A Feasible Ordered Statistic Decoding for Reed Solomon Codes with QAM Signaling

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ABSTRACT

This work presents a simple way of ordered statistic decoding (OSD) to decode Reed-Solomon codes with QAM signaling over additive white Gaussian channels. In such a way, there is a proposed erasure decoding employed in decoding instead of re-encoding process in OSD. Under consideration of decoding complexity, four nearest neighbors are used to replace each symbol in an erasure sequence during proposed decoding. Simulation results of (15,9,7) RS code show that order-1 OSD with proposed simple way can provide 1 dB coding gain, while full-order OSD only supports 2 dB coding gain but with lots of complexity. From those comparisons of complexity, this proposed simple way in OSD is achievable and easily implemented.

Keywords: Reed-Solomon decoding, erasure decoding, ordered statistic decoding

以 QAM 訊號傳送 Reed-Solomon 碼的可實現排序統計解碼法則之研究

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摘　要

本研究提出排序統計解碼的簡化方式，以作為 Reed-Solomon 碼使用 QAM 訊號在加法性高斯通道之解碼使用。此種研提方式使用抹去式解碼，取代原排序統計解碼中生成過程。在解碼複雜度之考量下，抹去式解碼方式使用週遭四點取代原點之方式。以(15,9,7) RS 為例，模擬結果顯示 order-1 的排序統計解碼可提供 1 dB 的編碼增益，然而 full-order 的排序統計解碼提供 2 dB 的編碼增益，但卻花費非常大的運算量。由此驗證，本研究研提之方式既可行又易實現。

關鍵詞：Reed-Solomon解碼，抹去式解碼，排序統計解碼

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Ⅰ. INTRODUCTION

There are two types to decode Reed Solomon (RS) codes: hard decision decoding is based on the algebraic properties of RS codes, and soft decision decoding is through received message and channel characteristics to calculate the reliability of each bit or symbol. For hard decision decoding, syndrome-based and interpolation-based are considered. Peterson-Gorenstein-Zierler algorithm[2], Berlekamp-Massey algorithm[1][2], Euclidean algorithm[1][2], frequency domain algorithm[1][2] and step-by-step decoding algorithm[3-6] are viewed as syndrome-based decoding. As for interpolation-based decoding, there mainly contains Welch-Berlekamp algorithm[7][8] and Guruswami-Sudan decoding algorithm[9].

The current RS soft decision decoding algorithms can be broadly divided into five classes, which contain generalized minimum distance (GMD) decoding, trellis-based decoding, re-encoding process, algebraic soft decision decoding and adaptive belief propagation (ABP) decoding. The first class is the GMD decoding algorithm[11]. The hard decision decoding algorithm and the ordered set of bytes' or symbols' reliability is employed in a successive erasure decoding. The second class of the decoding algorithm is on the code characteristics, such as the length of the code, the amount of information bits (or symbols). A construction of code lattice structure (trellis) is followed by decoding [12]. The third class is a re-encoding process [13] or reordering process or ordered statistic decoding [14-18], which the reliability values of symbols of the received signal vector ordered are to generate a code generation matrix, whereby to produce a series of codewords. Among these codewords, one of them as a decoded codeword is chosen with the smallest Euclidean distance. The fourth class is an algebraic soft-decision decoding algorithm (algebraic soft decision decoding)[10]. Based on Guruswami-Sudan decoding algorithm[11], each of the received signal vector signal first calculates its reliability between symbols over GF(2^m) and then converted into an interpolation point. By utilizing the characteristics of the interpolation method identifies a series of code words, and comparing them with metric of Euclidean distances, and finally taking the minimum one as the most appropriate codeword. The fifth class is adaptive belief propagation (ABP). In each iteration, based on the adapted parity-check matrix, first calculates the extrinsic log-likelihood ratio of a symbol, and then adds its original logarithmic ratio, and forms a new logarithmic ratio. Based on these logarithmic ratios, find the worst $N - K$ ones and corresponding columns in its code generation matrix, which Gaussian elimination is performed to produce a systematic form for the next iteration. Such a decoding is similar to that of maximum a posterior (MAP) decoding. The difference between both is MAP decoding to be binary-trellis-based, and ABP decoding to be parity-check matrix-based.

