Attitude Control for a Spacecraft with Uncertainties

Chiz-Chung Cheng[1*], Yew-Wen Liang[2], Sheng-Dong Xu[2], Der-Cherng Liaw[2] , and Hung-Liang Chen[1]

[1] *Department of Electrical Engineering, Lee-Ming Institute of Technologyi*
[2] *Department of Electrical and Control Engineering, National Chiao Tung University*

ABSTRACT

Issue regarding attitude tracking for a spacecraft with model uncertainties and external disturbances are addressed. By using the Sliding Mode Control (SMC) design technique, this paper presents a class of improved SMC schemes, which are shown to have the advantages of rapid response and robustness from the classic SMC designs. Besides, the new SMC law is continuous everywhere so that the chattering drawback of the sign-type SMC controller is greatly alleviated. This new scheme also improves the saturation-type SMC's weakness of only achieving uniformly ultimate boundedness performance to an asymptotic stability level. As a result, the improved SMC law enables the spacecraft to perform highly accurate pointing task. Numerical simulations demonstrate the benefits of the proposed SMC scheme

Keywords: nonlinear systems, robust control, sliding mode control, asymptotic stability.

衛星姿態之穩健性控制研究

鄭治中[1*]　梁耀文[2]　徐聖均[2]　廖德誠[2]　陳宏良[1]

[1] 黎明技術學院電機系
[2] 交通大學電機與控制系

摘要

本論文探討具有模式誤差及外界干擾時之衛星姿態控制議題。利用順滑模式技術，本論文提出一組改良式之順滑模式控制法，此改良順滑模式控制不但具有傳統順滑模式反應快速及穩健的優點，同時由於此改良順滑模式的連續特性，可大幅減輕傳統切換式順滑模式無可避免的切跳現象。更進一步，此改良順滑模式控制還能將傳統飽和式順滑模式控制僅能達到均勻終極有界的性能提升到漸進穩定的水準。因此，本控制律也可提供衛星之精確定位。模擬結果說明了所提控制律的優點。

關鍵字：非線性系統，穩健控制，順滑模式控制，漸進穩定

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I. INTRODUCTION

Recently, the study of attitude control and maneuvers for spacecraft has attracted lots of attention (see e.g., [1], [2]-[4], [5], [9], [11], and the references therein). The control objectives are expected to achieve highly accurate pointing, rapid maneuver and robust to model uncertainties, measurement noises and external disturbances. To fulfill the performance requirements, many methods have been proposed including Feedback Linearization [3], Nonlinear Optimal Control [5] and Sliding Mode Control (SMC) techniques [1], [2], [4], [9], [11]. Among the various approaches, it is known that the SMC schemes have the advantages of rapid response and low sensitivity to system uncertainties and disturbances [9], [10]. It has thus been widely applied to a variety of control problems (see e.g., [1], [2], [4], [6], [8] and the references therein), especially for attitude control of spacecraft [1], [2], [4], [9], [11]. The attitude control missions might be robust attitude stabilization [1], [11], or robust attitude tracking [2], [4]. For instance, Vadali [11] applied the SMC design technique to the large angle maneuver problem. He derived an optimal SMC law in the sense of a quadratic performance index. Under this SMC law, the resulted closed-loop system was shown to be insensitive to parameter variations and unmodelled effects. Bŏković et al. [1] explored the spacecraft robust stabilization problem while taking control input saturation into account. They also showed that the closed-loop system designed not only exhibit a rapid and accurate response but also is insensitive to external disturbances and parametric uncertainties. From the existing results, almost all the studies adopted either sign- or saturation-type functions when initiating SMC controller to guarantee that the system state would reach the selected sliding surface.

