A Further Evaluation of Orthometric Correction Using Measured Gravity for Each Set-up between Two Benchmarks

Szu-Pyng Kao[1] and Fang-Shii Ning[2]

[1]*Department of Civil Engineering, National Chung-Hsing University*
[2]*Department of Environmental Information and Engineering, Chung Cheng Institute of Technology, National Defense University*

ABSTRACT

Previous research in orthometric correction used gravity data measured on benchmarks only. It neglects the effect of the terrain and other factors along the leveling route between benchmarks. The objective of this study is to further show numerical experiments and results of field test for computing orthometric correction base on measured gravity at each set-up of the staff along the leveling route between two benchmarks. In this paper three sets of the first-order leveling and gravity data within the first-order leveling network of Taiwan, which represent the different terrains from lower altitude to higher altitude, were used in the experiment for the computation of the orthometric correction. Then a comparison is made between the orthometric height corrections by using gravity data at benchmarks only, and those using gravity data observed at each set-up of the staff between two benchmarks. The results of field tests show that, a difference of 0.1mm to 0.5 mm was observed between the orthometric corrections with the constant mass-density computed at benchmarks only and each set-up. From the obtained results of this study, it is concluded that, at the highest level of accuracy for leveling surveys, the orthometric corrections should be taken into account the measured gravity at each set-up along the leveling route between benchmarks.
Keywords: orthometric correction, measured gravity

利用水準點間每個觀測位置實測重力值進一步評估正高改正之研究

高書屏 [1]　　甯方璽 [2]

[1] 國立中興大學土木工程學系
[2] 國防大學理工學院環境資訊暨工程學系

摘　要

以往的正高改正研究所使用的重力數據皆僅使用在水準點上觀測之重力值，如此將忽略在水準點間之地形及沿著施測路線上其他的影響因素。因此，本研究於兩水準點間進行一等水準測量並同時於標尺擺設處實施重力測量，利用實測重力值進行正高改正之研究，針對計算出測線之正高改正結果進行分析。本文選擇台灣地區水準網中代表不同地形的 3 段水準測線進行一等水準測量及重力測量，以計算不同地形之正高改正進行研究。

本研究採用質量密度一致進行計算以比較利用兩水準點上之重力數據計算之正高改正與兩水準點間一等水準測量時各標尺擺設處實施重力測量 累積所計算之正高改正，其結果顯示正高改正有 0.1mm～0.5mm 之間的差異。最後根據本研究所獲得的結果顯示，於高精度水準測量進行正高改正時需以沿水準路線各標尺擺設位置之實測重力以進行計算。
關鍵字：正高改正，實測重力值

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I. INTRODUCTION

On the basis of former research results in orthometric corrections conducted by the authors [1], the further verification of measured gravimetric observation for each set-up between two benchmarks is carried on three leveling traverse of several first order benchmarks at various terrains in Taiwan island. The orthometric height can be approximately obtained by spirit leveling and gravity corrections. The height difference from leveling must be corrected for non-parallel equipotential surfaces using the orthometric correction in order to obtain orthometric height. As we know that the effect of orthometric corrections on leveling results is a systematic one that will accumulate over long leveling runs. The process of computing the orthometric height correction uses gravity at the two adjacent benchmarks only in the past [1]. It was also recommended that orthometric corrections, which take into account observed gravity and topographic mass-density, be considered in any future redefinition of the Australian Height Datum [2]. In this research a comparison will be made between orthometric height correction results using measured gravity data at benchmarks only, and using measured gravity data at set-up of the staff between benchmarks. These lines and one loop within the first-order leveling and gravity network of Taiwan were used that represent the different terrains to compute orthometric corrections via three formulae at only two benchmarks, and also at every set-up of staff. The objective of this research is to determine whether these are significant differences via three formulae at only benchmarks, and at every set-up of staff. Three different terrains, from flat area (with an average elevation 81.544m), mountain area (with an average elevation 1797.950m) to higher mountain area (with an average elevation 3165.233m), were chosen and used as the example test surveys to fully test the effect. Since the first-order leveling and gravimetric surveying used for this field experiment at every set-up of staff will cost a lot of money, time and man powers, therefore only three different terrains will be conducted to see the inherent difference between three different formulae. The leveling traverses and loop observed for this study followed part of a first-order leveling network of Taiwan and covers a distance of 2Km between existing benchmarks of Taiwan [3]. The results from such experiments are discussed in this paper.