Applying a soft decision decoding algorithm has about 3dB of coding gain over a hard decision decoding algorithm which uses algebraic algorithm in decoding[1]. In [14], ordered statistic decoding (OSD) contains three stages, (1) reordering, (2) re-encoding and (3) comparing. OSD reduces search space for maximum likelihood decoding (MLD) performance by taking advantage of the reliability information from the received symbols [6]. In this work, we propose an innovation way to reduce the complexity in re-encoding, which is a portion of OSD. The organization of this work as follows. The ordered statistic decoding of RS codes with QAM signal is proposed in Section II. For RS codes, an innovation decoding instead of re-encoding is proposed in Section III. Sections IV and V present simulation results and conclusions.

Ⅱ. ORDERED STATISTIC DECODING

Suppose that sequence $\boldsymbol{R}=\left(\boldsymbol{r}_0,\boldsymbol{r}_1,\cdots,\boldsymbol{r}_{N-1}\right)$ is a received sequence, where $\boldsymbol{r}_i=\left(r_i^{(\mathrm{I})},r_i^{(\mathrm{Q})}\right)$ is a point in the coordinate system. Let $\boldsymbol{S}^{\mathrm{QAM}}=\left(s_0^{\mathrm{QAM}},s_1^{\mathrm{QAM}},\cdots,s_{N-1}^{\mathrm{QAM}}\right)$ is a QAM sequence, where s_i^{QAM} is the nearest-field-point of $\boldsymbol{r}_i$. The inverse mapping of a signal vector $\boldsymbol{S}^{\mathrm{QAM}}$ is an *N*-tuple vector $\boldsymbol{Z}=\left(z_0,z_1,\cdots,z_{N-1}\right)$ where $z_i\in\mathrm{GF}(2^m)$. The reliability value for each symbol $\boldsymbol{r}_i$ of $\boldsymbol{R}$ is evaluated by computing the Euclidean distance $d\left(s_i^{\mathrm{QAM}},\boldsymbol{r}_i\right)$, $0\le i\le N-1$. The distance $d\left(s_i^{\mathrm{QAM}},\boldsymbol{r}_i\right)$ is smaller, which means $\boldsymbol{r}_i$ to be the symbol s_i^{QAM} with higher reliability. The value of $d\left(s_i^{\mathrm{QAM}},\boldsymbol{r}_i\right)$ is usually viewed as the reliability value of $\boldsymbol{r}_i$. Obviously, the smaller $d\left(s_i^{\mathrm{QAM}},\boldsymbol{r}_i\right)$ is, the more reliable $\boldsymbol{r}_i$ is. Based on the obtained reliability values, we decreasingly order positions of all elements z_i in $\boldsymbol{Z}$, positions of all symbols s_i^{QAM} in $\boldsymbol{S}^{\mathrm{QAM}}$ and r_i and $\boldsymbol{R}$, respectively. Such that the element with the highest reliability value is at the first position, and the element with the lowest reliability value is at the last position. An ordered vector $\tilde{\boldsymbol{S}}^{\mathrm{QAM}}=\left(\tilde{s}_0^{\mathrm{QAM}},\tilde{s}_1^{\mathrm{QAM}},\cdots,\tilde{s}_{N-1}^{\mathrm{QAM}}\right)$, $\tilde{\boldsymbol{R}}=\left(\tilde{\boldsymbol{r}}_0,\tilde{\boldsymbol{r}}_1,\cdots,\tilde{\boldsymbol{r}}_{N-1}\right)$, and satisfy this inequality

$$d\left(\tilde{s}_0^{\mathrm{QAM}},\tilde{\boldsymbol{r}}_0\right)\le d\left(\tilde{s}_1^{\mathrm{QAM}},\tilde{\boldsymbol{r}}_1\right)\le\cdots\le d\left(\tilde{s}_{N-1}^{\mathrm{QAM}},\tilde{\boldsymbol{r}}_{N-1}\right) \tag{1}$$