Although the SMC schemes have the advantages of rapid response and low sensitivity to system uncertainties and disturbances, the traditional sign-type SMC scheme often results in a chattering behavior because of its discontinuous switching nature. The chattering behavior has some drawbacks, including damage to mechanisms, excitation of unmodelled dynamics and waste of too much energy near the sliding surface (see e.g., [9]). On the other hand, it is known that the traditional boundary layer method with fixed boundary layer in SMC design can attenuate the degree of high frequency behavior, but it only achieves a uniformly ultimate boundedness performance. Due to the disadvantages of traditional SMC schemes, in this paper we will employ the SMC design technique to synthesize a class of continuous control laws. The proposed SMC laws will be shown to be able to alleviate the chattering behavior found in the sign-type SMC controller, and to mend the saturation-type SMC law's weakness of only achieving uniformly ultimate boundedness performance to an asymptotic stability level .

II.PROBLEM FORMULATION

The governing equations of motion for a rigid spacecraft are described by [4]：

$$\dot{\xi} = T(\xi)\omega \tag{1}$$

and

$$J\dot{\omega} = H\omega + u + d \tag{2}$$

Here, $\xi \in \Re^3$, $\omega \in \Re^3$ and $u \in \Re^3$ denote the Gibbs vector of Cayley-Rodrigues attitude parameters, the inertial angular velocity vector of the spacecraft with respect to body fixed frame and the control torque, respectively, d denotes the external disturbance torque including the gravity gradient torque, aerodynamics and solar radiation pressure torque and J is the moment of inertia matrix of the spacecraft, which is assumed to be symmetric and positive definite about the body fixed frame. In addition, the matrices $T(\xi)$ and H are defined as follows:

$$T(\xi) = \frac{1}{2}[I + \xi\xi^T + \xi\times] \tag{3}$$

and

$$H = [h\times] = \begin{pmatrix} 0 & -h_3 & h_2 \\ h_3 & 0 & -h_1 \\ -h_2 & h_1 & 0 \end{pmatrix} \tag{4}$$

where $I \in \Re^{3\times3}$ is the identity matrix, $h = J\omega$ is the angular momentum and the notation $\times$ ' denotes the vector cross product. In this study, the inertia matrix J is allowed to be uncertain. J and H are represented by

$$J = J_0 + \Delta J \quad (5)$$

and

$$H = H_0 + \Delta H \quad (6)$$

where J_0 and $H_0 = |(J_0\omega)\times|$ denote the nominal parts while ΔJ and $\Delta H = [(\Delta J\omega)\times]$ denote the uncertain parts.

The main goal of this paper is to synthesize a control law to achieve the tracking performance under model uncertainties and external disturbances. That is, to design u such that the closed-loop system (1)-(2) has the property: $\xi(t) \to \xi_d(t)$ as $t \to \infty$. Here, $\xi_d(t)$ is the desired attitude trajectory to be tracked.

III. CONTROLLER DESIGN

In order to achieve the main goal of this paper as stated in Section II, we employ the Sliding Mode Control (SMC) algorithm to perform the design task.

For the first step, suppose that $\xi_d(t)$ is the desired Rodrigues parameter trajectory to be tracked. We choose the sliding surface in the following form

$$s(t) = [\omega(t) - \omega_d(t)] + \alpha[\xi(t) - \xi_d(t)] = 0 \quad (7)$$

where α is a positive scalar and

$$\omega_d(t) = T^{-1}(\xi(t))\dot{\xi}_d(t)$$

It was shown in [2] that the tracking performance $\xi(t) \to \xi_d(t)$ and $\omega \to \omega_d(t)$ as $t \to \infty$ can be achieved if the system state keeps staying on the sliding surface.

The second step of the SMC design is to synthesize a control law in the form of

$$u = u^{re} + u^{eq} \quad (8)$$

where u^{re} forces the system state to reach the sliding surface in a finite time and keeps the sliding surface an invariant set. From Eqs. (1)-(2), (5)-(6) and (7),

$$\begin{aligned} J\dot{s} &= J\dot{\omega} - J\dot{\omega}_d + \alpha J(\dot{\xi} - \dot{\xi}_d) \\ &= (H_0 + \Delta H)\omega + u + d - (J_0 + \Delta J)\dot{\omega}_d \\ &\quad + \alpha(J_0 + \Delta J)[T(\xi)\omega - \dot{\xi}_d]. \end{aligned} \quad (9)$$

