Probability Distributions of Diversity Combiner Output in a Rayleigh Fading Environment

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ABSTRACT

Probability distribution of the power of diversity combiner output signals in a Rayleigh fading environment is presented in this paper. If diversity techniques are to be used in the systems, an anomaly, the misalignment of received symbols, can severely degrade system performance. The probability density functions of the output power reveal the fact that signals from different branches may very often cancel each other due to random phase shift during transmission. A misalignment-free receiver is proposed to overcome this anomaly. Even by choosing a small alignment window, successful removal of the anomaly in systems with low orders of diversity, such as second- and third-order systems, is demonstrated. Simulation results show that the error performance of the proposed receiver can be close to that of a coherent system.

Keywords: coherent system, diversity techniques, nonlinear phase synchronizer

在瑞立衰減環境中合併器輸出之機率分佈

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摘　　要

本論文主要討論在瑞立衰減環境中，分集合併器之輸出訊號能量之機率分佈。如果在系統中使用分集技術，就會產生所謂的錯位現象，並會明顯影響系統之錯誤率表現。由分集合併器輸出功率的機率密度函數，可發現從各個分枝傳到接收端的訊號，常因傳輸過程的隨機相位偏移，而導致互相抵消。本文提出一組無錯位接收器架構，並證實了在使用低階 (二階或三階) 分集技術的系統中，即使只使用很小的錯位視窗，仍然能將系統錯誤率降低至接近同調系統。

關鍵詞：非線性相位同步器，分集技術，同調系統

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Ⅰ. INTRODUCTION

Reliable synchronization is required in many communication systems. The phase-locked loop (PLL) is the most commonly used circuit for synchronization systems [1-3]. However, PLLs can exhibit several anomalous performance characteristics such as instabilities and hangup [4]. Recently, carrier synchronization using digital techniques has received a considerable amount of attention [5]. These techniques have been applied to carrier synchronization as an alternative to the PLL to prevent hangup.

A nonlinear open-loop phase estimator for a phase-shift-keying (PSK) modulated carrier was developed by Viterbi [6]. One of its primary applications is carrier synchronization in fading channels. The nonlinear phase estimator structure has been successfully utilized in time-division multiple access (TDMA) systems, but an anomalous behavior (misalignment) degrades the performance for communications in fading channels if diversity techniques are to be used. The misalignment anomaly results from phase ambiguity during phase estimation. Because the data is differentially encoded, the bit error probability (BEP) performance is unaffected when diversity techniques are not used in the system. However, when diversity techniques are applied, the BEP performance is severely degraded because the phase ambiguity associated with each of the channel branches is unknown, and the diversity combiner is unable to optimally use the information obtained from the branches. The purpose of this paper is to propose a misalignment-free nonlinear phase estimation structure.

The paper is organized as follows. Section 2 describes the signal, channel, and receiver models that are used in fading channel communications. Section 3 describes the nonlinear phase estimation-based demodulator for QPSK modulations. Diversity and combining techniques are also described. In Section 4, the presentations of misalignment, the cause of misalignment, and a simple example to demonstrate the degradation caused by misalignment are provided. Section 5 provides the misalignment-free nonlinear phase estimator architecture and simulation results for various sizes of alignment windows. The probability distributions of the output power of the combined symbols are presented in Section 6. Finally, Section 7 concludes this paper.

Ⅱ. Signal and Channel Models

According to the central limit theorem (CLT), signals received from fading channels without line-of-sight (LOS) paths tend to have Rayleigh-distributed magnitudes and uniform random phases [7,8]. A model of both the channel and detector of fading channel communication is shown in Fig. 1. For a fading channel with additive noise, the transmitted signals are multiplied by a complex channel gain $Re^{j\phi}$, which is random in both magnitude and phase. For a Rayleigh fading channel, the magnitude R of the multiplicative distortion is Rayleigh distributed and the phase is uniform over [0, 2). The noise is assumed to be additive white Gaussian noise, and the receiver is matched to the modulation pulse shape. The pulse shapes are assumed to satisfy the Nyquist condition, and intersymbol interference (ISI) is negligible.

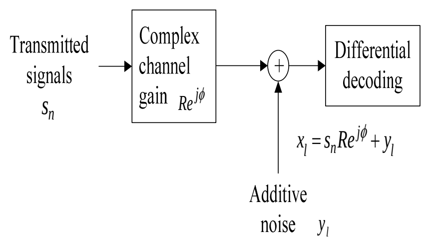

Fig.1. Model of fading channel communications.