Let Φ denote such a position permutation, then such a permutation is employed in the following operation, $\tilde{\boldsymbol{Z}}=\left(\tilde{z}_0,\tilde{z}_1,\cdots,\tilde{z}_{N-1}\right)=\Phi\left(\boldsymbol{Z}\right)$, $\tilde{\boldsymbol{S}}^{\mathrm{QAM}}=\Phi\left(\boldsymbol{S}^{\mathrm{QAM}}\right)$ and $\tilde{\boldsymbol{R}}=\Phi\left(\boldsymbol{R}\right)$. We see that the first *K* positions in $\tilde{\boldsymbol{Z}}$ are called the most reliable independent (MRI) positions, and the last *N*-*K* positions are called the least reliable independent (LRI) positions. As a consequence, $\tilde{\boldsymbol{Z}}$ can be rewritten as

$$\begin{aligned}\tilde{\boldsymbol{Z}} &= \left(\underbrace{\tilde{z}_0, \tilde{z}_1, \ldots, \tilde{z}_{K-1}}_{\text{MRI positions}}, \underbrace{\tilde{z}_K, \tilde{z}_{K+1}, \ldots, \tilde{z}_{N-1}}_{\text{LRI positions}} \right) \\ &= \tilde{\boldsymbol{Z}}^{(I)} \,||\, \tilde{\boldsymbol{Z}}^{(P)}\end{aligned} \quad (2)$$

where || denotes concatenation, and the superscripts I and P denote the K MRI positions and the other (N-K) LRI positions. Consider $\boldsymbol{G}$ be a generator matrix of an (N, K) RS code, and $\Phi(\boldsymbol{G})$ be the column permutation of $\boldsymbol{G}$ based on reliability values of symbols within received vector $\boldsymbol{R}$. Applying this column permutation, we obtain a new generator matrix $\bar{\boldsymbol{G}}$ which has the first K columns are linear independent columns. Denote Ω be elementary row operations such that it transforms $\bar{\boldsymbol{G}}$ into a systematic matrix $\tilde{\boldsymbol{G}}$. We may write that

$$\tilde{\boldsymbol{G}} = \Omega\left(\bar{\boldsymbol{G}}\right) = \left[\mathbf{I}_K, \mathbf{P}_{K\times(N-K)}\right] \quad (3)$$

where $\mathbf{I}_K$ is an identity matrix of size K.

In OSD algorithm, the strategy to identify the correct codeword is that we generate a list of potential codewords over GF(2^m) and choose one whose QAM signal vector is the most likely to received sequence $\boldsymbol{R}$. Such a codeword has the minimum discrepancy metric. Let $\boldsymbol{c}_q$ is a codeword generated by re-encoding of OSD, and

$$\begin{aligned}\boldsymbol{c}_q &= \Phi^{-1}\left(\tilde{\boldsymbol{Z}}_q^{(I)} \times \tilde{\boldsymbol{G}}\right) \\ &= \Phi^{-1}\left((\tilde{\boldsymbol{Z}}_0^{(I)} + \boldsymbol{E}_q^{(I)}) \times \tilde{\boldsymbol{G}}\right) \\ &= (\tilde{\boldsymbol{Z}}_0^{(I)} + \boldsymbol{E}_q^{(I)}) \times \boldsymbol{G}_1\end{aligned} \quad (4)$$

where

$$\tilde{\boldsymbol{Z}}_q^{(I)} = \tilde{\boldsymbol{Z}}_0^{(I)} + \boldsymbol{E}_q^{(I)}, \quad (5)$$

$$\boldsymbol{G}_1 = \Phi^{-1}(\tilde{\boldsymbol{G}}) \quad . \quad (6)$$

$\tilde{\boldsymbol{Z}}_q$ could be viewed as a new vector generated by replacing some elements of at some positions within subsequence $\tilde{\boldsymbol{Z}}_0^{(I)}$ with adding an error pattern vector $\boldsymbol{E}_q^{(I)}$. When $\boldsymbol{E}_q^{(I)}$ is a zero vector, it generates the initial codeword $\boldsymbol{c}_0$ in OSD.

OSD always takes a long time to finish huge amount of computing metrics of each candidate codewords to pick the most likely one up. Complexity reduction for OSD method is a challenge in decoding processing for whether BPSK or QAM signaling. More clearly, in this re-encoding process of OSD algorithm of order-L for an (N, K) RS code with M-QAM signaling, the whole decoding process generates up to a maximum of $\sum_{i=0}^{L} M^i \binom{K}{i}$ candidate codewords. It needs very large computational complexity to generate all whole codewords.

