable y, we obtain

$$\frac{dS}{dy} = \sqrt{1+(\frac{dx}{dy})^2} \tag{12}$$

$$(l-y)\frac{d^2x}{dy^2} = \frac{1}{\gamma}\frac{dS}{dy} \tag{13}$$

Substituting Eq. (12) into Eq. (13), we get

$$(l-y)\frac{d^2x}{dy^2} = \frac{1}{\gamma}\sqrt{1+(\frac{dx}{dy})^2} \tag{14}$$

If now we consider a general condition that the position of missile and target are given by $(x,y) = (x(t), y(t))$ and $(x_{T0}+V_T t, l)$, respectively, Here t is time to reach instantaneous position and x_{T0} is initial distance between missile and target in x'-direction. Note

$$\frac{dx}{dy} = \frac{dx/dt}{dy/dt} = \frac{x_{T0} \pm V_T t - x}{l-y} \tag{15}$$

From this, we see

$$(l-y)\frac{dx}{dy} = \pm V_T t - x \tag{16}$$

Differentiating Eq. (16) by variable y, we obtain

$$(l-y)\frac{d^2x}{dy^2} = \pm V_T\frac{dt}{dy} \tag{17}$$

From the definition of velocity, we have

$$V_M = \sqrt{(\frac{dx}{dt})^2 + (\frac{dy}{dt})^2} = \frac{dy}{dt}\sqrt{1+(\frac{dx}{dy})^2}$$

$$\text{Similarly, } \frac{dt}{dy} = \frac{1}{V_M}\sqrt{1+(\frac{dx}{dy})^2} \tag{18}$$

Substituting Eq. (18) into Eq. (17), we obtain the similar result as follows:

$$(l-y)\frac{d^2x}{dy^2} = \pm\frac{1}{\gamma}\sqrt{1+(\frac{dx}{dy})^2}$$
$$(+: outgoing, -: incoming) \tag{19}$$

Eq. (19) is an important differential equation to govern the behavior of pursuit course. Therefore, the missile path trajectories are a family of differential equations defined by Eq. (19) [7,8].

Assume that the missile is initially at rest at point (0,0). Then the initial condition of the governing equation is

$$x(0) = 0$$
$$x'(0) = 0$$

Then according to Eq. (14), we have

$$\frac{dx'}{\sqrt{1+(x')^2}} = \frac{1}{\gamma}\frac{dy}{(l-y)} \tag{20}$$

By integrating Eq. (20), we have

$$\ln[x' + \sqrt{1+(x')^2}] = \ln(l-y)^{-\frac{1}{\gamma}} + \ln c_1 \tag{21}$$

where c_1 is a constant, and then

$$x' + \sqrt{1+(x')^2} = c_1(l-y)^{-\frac{1}{\gamma}} \tag{22}$$

$$1+(x')^2 = c_1^{\,2}(l-y)^{-\frac{2}{\gamma}} -2c_1(l-y)^{-\frac{1}{\gamma}}x' + (x')^2 \qquad (23)$$

Let Eq. (23) expansions for $x(y)$ be

$$x' = x'(y) = \frac{c_1^{\,2}(l-y)^{-\frac{2}{\gamma}} - 1}{2c_1(l-y)^{-\frac{1}{\gamma}}} = \frac{1}{2}[c_1(l-y)^{-\frac{1}{\gamma}} - \frac{1}{c_1}(l-y)^{\frac{1}{\gamma}}] \qquad (24)$$

Solving for x' from $x'(0)=0$, we obtain

$$x'(0) = \frac{1}{2}(c_1 l^{-\frac{1}{\gamma}} - \frac{1}{c_1} l^{\frac{1}{\gamma}}) = 0, \quad c_1 = l^{\frac{1}{\gamma}}$$