Following the SMC design procedure, u^{eq} is obtained as

$$u^{eq} = -H_0\omega + J_0\dot{\omega}_d - \alpha J_0[T(\xi)\omega - \dot{\xi}_d]. \quad (10)$$

It follows that

$$J\dot{s} = u^{re} + \delta \quad (11)$$

where

$$\begin{aligned} \delta &= \delta(\xi, \omega, \dot{\xi}_d, \dot{\omega}_d, \Delta J, d) \quad (12) \\ &= \Delta H\omega + d - \Delta J\dot{\omega}_d + \alpha\Delta J[T(\xi)\omega - \dot{\xi}_d] \quad (13) \end{aligned}$$

In order to design u^{re} to force the system state reaching the sliding surface in a finite time, we impose the following assumption on δ:

Assumption 1 There exist three nonnegative scalar functions $\rho_i(\xi, \omega, t)$ such that

$$|\delta_i| \le \rho_i(\xi, \omega, t) \text{ for all } i = 1, 2, 3 \quad (14)$$

where δ_i is the *i*th component of δ.

To guarantee the reaching condition, one might select u^{re} from two classic SMC designs as given in Eqs. (15)-(16) below (see e.g., [9]):

$$u_i^{re} = -[\rho_i(\xi, \omega, t) + \eta_i] \cdot \mathrm{sgn}(s_i) \quad (15)$$

$$u_i^{re} = -[\rho_i(\xi, \omega, t) + \eta_i] \cdot \mathrm{sat}(s_i / v_i) \quad (16)$$

for $i = 1, 2, 3$. In Eqs. (15)-(16) above, η_i and ν_i are positive constants, and $\text{sgn}(\cdot)$ and $\text{sat}(\cdot)$ denote the sign and the saturation functions, respectively. The constants ν_i are referred as the boundary layer widths of the sliding surfaces for saturation-type control. The overall control for SMC design is then constructed in the form of (8) with u_i^{eq} given by (10), and u_i^{re} given by (15) and (16). Though these two types of controls may attain the desired performance, they exhibit some drawbacks. First, the discontinuity ofsign-type control as given in (15) leads to the chattering phenomenon of system dynamics. In practical applications, chatter is generally undesirable since it involves extremely high control activity and might further excite high frequency dynamics neglected in the course of modeling. Second, though the saturation-type controller as given in (16) is a continuous one, the boundary layer has a constant width and the magnitude of u^{re}, as given in (16), reduces to zero as the system state approaches the sliding surface. This might result in the magnitude of the control effort being smaller than those of disturbances when system state lies within the boundary layer. Thus, the saturation-type control can only guarantee a uniformly ultimate bounded property.

Instead of using u_i^{re} in the form of (15)-(16), a continuous control law to alleviate the chattering behavior while retaining the system performance of asymptotic stability is proposed in [7] having the form of (8) with u^{eq} being by (10) and

$$u_i^{re} = -[\rho_i(\xi,\omega,t) + \eta_i] \cdot g_i(s), \tag{17}$$

where $g_i(s) := 2s_i / (|s_i| + \phi(t))$ for $i = 1, 2, 3$. Here, $\phi(t)$, $t > 0$, is a smooth, positive, and strictly decreasing function having the properties of $\phi(t) \to 0$ and $\dot{\phi}(t) \to 0$ as $t \to \infty$. Two classes of $\phi(t)$ are $\varepsilon e^{-\gamma t}$ and $\varepsilon t^{-\gamma}$, where $\varepsilon > 0$ and $\gamma > 0$. Noted that, the function $g_i(s)$ given in (17) behaves like sign and saturation functions when $|s_i| >> \phi(t)$ and $|s_i| << \phi(t)$, respectively. To ensure that $g_i(s)$ behaves like sign function when $|s_i| \geq \phi(t)$, the coefficient in the numerator of $g_i(s)$ is selected to be 2. Since the proposed law is continuous everywhere, including on the sliding surface, the chatter can thus be greatly alleviated.Moreover, under this modified law,it was shown in [7] that the system state will satisfy the relation $|s_i| \leq \phi(t)$ after a finite amount of time and achieve the asymptotic stability performance.