In decoding processing, there are some ways to propose to modify the decoding process for complexity reduction, such as reduction of error patterns checked in re-encoding, reduction of codeword generation complexity of re-encoding, and reduction of the value of order-L and so on. A way to reduce codeword generation complexity in re-encoding is proposed in the following section.

Ⅲ. FEASIBLE ORDERED STATISTIC DECODING OF RS CODES

3.1 Proposed Erasure Decoding

Erasure decoding of a code with minimum distance $d_{\min}$ is capable of correcting all combination of v errors and e erasures provided that $2v+e+1 \le d_{\min}$ [1]. An (N, K) RS code with minimum distance $D_{\min}$ is with a property of maximum distance separable code (MDS), and then $D_{\min} = N - K$. By using erasure decoding, assuming that $v = 0$ and the number of erasures $e = D_{\min} - 1 = N - K$, an erasure sequence $\boldsymbol{R}^{(er)}$ contains all symbols in the original received sequence $\boldsymbol{R}$, but $N-K$ symbols, which are erased to be 0, with the less symbol reliability than the other K ones. Instead of re-encoding, the proposed way of adopting erasure decoding [2] is presented in the following.

Theorem 1: For $e = D_{\min} - 1 = N - K$, $X^{j_1}, X^{j_2}, \cdots, X^{j_e}$ are erasure locations and $\alpha^{j_1}, \alpha^{j_2}, \cdots, \alpha^{j_e}$ are their erasure-location numbers. Steps of proposed decoding are presented as

1) Compute erasure-location polynomial $\beta(X) = \prod_{l=1}^{e}(1-\alpha^{j_l}X)$.

2) Compute syndrome polynomial, $S(X) = S_1 + S_2X + S_{2t}X^{2t-1}$, and $S_i = R^{(e)}(\alpha^i)$, $1 \le i \le 2t$.

3) Evaluate values of the erased symbols, $f_{j_l} = \dfrac{[\beta(X)S(X)]_{2t}\big|_{X=\alpha^{-j_l}}}{\beta'(X)\big|_{X=\alpha^{-j_l}}}$, $1 \le l \le e$, where $[\beta(X)S(X)]_{2t} = T_0 + T_1X + T_2X^2 + \cdots T_{2t-1}X^{2t-1}$ and

$$\beta'(X) = \frac{d}{dX}\beta(X) = \sum_{l=1}^{e}\alpha^{j_l}\prod_{i=1, i\ne l}^{e}(1-\alpha^{j_i}X)$$

4) Obtain the initial codeword $\boldsymbol{c}_0$ by updating the values of erasure positions in the erasure sequence $\boldsymbol{R}^{(er)}$ with erased symbols f_{j_l}, $1 \le l \le e$.

5) Evaluate new values of the erased symbols, $f_{j_l}^{(i)} = f_{j_l} + \Delta f_{j_l}^{(i)}$, $1 \le l \le e$, where

$$\Delta f_{j_l} = \frac{[\beta(X)\Delta S_i(X)]_{2t}\big|_{X=\alpha^{-j_l}}}{\beta'(X)\big|_{X=\alpha^{-j_l}}},$$

and syndrome discrepancy polynomial is defined as $\Delta S_i(X) = \Delta S_{i,1} + \Delta S_{i,2}X + \cdots \Delta S_{i,2t}X^{2t-1}$, and $\Delta S_{i,j} = \boldsymbol{E}_i(\alpha^j)$, and a new error pattern polynomial $\boldsymbol{E}_i(X) = e_{i,0} + e_{i,1}X + \cdots + e_{i,N-1}X^{N-1}$ is generated in each iteration, and its vector is denoted as $\boldsymbol{E}_i = (e_{i,0}, e_{i,1}, \cdots e_{i,N-1})$. For these erasure positions in $\boldsymbol{R}^{(er)}$, their coefficients of $\boldsymbol{E}_i$ are 0.

6) Obtain a codeword $\boldsymbol{c}_i$ by updating the values of erasure positions in the erasure sequence $\boldsymbol{R}^{(er)}$ with addition of an error pattern $\boldsymbol{E}_i$, whose erased symbols f_{j_l}, $1 \le l \le e$, are not equal to all zeros.
7) Go to step 5) until all error pattern polynomials have been generated, and find a codeword with the smallest Euclidean distance to $\boldsymbol{R}$.