By applying $c_1 = l^{\frac{1}{\gamma}}$,

Eq. (24) can be rewritten as

$$x' = x'(y) = \frac{1}{2}[l^{\frac{1}{\gamma}}(l-y)^{-\frac{1}{\gamma}} - l^{-\frac{1}{\gamma}}(l-y)^{\frac{1}{\gamma}}] \qquad (25)$$

Integrating Eq. (25), we obtain

$$x = x(y) = \frac{1}{2}[\frac{1}{1+1/\gamma} l^{-\frac{1}{\gamma}}(l-y)^{1+\frac{1}{\gamma}} - \frac{1}{1-1/\gamma} l^{\frac{1}{\gamma}}(l-y)^{1-\frac{1}{\gamma}}] + c_2 \qquad (26)$$

where c_2 is a constant.

By applying $x(0)=0$, Eq. (26) becomes

$$x(0) = \frac{1}{2}[\frac{1}{1+1/\gamma} l^{-\frac{1}{\gamma}}(l-y)^{1+\frac{1}{\gamma}} - \frac{1}{1-1/\gamma} l^{\frac{1}{\gamma}}(l-y)^{1-\frac{1}{\gamma}}] + c_2 = 0$$

, get $c_2 = \frac{1}{2}(\frac{1}{1-1/\gamma} - \frac{1}{1+1/\gamma})l$

Again, by substituting

$$c_2 = \frac{1}{2}(\frac{1}{1-1/\gamma} - \frac{1}{1+1/\gamma})l,$$

Eq. (26) can be converted into the form as follows:

$$x(y) = \frac{1}{2}[\frac{1}{1+1/\gamma} l^{-\frac{1}{\gamma}}(l-y)^{1+\frac{1}{\gamma}} - \frac{1}{1-1/\gamma} l^{\frac{1}{\gamma}}(l-y)^{1-\frac{1}{\gamma}}] + \frac{1}{2}(\frac{1}{1-1/\gamma} - \frac{1}{1+1/\gamma})l \qquad (27)$$

By applying the result of Eq. (27), when y closes to l and $l-y \to 0^+$, we note that

$$x \to \frac{1}{2}(\frac{1}{1-1/\gamma} - \frac{1}{1+1/\gamma})l = \frac{\gamma}{\gamma^2-1}l$$

Homing problem is inherently a "final value", i.e., nearly intercept. The missile hits the target at the point $(\frac{\gamma}{\gamma^2-1}l, l)$.

Since

$$V_T t = \frac{V_T}{V_M}S = \frac{S}{\gamma} = \frac{\gamma}{\gamma^2-1}l,$$

we can see

$$S = \frac{\gamma^2}{\gamma^2-1}l \qquad (28)$$

Eq. (28) could be derived by using the properties of arc length given in Eq. (9) readily. From Eq. (9) and Eq. (25), we deduce

$$S = \int_0^l \sqrt{1+(x')^2}\,dy = \int_0^l \{1 + \frac{1}{4}[(\frac{l-y}{l})^{\frac{1}{\gamma}} - (\frac{l}{l-y})^{\frac{1}{\gamma}}]^2\}^{\frac{1}{2}} dy$$

$$=\frac{1}{2}\int_0^l [l^{-\frac{1}{\gamma}}(l-y)^{\frac{1}{\gamma}} + l^{\frac{1}{\gamma}}(l-y)^{-\frac{1}{\gamma}}]dy$$

$$=\frac{1}{2}[l^{-\frac{1}{\gamma}}\frac{(l-y)^{\frac{1}{\gamma}+1}}{1/\gamma+1} + l^{\frac{1}{\gamma}}\frac{(l-y)^{-\frac{1}{\gamma}+1}}{-1/\gamma+1}]_{y=0}^{y=l}$$

$$=\frac{1}{2}(\frac{1}{1+1/\gamma}+\frac{1}{1-1/\gamma})l = \frac{\gamma^2}{\gamma^2-1}l \qquad (29)$$

Eq. (28) ≡ Eq. (29).