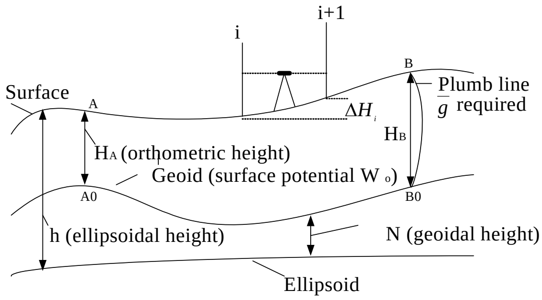

Fig.1. The scheme of precise leveling set-up between benchmark A and B

II.METHODOLOGY

The separation of benchmarks in Taiwan is approximately two kilometers. Precise leveling takes place from benchmark A to benchmark B. The geodetic height is the distance of a point above an ellipsoidal surface. The difference between a point's geodetic height and its orthometric height equals the geoidal height. The level run is divided into set-up as shown in Figure 1. This study uses all gravity measurements and precise leveling to compute the orthometric height corrections of each section. It shows that the summation of orthometric corrections of the sections at every set -up of the staff between the two benchmarks compares well with doing the orthometric correction at two benchmarks only. Orthometric correction equations and the more precise equations reported in this paper are used to compute three leveling traverses, and one leveling loop. The results are compared and analyzed. The adopted formulae in this section were quoted from the authors [1] as follows:

(I) The orthometric height correction formula used normal gravity [4].

$$OC_1 = \frac{1}{\gamma_0}\left[h_A(\gamma_A - \gamma_B) + \Delta h_{AB}(\gamma_{AB} - \gamma_B)\right](mm) \quad (1)$$

where γ_{AB} is $\frac{1}{2}(\gamma_A + \gamma_B)$.

The conventional abbreviated series describing the variations in normal gravity on the ellipsoid with geodetic latitude (φ), and after substituting the constants for GRS80 is[5]:

$$\gamma = 978032.7(1 + 0.0053024 \sin^2 \phi$$
$$+0.0000058 \sin^2 2\phi \ )$$

h_A is height of benchmark A and Δh_{AB} is height difference between benchmarks A an B.

(II) The orthometric correction calculated by observed gravity[4].

$$OC_2 = \frac{1}{\gamma_0}\left[H_A(\overline{g_A} - \overline{g_B}) + \Delta H_{AB}(g_{AB} - \overline{g_B})\right](mm) \quad (2)$$

where: $\overline{g_A} = g_A + 0.424\,H_A$

$\overline{g_B} = g_B + 0.424\,H_B$

$g_{AB} = \frac{1}{2}(g_A + g_B)$

g_A, g_B is observed gravity of benchmark A and B.

(III) The very approximate formula for the orthometric height correction is[6] :

$$OC_3 = 0.114\cdot 10^{-3}\cdot H_m\cdot \Delta H_{AB} - 1.02\cdot 10^{-3}(\Delta g_B^B - \Delta g_A^B)\cdot H_m - 0.83\cdot 10^{-3}\cdot \sin 2\varphi_m \cdot S\cdot H_m(mm) \quad (3)$$

Where φ_m, and H_m are the mean latitude and, mean height between two bench marks. Δg_A^B , Δg_B^B represents the Bouguer gravity anomalies of points A and B, respectively. S represents the north-south distance of two set-ups or benchmarks.

III. DESCRIPTION OF THE TEST SUBJECTS

In this study, three leveling lines within the first-order leveling network of Taiwan, each approximately 2 kilometers long, are selected as the subjects for the numerical tests to examine how the density of the gravity measurements influence orthometric corrections between two adjacent benchmarks(B.M.). The three representative sections are: flat section (Q053~Q054), mountain section (C045~C046) and high mountain section (Q060~Q062). The elevations vary irregularly from 81m to 3165m stretching over plain, hill and mountain. In high mountain area, three benchmarks connected as a loop were also used for the numerical checks of loop closure. The first-order leveling was measured by using Leica NA3003 digital level and gravimetric measurements were measured by using Lacoste & Romberg G model gravimeter. The data of the respective sections of leveling lines and loop are shown in Table 1.

IV. ANALYSES OF THE RESULTS

4.1 CASE OF THE LEVELING LINE IN FLAT AREA

The survey results for the flat area Q053~Q054 is shown below. The profile and the results of the Q053~Q054 section of leveling line are illustrated in Table 2and Figure 2, respectively. The leveling section of Q053~Q054 benchmarks computed the accumulated orthometric correction just on benchmarks and every set-up of staff out of the three formulae mentioned above to find the difference. Table 2 lists the computed orthometric corrections out of the three formulae. No significant differences are found within the results out of Eq(1), Eq(2) and Eq(3) itself with on benchmarks and every set-up of staff.. However, the orthometric corrections by Eq(2) and Eq(3) are many times larger than Eq(1). A difference of 0.1mm and opposite sign was also observed between Eq(2) and Eq(3).