Proof: This proof is quite straightforward. For a modified erasure sequence $\boldsymbol{R}_i^{(er)}(X) = \boldsymbol{R}^{(er)}(X) + \boldsymbol{E}_i(X)$, its corresponding syndrome polynomial is computed as $S_i(X) = S(X) + \Delta S_i(X)$ and

$$\Delta S_i(X) = \Delta S_{i,1} + \Delta S_{i,2} X + \cdots \Delta S_{i,2t} X^{2t-1}$$

and $\Delta S_{i,j} = \boldsymbol{E}_i(\alpha^j)$. The new values of the erased symbols are evaluated as

$$f_{j_l}^{(i)} = \frac{[\beta(X)S_i(X)]_{2t}\Big|_{X=\alpha^{-j_l}}}{\beta'(X)\Big|_{X=\alpha^{-j_l}}}$$

$$= \frac{[\beta(X)(S(X)+\Delta S_i(X))]_{2t}\Big|_{X=\alpha^{-j_l}}}{\beta'(X)\Big|_{X=\alpha^{-j_l}}}$$

$$= \frac{[\beta(X)S(X)]_{2t}\Big|_{X=\alpha^{-j_l}} + [\beta(X)\Delta S_i(X))]_{2t}\Big|_{X=\alpha^{-j_l}}}{\beta'(X)\Big|_{X=\alpha^{-j_l}}}$$

$$= f_{j_l} + \Delta f_{j_l}^{(i)}$$

#

Since a field multiplication takes about 5 times execution instruction numbers of a field addition, the comparison of how many times of field multiplications needed in the proposed algorithm and OSD is listed in Table 1. The most complexity needed at matrix operations for obtaining the generation matrix $\boldsymbol{G}_1$ in re-encoding takes KN field multiplications, while the syndrome computation in the proposed algorithm takes $N(N-K)$ ones. The complexity of finding the initial codeword $\boldsymbol{c}_0$ is with $\frac{K^2(2N-K+1)}{2}$ + KN field multiplications in re-encoding and with $\frac{(N-K)(9N-7K+10+(N-K)^2)}{2} - 2$ ones in the proposed algorithm. For further more decoding, the complexity of finding the other codeword $\boldsymbol{c}$ is with KN field multiplications in re-encoding and with $\frac{(N-K)((N-K)^2+5(N-K)+2N+8)}{2}$ ones without computing erasure-location polynomial $\beta(X)$ in the proposed algorithm. By using computation to find the amount of field multiplications of a successive codeword $\boldsymbol{c}$ in both decoding algorithms, the proposed decoding algorithm can be adopted in some RS codes listed in Table 2 needs less complexity than re-encoding of OSD. For example, (255, 213) RS code with the smallest code over GF(2^8) is suitable to adopt the proposed decoding algorithm. The (255, 223) RS code specified in deep space communication is also suitable to employ this proposed decoding algorithm, provided that this code is transmitted over channels with QAM signaling.

Example: Consider a single-error correction (7, 5, 3) RS code over GF(2^3), which created by a minimal polynomial $1+X+X^3$. If a codeword $\boldsymbol{V} = (\alpha^2, 1, \alpha, 1, 1, 0, 0)$ is transmitted with 16-QAM signaling, then the received erasure sequence is $\boldsymbol{R}^{(er)} = (\alpha^2, 1, \alpha, *, *, 0, 0)$ and two lost field symbols (or erasure symbols) at positions 3 and 4 are denoted by *, which can be viewed as 0 in computation of the syndrome polynomial $S(X)$. The syndrome polynomial and erasure-location polynomial are $S(X)$ = $\alpha^6 + \alpha^5 X$ and $\beta(X) = 1 + \alpha^6 X + \alpha^7 X^2$, respectively.