IV. Illustrative Example

In this section, we consider some realistic examples to illustrate the performance of the proposed skill and compare it to the Howe's closed form and the exact solution. The tests use three scenarios. Three types of target engagement: an outgoing target ($\beta_0 = \pi/2$), an outgoing target ($\beta_0 = \pi/4$) and an incoming target ($\beta_0 = \pi/4$) are considered. In the numerical example, we are also concerned with the arc length as well as with the effectiveness of the proposed schemes of the simple closed form solution.

A. Outgoing target ($\beta_0 = \pi/2$)

Assume that the target moves with fixed velocity, $V_T = 100\,m/\sec$, and fixed heading, $\theta_T = 0$. Assume that the missile with fixed velocity, $V_T = 300\,m/\sec$, is launched at an initial range, $R_0 = 2000m$, and initial line-of-sight angle, $\beta_0 = \frac{\pi}{2}$. Notice that when Howe's skill is employed [9-11], substituting $\gamma = V_M/V_T = 3$ and $\sin\beta = 2\sin\frac{\beta}{2}\cos\frac{\beta}{2}$ into the Eq. (7), we obtain

$$\dot\beta = -\frac{4V_T\sin^2\frac{\beta}{2}\cos^2\frac{\beta}{2}}{K\left(\sin\frac{\beta}{2}\Big/\cos\frac{\beta}{2}\right)^3}$$

$$= -\frac{4V_T\cos^5\frac{\beta}{2}}{K\sin\frac{\beta}{2}} \qquad (30)$$

Then as $\beta_0 = \pi/2, \sin\beta_0 = 1$ and $\tan\beta_0/2 = 1$, Eq. (6) becomes

$$R_0 = K \qquad (31)$$

Substituting the Eq. (31) into the Eq. (30), the LOS rate, $\dot\beta$, is

$$\dot\beta = -\frac{4V_T\cos^5\frac{\beta}{2}}{R_0\sin\frac{\beta}{2}} = -\frac{4V_T\cos^6\frac{\beta}{2}}{R_0\sin\frac{\beta}{2}\cos\frac{\beta}{2}} \qquad (32)$$

By noting that.

$$\cos^2\frac{\beta}{2} = \frac{1+\cos\beta}{2} \quad \text{and} \quad \sin\frac{\beta}{2}\cos\frac{\beta}{2} = \frac{\sin\beta}{2}$$

Eq. (32) can be written as:

$$\dot\beta = -\frac{V_T(1+\cos\beta)^3}{R_0\sin\beta}$$

$$\text{or}\ \frac{\sin\beta}{(1+\cos\beta)^3}d\beta = -\frac{V_T}{R_0}dt \qquad (33)$$

Integrating both side of Eq. (33), then we can write

$$\frac{1}{2(1+\cos\beta)^2}\Big|_{\beta_0}^{\beta} = -\frac{V_T t}{R_0}\Big|_0^t$$

$$\Rightarrow \frac{1}{2(1+\cos\beta)^2} - \frac{1}{2} = -\frac{V_T t}{R_0}$$

$$\Rightarrow \cos\beta = \sqrt{\frac{1}{1-\frac{2V_T t}{R_0}}} - 1$$

$$\Rightarrow \beta = \cos^{-1}\left(\sqrt{\frac{1}{1-\frac{2V_T t}{R_0}}} - 1\right) \qquad (34)$$

Substituting Eq. (31) into Eq. (6), we have

$$R = \frac{R_0 (\tan \frac{\beta}{2})^{\gamma}}{\sin \beta} \qquad (35)$$

In this case, by using the Eqs. (34) and (35) repeatedly, we can calculate the position of target and missile relative to target. The plot of the trajectories is drawn in Fig. 4. To implement the simple closed form of pursuit course, Eq. (27) is employed directly. Numerical results have been obtained using a computer program and plotted in Fig.4.