IV. SIMULATION RESULTS

Here we present an example to demonstrate the effectiveness of the proposed SMC scheme. In this example, the nominal inertia matrix J_0, the desired Rodrigues parameter trajectory $\xi_d(t)$, the initial states $\xi(0)$ and $\omega(0)$, the widths of the boundary layer ν_i for $i = 1, 2, 3$ given in saturation-type control, the disturbance d and the uncertainty ΔJ are adopted from [2] as follows:

$$J_0 = \text{diag}\{87.212, 86.067, 114.562\}$$
$$\xi_d(t) = (\sin(\pi t/50), -\sin(\pi t/50), 0.5\cos(\pi t/50))^T$$
$$\xi(0) = (1, 1, -1)^T$$
$$\omega(0) = (0.001, -0.005, 0.001)^T$$
$$\nu_i = 0.05 \quad \text{for} \quad i = 1, 2, 3$$
$$d = (-0.005\sin(t), 0.005\sin(t), -0.005\sin(t))^T$$

and ΔJ is in diagonal form with

$$\text{diag}\{-8.7212, -4.3034, -17.1843\} < \Delta J < \text{diag}\{8.7212, 4.3034, 17.1843\}$$

Under these settings, the three nonnegative functions given in (14) can be calculated as

$$\rho_1(\xi,\omega,t) = 21.488 \cdot |\omega_2\omega_3| + 0.005 + 8.721 \cdot |\dot{\omega}_{d1}| + 4.3605 \cdot (|T_{\omega 1}| + |\dot{\xi}_{d1}|)$$
$$\rho_2(\xi,\omega,t) = 25.905 \cdot |\omega_1\omega_3| + 0.005 + 4.304 \cdot |\dot{\omega}_{d2}| + 2.152 \cdot (|T_{\omega 2}| + |\dot{\xi}_{d2}|)$$

$$\rho_3(\xi,\omega,t)=13.025\cdot|\omega_1\omega_2|+0.005+17.184\cdot|\dot{\omega}_{d3}|$$
$$+8.592\cdot(|T_{\omega 3}|+|\dot{\xi}_{d3}|)$$

where ω_i , $\dot{\omega}_{di}$, $\dot{\xi}_{di}$ and $T_{\omega i}$ for $i=1,2,3$ denote the ith component of ω , $\dot{\omega}_d$, $\dot{\xi}_d$ and $T(\xi)\omega$, respectively. In addition, the parameters of the proposed SMC scheme are chosen to be $\alpha=0.5$, $\varepsilon=0.25$, $\gamma=0.007$, $\eta_1=\eta_2=\eta_3=1$.

Simulation results are summarized in Figs. 1-2. Each of these figures contains simulation results with regard to three different type SMC schemes. The three SMC controllers are in the form of (8) with u^{re} being given by sign-type (15), saturation- type (16) and improved SMC scheme (17). In order to inspect the asymptotic behavior, Fig. 1 shows the time responses of the norm of the state error $\|e(t)\|=\|\xi(t)-\xi_d(t)\|$ in a magnified scale. Figure 2 displays the time histories of the norm of control efforts. It is observed from Fig. 1(a) that the norm of the error state for the system with sign-type controller exhibits the smallest magnitude and, from Fig. 1(b), the saturation-type controller does not reduce tracking error to zero as expected [2]. On the other hand, it is seen from Fig. 1(c) that the norm of the error state for the system with the improved SMC scheme clearly has the tendency of decay to zero. This means that the tracking performance $\xi(t)\to\xi_d(t)$ will be achieved asymptotically. These results agree with the theoretical results as proposed in Section III. Moreover, it is observed from Fig. 2(a) that the sign-type control exhibits chatter, while those of the other two controls do not. The energy consumption is found to be very close by saturation-type and improved SMC designs. In contrast, due to the chatter effect, the energy consumption by sign-type SMC controller is obviously higher than those of the other two SMC schemes, especially for system state near the sliding surface. Thus the improved SMC design does not create an extra burden on control efforts.