Since

$$[\beta(X)S(X)]_{2t} = [\beta(X)S(X)]_2 = \alpha^6$$

and $\beta'(X) = \alpha^6$, then

$$f_{\alpha^3} = \frac{\alpha^6}{\alpha^6} = 1 = f_{\alpha^4}$$

The decoded output is $\boldsymbol{c}_0 = (\alpha^2, 1, \alpha, 1, 1, 0, 0) = \boldsymbol{V}$. Suppose the 5-th field symbol, 0, in $\boldsymbol{R}^{(er)}$ is the least reliable and is replaced by 1. That makes a updated erasure sequence $\boldsymbol{R}_1^{(er)} = (\alpha^2, 1, \alpha, *, *, 1, 0)$ and the corresponding error pattern polynomial is $\boldsymbol{E}(X) = X^{\mathbf{5}}$. The corresponding syndrome discrepancy polynomial becomes $\Delta S(X)$ = $\alpha^5 + \alpha^3 X$. After having computed the following equation,

$$[\beta(X)\Delta S(X)]_2 = \alpha^5 + \alpha^6 X,$$

we obtain two discrepancies $\Delta f_{\alpha^3} = \alpha^3$ and $\Delta f_{\alpha^4} = \alpha^4$. And the new values of erased symbols becomes

$$f'_{\alpha^3} = f_{\alpha^3} + \Delta f_{\alpha^3} = 1 + \alpha^3 = \alpha$$

$$f'_{\alpha^4} = f_{\alpha^4} + \Delta f_{\alpha^4} = 1 + \alpha^4 = \alpha^5$$

The new decoded output is $\boldsymbol{c}_1 = (\alpha^2, 1, \alpha, \alpha, \alpha^5, 1, 0)$. We can also verify these decoded outputs to be codewords. If the re-encoding process is adapted, the original generator matrix of (7, 5, 3)RS code is

$$\boldsymbol{G} = \begin{bmatrix} \alpha^3 & \alpha^4 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ \alpha^3 & \alpha^5 & 0 & 0 & 1 & 0 & 0 \\ \alpha & \alpha^5 & 0 & 0 & 0 & 1 & 0 \\ \alpha & \alpha^4 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

Based on coordinates in $\boldsymbol{R}_1$, the corresponding generator matrix is adjusted to the following

$$\boldsymbol{G}_1 = \begin{bmatrix} 1 & 0 & 0 & \alpha^3 & \alpha^5 & 0 & 0 \\ 0 & 1 & 0 & \alpha & \alpha^5 & 0 & 0 \\ 0 & 0 & 1 & \alpha & \alpha^4 & 0 & 0 \\ 0 & 0 & 0 & 1 & \alpha^4 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 \end{bmatrix}.$$

The corresponding codeword is $\boldsymbol{c}_0 = \boldsymbol{U}_0 * \boldsymbol{G}_1 = (\alpha^2, 1, \alpha, 1, 1, 0, 0)$, and $\boldsymbol{U}_0 = (\alpha^2, 1, \alpha, 0, 0)$. As $\boldsymbol{E}_1^{(I)} = (0, 0, 0, 1, 0)$, $\boldsymbol{U}_0 + \boldsymbol{E}_1^{(I)} = \boldsymbol{U}_1 = (\alpha^2, 1, \alpha, 1, 0)$ is used in decoding, the corresponding codeword is $\boldsymbol{c}_1 = (\alpha^2, 1, \alpha, \alpha, \alpha^5, 1, 0)$. Assume all inverse elements for each field symbol are stored in memory within decode proceeding and fetch them would not take much complexity, which can be ignored. Comparing both decoding, the proposed erasure decoding algorithm needs 19 field symbol multiplications for $\boldsymbol{c}_0$, and 8 field symbol multiplications for $\boldsymbol{c}_1$. However, for re-encoding process, there are 125 field symbol multiplications for $\boldsymbol{G}_1$, and 35 field symbol multiplications for $\boldsymbol{c}_0$ and $\boldsymbol{c}_1$. This proposed erasure decoding algorithm does indeed save a lot of complexity during the decoding process.

3.2 Simple Scheme for Ordered Statistic Decoding

In this study, base on the three-sigma rule [8] theory, we propose simple scheme for searched points. If a QAM signal is transmitted through an AWGN channel, value of noise abides by Gaussian distribution with mean $\mu = 0$ and standard deviation $\sigma = \sqrt{N_0 / 2}$ for vertical or horizontal coordinate, respectively. It means almost values of noise distribute between $-\sqrt{2N_0}$ and $\sqrt{2N_0}$ for each element of a QAM signal in general. A proposed scheme of the nearest neighbors is proposed as follows.