Fig. 4. depicts the trajectories of the Howe's solution and our proposed solution, which are identified by the symbols "-" and "o" respectively. Numerical results show that trajectory obtained by the simple closed form is same to that obtained by the Howe's solution. According to proposed skill, the interception point is $(\frac{\gamma}{\gamma^2-1} l, l) = (750, 2000)$.

We may draw a cross point upon the line of missile path and target path. Of the two observers shown, the position of the interception point is (750,2000).

As expected, the position of the interception point is very close to the predictive position, $(\frac{\gamma}{\gamma^2-1} l, l)$.

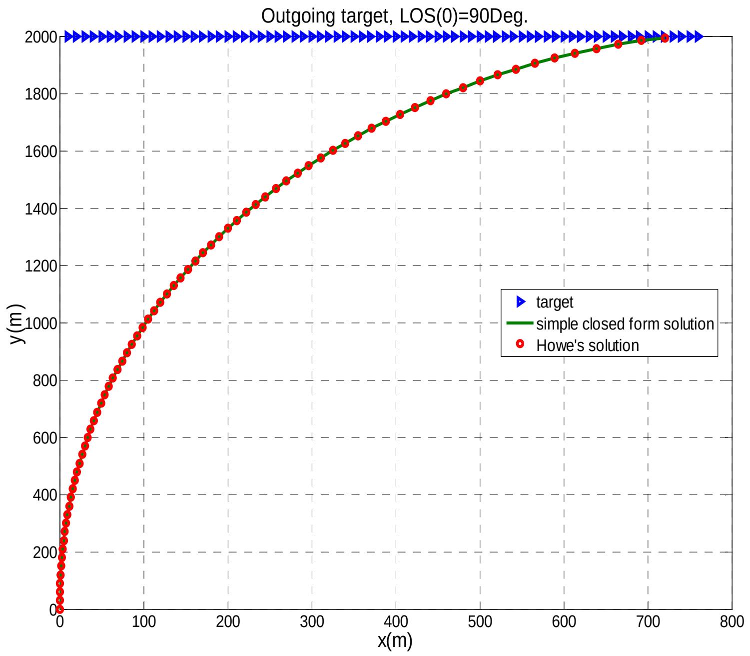

Fig. 4. Trajectory for outgoing target with β_0 =90 Deg.

This pattern, which was manifest during all tests, is precisely that the proposed solution predicts. Knowing $\frac{\gamma}{\gamma^2-1} l$, we can calculate $\frac{\gamma^2}{\gamma^2-1} l$, which is arc length along the trajectory, S. In the case A, $S = 2250m$, the proposed solution is simpler and more convenient than the

Howe's solution.

B. Outgoing target ($\beta_0 = \pi/4$)

The missile velocity is chosen as that in the case A. The initial conditions of the engagement are given by $\theta_0 = 45\deg$, $x_{T0} = 2000m$. The target is outgoing with a constant speed same to that in the case A. From Eq. (6), given the missile initial range R_0 and the initial LOS angle θ_0, we can compute the constant K as

$$K = \frac{\sqrt{2} \times 2000 \times \sin\frac{\pi}{4}}{(\tan\frac{\pi}{8})^3} = 28142. \quad (36)$$

From Eq. (30),

we have $\frac{1}{2(1+\cos\beta)^2}\Big|_{\frac{\pi}{4}}^{\beta} = -\frac{V_T t}{K}\Big|_0^t$ or the functional relationships.

$$\beta = \cos^{-1}\left[\sqrt{\frac{1}{0.3432 - \frac{2V_T t}{28142}}} - 1\right] \quad (37)$$

Substituting Eq. (37) into Eq. (6) gives

$$R = \frac{28142(\tan\frac{\beta}{2})^3}{\sin\beta} \quad (38)$$

The recursive target location and missile position algorithm for the pursuit trajectory are employed as Eqs. (37) and (38), respectively. We can gain information about the shape of the trajectory graph if we know the Eqs. (37) and (38). Finally, we plot the points and use the information about how the LOS line shortens and rotates to complete the sketch, as shown in Fig.5.