At the receiver, the received symbols are derotated by the phase estimate $\hat{\phi}$ of the complex channel gain; that is, $e^{-j\hat{\phi}}$ is multiplied by the detected signals to remove the phase rotation caused by the channel during transmission. The phase estimate is obtained using a nonlinear phase estimator. Because phase ambiguity inevitably occurs, which is a multiple of 2/*M* for *M*-ary PSK modulated systems during nonlinear phase estimation, the data are typically differentially encoded. Assuming that differentially encoded QPSK modulated signals are transmitted, the derotated symbol is mapped by the analog-to-digital converter (ADC) to a two-bit digital symbol before being sent to the differential decoder. Then, two successive digital symbols are compared and the estimate of the data signal is determined by a decoding rule.

Ⅲ. Nonlinear Phase Estimation and Diversity Combining

The nonlinear phase estimator for QPSK-modulated signals is shown in Fig. 2. It is also called an open-loop synchronizer because the structure does not contain a feedback path as in PLL synchronizers. The received symbols from the l-th diversity branch at time instant n is

$$x_l = s_n R e^{j\phi_l} + y_l \quad (1)$$

where s_n is the transmitted signal, y_l denotes an additive noise, and $R_l e^{j\phi_l}$ represents the multiplicative distortion caused by the fading channels. Because the AWGN is assumed to have a zero mean, the phase estimator is unbiased when the fading process is assumed constant. However, due to the randomness of the additive noise and the fading process, the phase estimate cannot be exactly equal to the true phase for finite-sized synchronizers. According to the central limit theorem, the phase estimate can be made arbitrarily accurate by using a large-sized synchronizer window if the multiplicative distortion does not change over time. However, this is not possible because of the time variation of the fading process. For slow-fading channels, the fading process is assumed to be almost constant during several signal periods. Under this assumption, the nonlinear phase synchronizer can provide an unbiased phase estimate if the size of the synchronizer window is chosen so that the fading process remains constant during 2M + 1 signal periods.

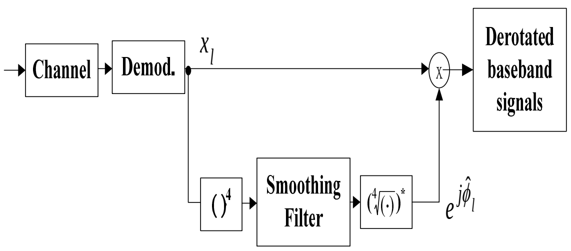

Fig. 2. The nonlinear phase estimator for QPSK modulated signals.

Communications using a fading channel are considerably less accurate than those using a non-fading channel [9]. Increasing diversity is a practical method of improving the performance of a fading channel. In a system with L-th order diversity, the total transmission power is the sum of the individual power transmitted through each channel branch. The block diagram of a diversity combiner is shown in Fig. 3. The phase estimate $\hat{\phi}_l$ of the l-th diversity branch for the receiver structure shown in Fig. 2 is

$$\hat{\phi}_l = \arg\left\{\left[-\sum_{i=-M}^{M}\left(x_i\right)^4\right]^{1/4}\right\}, \; l=1, 2,\ldots, L,$$

where [-M, M] is the synchronizer window with a size of 2M+1, and x_i is the received symbols at time i. The diversity combiner collects the information from the L branches and performs a linear combination of these data. Assuming that the diversity branches are independent of each other, the phase distortion in the branches is also independent. Therefore, the phase ambiguity associated with the L inputs to the diversity combiner is independent. This is problematic because the undesirable cancellation of the data may occur if combining is performed directly on these inputs. This is called the misalignment of data and is described in the following section.

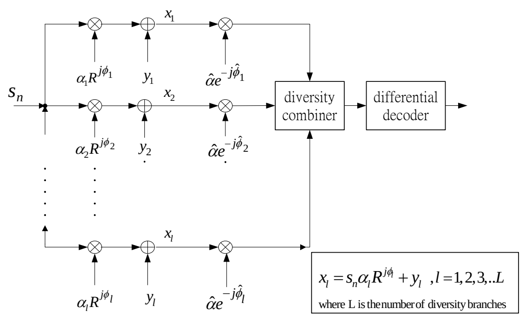

Fig. 3 .The block diagram of a diversity combiner.