Table 1.Data Sheet of points in section of leveling line and loop

Terrain	B.M. ID	Orthometric Height (m)	Observed gravity (mgal)	Average elevation (m)	No. of set-ups Between B.M.	Distance (Km)
Flat area	Q053	69.279	978786.6710	81.544	20	1.8
	Q054	93.808	978785.5678			
Mountain Area	C045	1734.378	978487.8239	1797.950	42	2.0
	C046	1861.521	978461.8321			
High Mountain Area	C060	3090.096	978221.8410	3182.900	74	1.9
	C061	3275.704	978184.6272			
				3202.802	58	2.0
	C062	3129.900	978223.3992			

Table 2. The total orthometric correction for each equation (between benchmarks Q53~Q54).

Spacing of used for orthometric correction	Total orthometric correction of two benchmarks (mm)		
	Eq(1)	Eq(2)	Eq(3)
Every staff setting up	-0.022	-0.082	0.183
On benchmark only	-0.021	-0.081	0.187

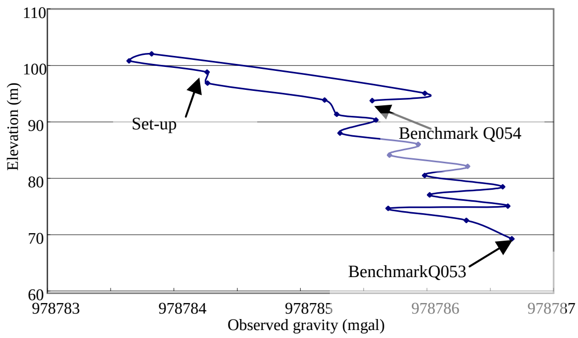

Fig. 2. Elevation and observed gravity of every set-up of leveling line in flat area. (Q53~Q54)

4.2 CASE OF THE LEVELING LINE IN MOUNTAIN AREA

In the mountain area, leveling observations from benchmark C045 to C046, gravity is surveyed at every staff set-up. There are 42 set-ups totally between C045 and C046 benchmarks. The profile and the results of this leveling line are illustrated in Table 3 and Figure 3.

The average height in mountain area is 1797.950m high and height difference around 120m so orthometric correction is large than flat area. The orthometric corrections show the more sensitive to the height and change in height. [1,2]. The summation of orthometric corrections of every staff setting up are almost the same as only computing benchmarks on the two ends of leveling line, in Eq(1) and Eq(2). From the Table 3 obtained results, that Eq(1) with large difference than Eq(2) and Eq(3), concluded that in high elevation area observed gravity should be used to compute orthometric correction rather than normal gravity. Again the greater height difference and the higher mean elevation between two benchmarks incur the larger correction pointed out by Kao et. al was also found in Table3 .

Table 3. The total orthometric correction for each equation. (between benchmarks C045~C046)

Spacing of used for orthometric correction	Total orthometric correction of two benchmarks (mm)		
	Eq(1)	Eq(2)	Eq(3)
Every staff setting up	-0.022	-0.082	0.183
On benchmark only	-0.021	-0.081	0.187

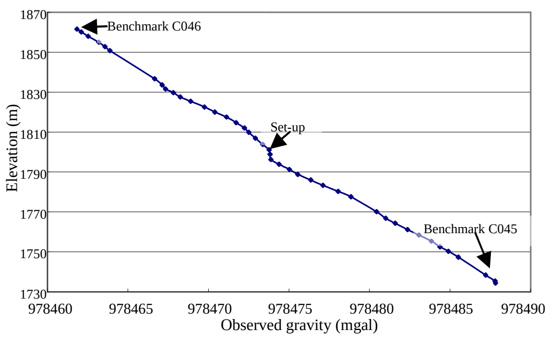

Fig. 3. Elevation and observed gravity of every set-up of leveling line in mountain area. (between benchmarks C045~C046)

4.3TEST LOOP IN HIGH MOUNTAIN AREA

For this research purpose in a high mountain area a leveling loop from benchmarks C060 to C062 via C061, then back to C060 was used. Tables 4, 5, 6 and Figure 4, 5 show results of the testing loop. When the leveling is complete, the survey resumes at benchmark C062, then passing different routes, except benchmark C061, and closes to benchmark C060. When conducting a level survey from C060 to C062, a gravity survey is also used on each staff set-up. Again, the sums of the orthometric correction from Eq(1) to Eq(3) are calculated and displayed in Tables 4 and 5.