Simple Scheme: In this scheme, only four nearest neighbors of a received symbol are selected in each element replacement during in the proposed OSD. In other words, some symbols in an erasure sequence are replaced by their four-nearest neighbors. For example, for order-J OSD to decode an (N, K) RS code with M-QAM signaling, there are $\sum_{i=0}^{J} 4^i \binom{K}{i}$ error patterns, instead of $\sum_{i=0}^{J} M^i \binom{K}{i}$, whose decoding complexity reduces greatly at expense of a little lose of coding gain.

IV. NUMERICAL RESULTS AND COMPARISON

Figure 1 depicts the error performances of the (15,9,7) RS code with 16-QAM signaling over AWGN channels. The simulations are shown for the proposed simple scheme. In such a way, there are four different amounts of error patterns checked in decoding. In order-1 decoding with proposed simple scheme, there are $\sum_{i=0}^{1} 4^i \binom{9}{i} = 37$ error patterns checked, and a coding gain between hard decision decoding is about 1dB at word error rate (WER) 10^{-4}. And order-2 decoding with complexity of $\sum_{i=0}^{2} 4^i \binom{9}{i} = 613$ error patterns checked provides a coding gain is about 1.4 dB at WER 10^{-4}, while order-3 decoding provides 1.5 dB at expense of $\sum_{i=0}^{3} 4^i \binom{9}{i} = 5,989$ As for the full-order decoding with proposed simple scheme, since the amount of $\sum_{i=0}^{9} 4^i \binom{9}{i} = 1,953,125$ error patterns is checked during in each decoding process, there is a coding gain of 2 dB at WER 10^{-4}. From these simulations, if the decoding complexity is limited, the value of order-J in such a proposed decoding scheme can be chosen as a low value. If the coding gain is important, the value of order-J could be chosen as a high value. In comparison of decoding complexity and coding gain, although coding gain of order-1 OSD is worse than that of full-order OSD by about 1 dB, however, the speed of order-1 decoding is faster about 50,000 times than full-order does.

V. CONCLUSIONS

In this study, ordered statistic decoding of RS codes with QAM signaling has been investigated. A proposed erasure decoding presented is to replace re-encoding in OSD. To decode a proper RS codes with this proposed way, which is listed in Table 1, would take less complexity than re-encoding. From simulations, by choosing a proper value of order-J in OSD, this proposed algorithm is realized feasibly and provides significantly good results.

Table 1. Comparisons of the worst case in a codeword generation complexity of OSD and the proposed algorithm

	OSD		Proposed Algorithm	
	G_1 obtained in re-ordering	a codeword generation in re-encoding		
Field Multiplication	$\frac{K^2(2N-K+1)}{2}$	KN	$S(X)$, $\Delta S_l(X)$	$N(N-K)$
			$\beta(X)$	$(N-K+2)(N-K-1)$
			f_λ, $f_\lambda^{(l)}$, $1 \le l \le e$	$(\frac{(N-K+5)(N-K)}{2}+4)(N-K)$

Table 2. The proposed algorithm employed in decoding in following RS codes needs less complexity than OSD

Code length, N symbols	Shortest information length, K symbols	Code rate = K/N
15	11	0.73
31	23	0.74
63	49	0.77
127	103	0.81
255	213	0.83
511	441	0.86
1023	909	0.88

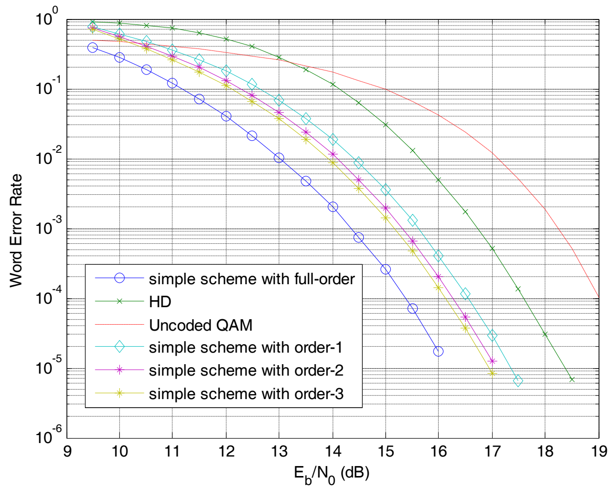

Figure 1. With 16-QAM signaling, error performances of (15,9,7) RS code by employing the proposed OSD simple scheme.

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