Ⅳ. Misalignment for Diversity Combining

The phase ambiguity incurred during nonlinear phase estimation can be removed by differentially encoding and decoding. However, if diversity techniques are to be used, the misalignment of the receiver output from all channel branches gives rise to an anomaly. Misalignment means that the inputs to the diversity combiner are not well aligned and the combination typically does not enable the optimal use of the information from each channel branch. This anomaly is also due to the ambiguity produced by nonlinear phase estimation. Because the phase estimator can only detect the relative phase between successive received symbols, the entire absolute phase is missing during estimation. Therefore, even if the phase is well recovered in each channel branch, the receiver output of a diversity branch may remain entirely "out of phase" with the others. This can be a severe problem even when the signal-to-noise ratio (SNR) is high. For example, assume that two successive symbols $s_{-1} = 1 + j$, $s_0 = -1 + j$ are transmitted through a slow-fading channel to a receiver with second-order diversity. Additive noise is negligible and, consequently, nonlinear phase estimation is accurate except that a phase ambiguity exists for each diversity branch. The multiplication distortion associated with the two branches are 1 and $e^{j\pi}$, respectively. The fourth power of all the received symbols have zero phases; therefore, the phase estimates for both channel branches is zero. Let $x_{l,k}$ denote the synchronizer output of the *l*-th branch at the time instant *k*. Then, $x_{1,-1} = 1 + j$, $x_{1,0} = -1 + j$ and $x_{2,-1} = -1 - j$ and $x_{2,0} = 1 - j$ are obtained. If no proper aligning procedure is conducted before the combining, these symbols cancel each other at the time instants $k = -1, 0$, and the output of the diversity combiner is zero. This implies that the signal information is completely missing, and the conditional error probability under this circumstance is 0.5. One approach to solve this problem is to use selection diversity combining, in which only one of the channel branch outputs is used to determine the signal information at each time instant and all others are discarded. This can prevent the effect caused by phase misalignment. However, selection diversity combining performs less favorably than maximal-ratio combining for high orders of diversity does because it involves disregarding a large portion of the transmitted information. Brennan [10] showed that, for fourth-order diversity, maximal-ratio diversity combining exceeds selection combining by 6.02 dB, which is a significant difference. Regarding error performance, an alignment scheme for the various branches is preferable to disregarding a large amount of valuable information.

Ⅴ. A Misalignment-Free Diversity Combiner

For noiseless communications, alignment can be accomplished by simply rotating the synchronizer outputs in each channel branch in such a way that the symbols at any time instant appear at the same quadrant. For the example with second-order diversity given in Section 4, the data can be rotated from the second branch by radians and the synchronizer outputs are perfectly aligned. However, in practice, this method is ineffective because additive noise can sometimes cause erroneous alignment. This problem is particularly severe under low SNR environments. A more practical approach involves observing a sequence of synchronizer output symbols, fixing the symbols from one of the channel branches, and then determining the amount of rotation for all other branches that maximizes the output energy of the diversity combiner. Therefore, the proposed receiver structure is called the maximal-energy selection diversity combiner (MESC).

Let r_l be the received signal at the *l*-th diversity branch, *l*= 1,2,…, *L*, then

$$r_l = \alpha_l e^{j\phi_l} s + Z_l \tag{2}$$

where *s* is the transmitted signal, and Z_l denotes the additive noise. The complex multiplication factor associated with the *l*-th diversity branch is written as $\alpha_l e^{j\phi_l}$, where α_l and ϕ_l represent the scalar attenuation factor and phase distortion of the *l*-th branch, respectively. The received signals in each branch are derotated using phase estimates obtained from nonlinear synchronizers. Therefore,

$$x_l = \left[\alpha_l e^{j\phi_l} s + Z_l \right] \cdot e^{-j\hat{\phi}_l} \tag{3}$$

To determine the optimal alignment, the signals

$$r_{l,k_l} = x_l e^{jk_l \frac{\pi}{2}} \tag{4}$$

are computed for $k_l = 0,1,2,3$ and $l = 2,3,4$. Next, the derotated signals are fixed from the first branch to reduce the computational load. Then, the maximum of the random variables

$$Z_{k_2,k_3,\ldots,k_L} = x_1 + \sum_{l=2}^{L} r_{l,k_l} \tag{5}$$

is computed using a selector. That is,the output signal from the MESC is.

$Z = \max_{k_2,k_3,\dots,k_L}\left\{Z_{k_2,k_3,\dots,k_L}\right\}$,The block diagram of a misalignment-free diversity combiner is shown in Fig. 4. This approach can accurately align signals from channel branches that are not deeply faded. However, for deeply faded signals, the conditional density functions of the received signals are close to zero-mean complex Gaussian densities because of the low SNR. Therefore, an error might occur in the alignment procedure, and the symbols transmitted through deep fading channels might be incorrectly rotated. Fortunately, this has little influence on the overall performance because the symbols transmitted through channel branches with a high SNR dominate in the diversity combiner output.

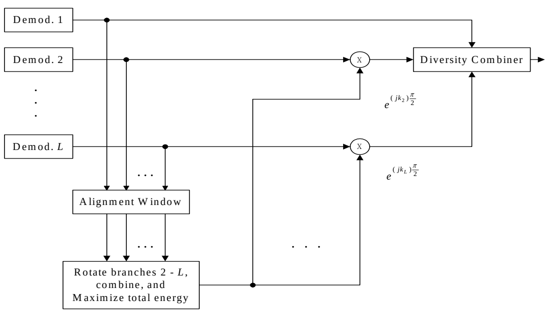

Fig.4. The block diagram of a misalignment-free diversity combiner.