About the proposed skill, we have established that the trajectory of the pursuit course is Eq. (24) for some values of c_1. To find out them, we substitute the initial conditions $x(0) = 0$ and $x'(0) = 1$ into Eq. (24). The solution we seek is therefore

$$x(y) = \frac{1}{2}\Big[\frac{1}{1-\sqrt{2}}\frac{1}{1-\frac{1}{\gamma}} l^{\frac{1}{\gamma}}(l-y)^{1-\frac{1}{\gamma}} - \frac{1-\sqrt{2}}{1+\frac{1}{\gamma}} l^{-\frac{1}{\gamma}}(l-y)^{1+\frac{1}{\gamma}} + \Big(\frac{1-\sqrt{2}}{1+\frac{1}{\gamma}} - \frac{1}{1-\sqrt{2}}\frac{1}{1-\frac{1}{\gamma}}\Big)l\Big] \quad (39)$$

where $\gamma = 3$ and $l = 2000m$.

We sketch the curve of Eq. (39) in Fig. (5). Substituting the final value into Eq. (39), the interception point gives:

$$(x_f, y_f) = \Big(\frac{\gamma^2 + \sqrt{2}\gamma}{\gamma^2 - 1} l, l\Big) = (3310.66, 2000) \quad (40)$$

Since

$$x_{T0} + V_T t = l + \frac{S}{\gamma} = \frac{\gamma^2 + \sqrt{2}\gamma}{\gamma^2 - 1} l,$$

the arc length of pursuit course in the case B, S, is $\frac{\sqrt{2}\gamma^2 + \gamma}{\gamma^2 - 1} l$ and therefore we obtain $S = 3931.98m$.

Fig. 5 shows that the curves for the Howe's solution and the proposed solution are almost the same. Again the position of the interception point is almost the same as that of the proposed solution.

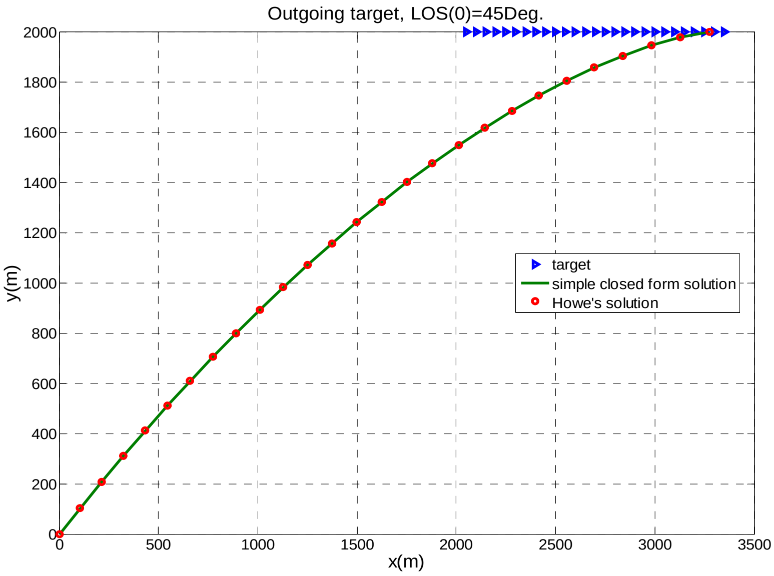

Fig. 5. Trajectory for outgoing target with β_0 =45 Deg.