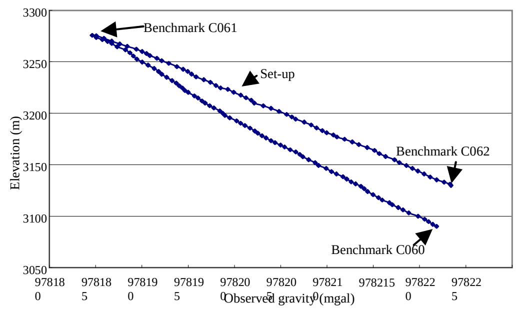

Fig. 4. Elevation and observed gravity of every set-up of leveling line in mountain area. (C060~C061~C062)

The Table 4 shows the summation of orthometric corrections of every staff setting up are almost the same as only computing benchmarks on the two ends of leveling line in orthometeric correction equation of applying Eq(1) formula. A difference of 0.1mm was observed between Eq(2) and a difference of 0.5mm was observed between Eq(3). And one can find out that the route of leveling line is going up (around 3090m to 3275m), so the orthometric height corrections are positive. From Table 4, it was showed that orthometric corrections are significant for Eq(2) and Eq(3). This problem is ture of spirit leveling lines that traverse(east-west) across north-south oriented mountain ranges[1,7].

Table 4. The total orthometric correction for each equation (between benchmarks C0060~C061)

Spacing of used for orthometric correction	Total orthometric correction of two benchmarks (mm)		
	Eq(1)	Eq(2)	Eq(3)
Every staff setting up	3.242	69.107	65.733
On benchmark only	3.246	69.208	66.287

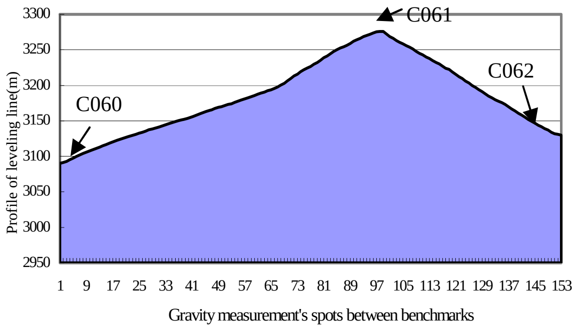

Fig. 5. Profile of leveling line in high mountain area (between benchmarks C060~ C062)

The Table 5 shows the same trend of Table 4 that every staff setting up are almost the same as only computing benchmarks on the two ends of leveling line in Eq(1). A difference of 0.3 mm for Eq(2) and 0.4mm for Eq(3) was found. And one can also find out that the route of leveling line is going down (around 3090m to 3275m), so the orthometric height corrections are negative.

From Table 6 test loop using Eq(1) to Eq(3) processed gravity measurements at each staff set-up versus at benchmarks only. The results of the two methods are very close. For example, when comparing the sum of the orthometric corrections between staffs, and applying orthometric correction at benchmarks only, a difference of 0.1mm for normal gravity formula Eq(1), 4.7mm for observed gravity formula Eq(2) and 4.9mm for very approximate formula Eq(3). The difference of both are within the specification of the first order leveling closing error (3mm $\sqrt{K}$ K is distance of leveling line in kilometer.).

However it was pointed out that the orthometric correction is a systematic effect and thus should not be compared with spirit leveling tolerances [2]. The loop test also can find the sign of the orthometric correction is path-dependent when the test line going up the sign is positive otherwise is negative.

Table 5. The total orthometric correction for each equation (between benchmarks C0061~C062)

Spacing of used for orthometric correction	Total orthometric correction of two benchmarks (mm)		
	Eq(1)	Eq(2)	Eq(3)
Every staff setting up	3.242	69.107	65.733
On benchmark only	3.246	69.208	66.287

Table6. The loop Misclosure of total orthometric correction for each equation(between benchmarks C060~C061~C062~C060)

Distance of using orthometric height correction	Closing error of orthometric corrections in test loop (mm) (The loop closing error should be under $\pm 3mm\sqrt{K} = \pm 8.48$ mm)		
	Eq(1)	Eq(2)	Eq(3)
Every staff setting up	0.000	-3.747	-6.629
On benchmark only	-0.098	-4.713	-4.907

V. CONCLUSION

The main goal of this study is to further show numerical experiments and results of field test for computing orthometric correction by every set-up of staff and just at benchmarks of leveling line. From the experiments carried out on leveling lines and leveling loop, it can be seen that there is a difference of 0.1mm to 0.5mm was observed between the two methods. So it is concluded that orthometric correction can be suitably done on benchmarks only. It also revealed the benchmarks at high altitude and large height difference between benchmarks had big orthometric height correction.

At the highest level of accuracy for leveling surveys, the orthometric correction should be taken into account the measured gravity and density at each set-up along the leveling route between benchmarks.

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