Ⅵ. Probability Distributions of the Diversity Combiner Output

The diversity combiner output energy and the total energy before combination for systems with and without MESC are shown in Fig.5. and Fig.6., respectively. Fig.5. shows that the misalignment phenomenon results in severe power cancellation during combination. Even at a low order of diversity, such as L = 2, the power loss resulting from misaligned combination can be substantial. For example, the signal power drops from 35 to 21.8 after the misaligned diversity combination. Therefore, the total power loss is (35-21.8)/35=38%. When the order of diversity increases, the loss increases. For fourth-order diversity, the total diversity combiner output energy is only 15.7, which is less than 45% of the original transmitted power. The simulation result depicted in Fig. 6. demonstrates that the MESC can effectively align the output signals from all diversity branches because the diversity combiner output power is relatively close to the original total power.

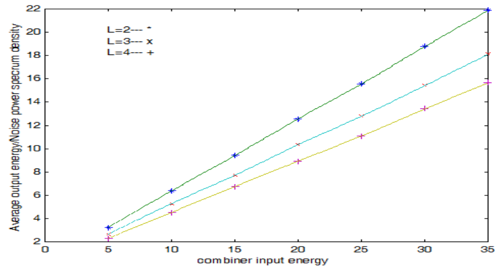

Fig. 5. the output energy vs. total input energy of the diversity combiner without MESC.

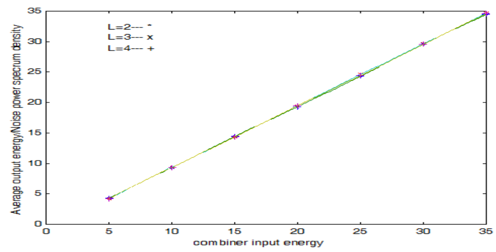

Fig. 6. The output energy vs. total input energy of the diversity combiner with MESC.

The probability density functions (pdf) of the output energy from the diversity combiner with and without the MESC are shown in Fig. 7 and Fig. 8, respectively. According to Fig. 7, the probability of extremely low output energy is very high, which is the consequence of misaligned combining. With the application of the MESC, the pdf at a low output energy is essentially zero, as shown in Fig. 8, which demonstrates the effectiveness of the proposed strategy.

The simulation results for second- and third-order diversity are shown in Fig. 9 and Fig. 10, respectively. The dotted lines in the figures are the performance curves for perfect alignment; that is, the optimal performance. $2K+1$ is defined as the length of the sequence used for aligning the data. Better alignment is achieved when a large K value is chosen. However, choosing a large K value is unfavorable because the complexity of the system increases exponentially when the value of K increases. The figures show that, for low orders of diversity, even a small K can provide substantial improvements of error performance. Therefore, the misalignment-free receiver is a practical design for fading channel communications with diversity.

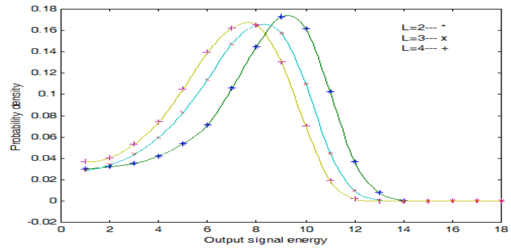

Fig. 7. The pdf of the diversity combiner output energy without MESC.

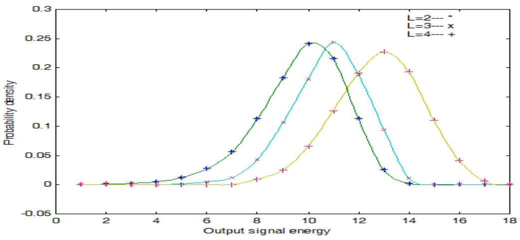

Fig. 8. The pdf of the diversity combiner output energy with MESC.

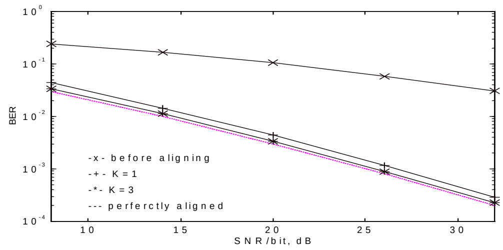

Fig.9. Simulation results for second order diversity.

Ⅶ. Conclusion

This paper presents the probability distribution of the power of diversity combiner output signals in a Rayleigh fading environment. Simulation results show that the misalignment anomaly of the nonlinear phase estimation can severely degrade system performance when diversity techniques are used. We have proposed a misalignment-free nonlinear phase estimator which can effectively remove the misalignment anomaly. This is crucial for fading channel communications because the commonly used decision-directed phase estimator has several anomalies that reduce its efficiency in the fading channels. Future work will involve an analytical description of the misalignment and the choice of the alignment window sizes for different sizes of synchronization windows.

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