C. Incoming target ($\beta_0 = \pi/4$)

Referring to Ref. 4, the relationship to Eq. (6) leads to the property of pursuit guidance which terminates in a tail chase, that is, $\beta(t_f)=0$, for all case except the head-on case. Note that $\dot{\beta}=0$ if $\beta=0,\pi$. Therefore $\dot{\beta}$ will always be varying unless $\beta=0,\pi$, which implies a head-on or tail chase path. Consequently, since $\theta=\beta$, the missile must be continuously turning if $\beta\neq 0,\pi$, a situation which is singular. When the pursuit equation is solved as Eq. (38), $\beta=\cos^{-1}(\bullet)$, $-1\leq(\bullet)\leq 1$ is assumed. In this case we shall approach head-on problems that have some nonlinearity in the Howe's solution. Of course, the presence of nonlinearity does not, in itself, insure that an interception point exists. [12,13] The proposed solution was validated using a scenario of incoming target with initial LOS angle, $\beta_0=\pi/4$. The first detection of position (2000m, 2000m) and LOS angle ($\pi/4$) which are adopted as the "lock on" point and the initial values. According to the incoming condition of Eq. (19), the pursuit equation can be rewritten as:

$$x' = x'(y) = \frac{1}{2}[c_1(l-y)^{\frac{1}{\gamma}} - \frac{1}{c_1}(l-y)^{-\frac{1}{\gamma}}] \tag{41}$$

By applying initial values $x(0)=0$ and $x'(0)=1$,
we have

$$x(y) = \frac{1}{2}[\frac{1}{1+\sqrt{2}}\frac{1}{1-\frac{1}{\gamma}}l^{\frac{1}{\gamma}}(l-y)^{1-\frac{1}{\gamma}} - \frac{1+\sqrt{2}}{1+\frac{1}{\gamma}}l^{-\frac{1}{\gamma}}(l-y)^{1+\frac{1}{\gamma}}] + \frac{1}{2}[\frac{1+\sqrt{2}}{1+\frac{1}{\gamma}} - \frac{1}{1+\sqrt{2}}\frac{1}{1-\frac{1}{\gamma}}]l \tag{42}$$

When

$y \to l^-$,

$$x(l) = \frac{\gamma^2 - \sqrt{2}\gamma}{\gamma^2 - 1}l,$$

the position of the interception point is $(\frac{\gamma^2-\sqrt{2}\gamma}{\gamma^2-1}l, l)$.

Since

$$x_{T0}-V_T t = x_{T0}-\frac{V_T}{V_M}S = l-\frac{S}{\gamma}=\frac{\gamma^2-\sqrt{2}\gamma}{\gamma^2-1}l,$$

the arc length along the trajectory can be obtained from the interception point as:

$$S=\frac{\sqrt{2}\gamma^2-\gamma}{\gamma^2-1}l \quad (43)$$

We plot Eq. (42) over $0 \le y \le 2000m$ to check its shape and the interception point. The graph goes up, as shown in Fig. 6.

Fig. 6 shows that the position of the interception point is almost the same as that of the proposed solution. As we can see in Fig. 6, y increases as x increases in the initial time, but the position of the curve that lies over the point (slop=∞) changes to different ways, and then y decreases as x increases. The graph of Fig. 6 is concave up with the tangent turning counterclockwise over $0 \le \beta \le \pi$. This is the major difference between Fig. 6, Fig. 4, and 5 . The Figs. 4 and 5 are concave down with tangent turning clockwise over $0 \le \beta \le \pi/2$.

We plot Fig. 7 and put in enough detail to identify the truth that Fig. 6 appears for incoming engagement. The results demonstrate that the velocities of missile and target are same under given conditions. The path angle follows the LOS angle, $\theta=\beta$, which clearly shows that the proposed solution is very close to the ideal pursuit course.