From this example, it is concluded that the proposed SMC scheme not only inherits the advantages of rapid response and robustness performance from classic SMC designs, but also improves the drawbacks of chattering behavior and only uniformly ultimate boundedness performance of sign-type and saturation-type SMC schemes, respectively. These agree with the theoretical conclusions as proposed in Section III.

V. CONCLUSIONS

This paper has dealt with the spacecraft attitude tracking problem via a novel SMC scheme considering model uncertainties and external disturbances. The proposed SMC scheme works like the sign-type and the saturation-type SMC schemes when the system state is far from and close to the selected sliding surface, respectively. Besides, the proposed SMC scheme was shown to inherit the merits of rapid response and robustness from the SMC designs, without exerting an extra burden on control efforts. This scheme is also shown to be able to achieve the asymptotic stability property rather than only a uniformly ultimate boundedness one as those by classic saturation-type SMC controllers, even when system is corrupted with disturbances. Plus, the improved SMC law is continuous, which can greatly alleviate the chattering behavior. Simulation results have demonstrated the usage and the benefits of the proposed scheme.

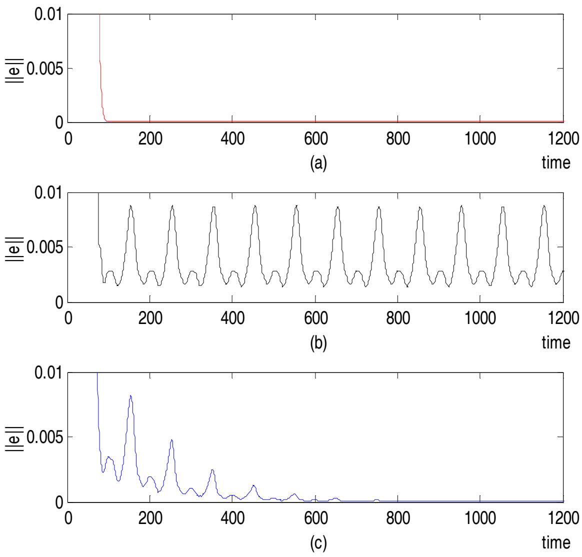

Fig. 1. Norm of error in magnified scale by (a) sign-type (b) saturation-type and (c) improved SMC designs.

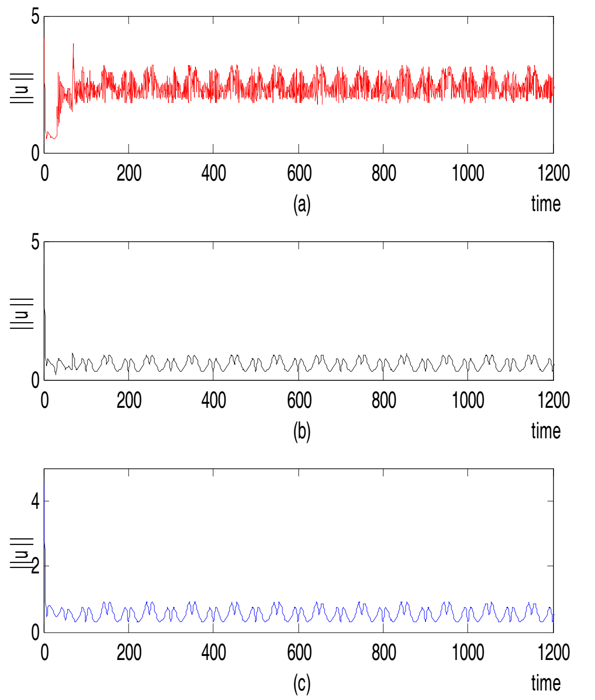

Fig. 2. Norm of control effort by (a) sign-type (b) saturation-type and (b) improved SMC designs.

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