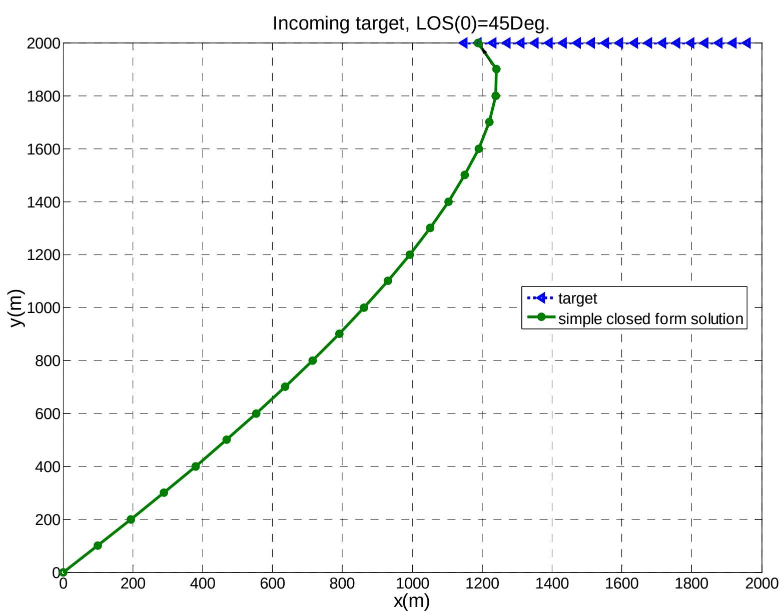

Fig. 6. Trajectory for incoming target with β_0 =45 Deg.

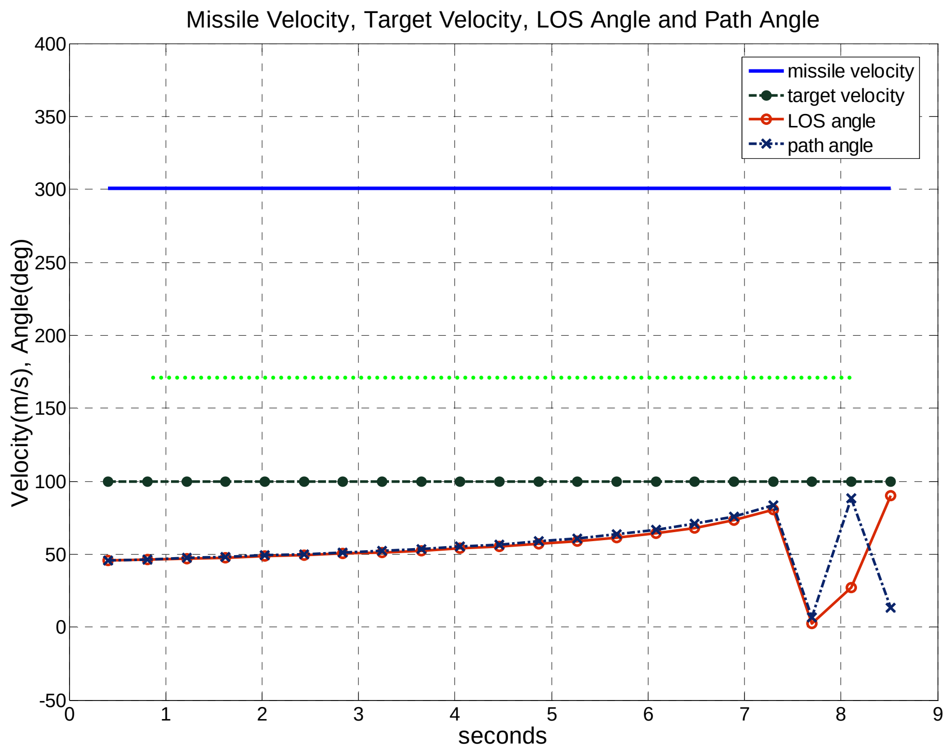

Fig. 7. Computation results of case C by proposed solution.

V. CONCLUSIONS

The algorithms presented a novel approach, instead of a classical solution derived by Howe, to obtain a good solution in the pursuit guidance problem. The proposed method is sufficiently robust to face the lack of a head-on solution, and the computational cost is affordable. This paper has demonstrated that the essential technique has been applied in the design of a pursuit problem. The methods actually offer great advantages of using the Cartesian coordinate variables to determine the trajectory, inception point and path length of pursuit course.

ACKNOWLEDGEMENT

This study was supported by National Science Council, Taiwan, Republic of China under the Grant NSC 92-2623-7-014-029.

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