Study on MIMO Beamforming Deploying with 3D Fading Channels

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ABSTRACT

Both AoA (angle of arrival) and AoD (angle of departure) in deployment of 3D (three dimensions) channel model, which impact the MIMO (multi-input multi-output) beamforming system over the spatial correlated-fading channel, is examined in the report. The channel correlation coefficient is applied to obtain the numerical results alternatively. The scenario is with corresponding to correlated Nakagami-*m* distributions are characterized as the fading specification in the analysis. The CF (characteristic function) is applied to determine the channel capacity for avoiding the difficulty in calculating the pdf (probability density function) of SNR (signal to noise ratio) traditionally. Furthermore, there are numerical results are offered for validating the accuracy of the derived theoretical formulas. Eventually, plots work out from combination with different number of transmitter and receiver for comparison is studied. The increment in AoA parameters, which is alternatively expressed as the channel correlation, definitely generates the impact of the system performance when the consideration of 3D channel.
Keywords: AoA; CF; Nakagami-*m* distributions; MIMO Beamforming system.

佈署 MIMO 波束成型機制於 3D 衰落通道之研究

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摘　要

本論文，研究通道相關特性之衝擊於 MIMO 波束成型系統，考慮其工作在空域或相關衰落通道中，並且假設三維通道模型的佈署，另外，也討論到信號到達角度(AoA)與信號離開角度(AoD)。分析中，藉相關性中上(Nakagami-*m*)分布作為衰落通道場景。避免傳統方式困難，透過特徵函數(CF)對訊雜比推導的機率密度函數(pdf)；之後，提供數值結果分析，經組合不同發射與接收天線數目的多少進行通道容量探討；研究發現，若以通道相關替代 AoA 之係數，當通道容量以相關係數作為討論時，對系統效能惡化的影響，不會低於 MIMO 系統中，假設三維通道模型對於通道容量所產生之衝擊結果。

關鍵詞：到達角度，特徵函數，中上(Nakagami-*m*)分布，MIMO波束成形。

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I. INTRODUCTION

Accordingly, it is known that the infrastructure is assigned with 5 layers for the 5G (5^{th} generation) wireless system. Physical and Medium Access Control layers are definitely impacting the system performance of a 5G cellular system. Extensive AoD (angle of departure) and AoA (angle of arrival), path loss, and multipath time delay spread measurements were conducted for steerable beam antennas of differing gains and beam widths for a wide variety of transmitter and receiver locations. Coverage outages and the likelihood of outage with steerable antennas were also measured to determine how random receiver locations with differing antenna gains and link budgets could perform in future cellular systems. The paper also provides measurements and models that may be used to design future 5G MMW (milli-meter wave) cellular networks and given insight into antenna beam steering algorithms for these systems. The channel correlation has been proved and measured definitely impact the channel capacity of a MIMO (multiple-input multiple-output) system [1-2]. In addition to, [3] in which the authors described that the potential capacity gain is highly dependent on the multipath richness, since a fully correlated MIMO radio channel only offers one subchannel, while a completely decorrelated radio channel potentially offers multiple subchannel depending on the antenna configuration. So far as, a huge numbers of research explored the issue about MIMO. In [4] authors studied the optimal transmission strategy of MIMO considered single user with beamforming and covariance feedback, and derived results shown that the optimal transmission strategy was generated in the direction of the eigenvectors of the transmit correlation matrix. Authors in [5] expressed the exact BER (bit error rate) analysis for different modulation schemes in a correlated Rayleigh MIMO channel were presented. In [6] the capacity performance has been investigated for a multiple antennas system which was proposed to switch among different MIMO transmission schemes include statistical beamforming, double space-time transmit diversity, and spatial multiplexing over spatially correlated channels.

Authors claimed that exact BER (bit error rate) analyzed for different modulation schemes in a correlated Rayleigh MIMO channel were presented in [7]. In [8] the capacity performance has been investigated for a multiple antennas system which was proposed to switch among different MIMO transmission schemes include statistical beamforming, double space-time transmit diversity, and spatial multiplexing over spatially correlated channels.

The exact closed-form expression of a CF (characteristic function) of mutual information for MIMO channels both are with interference in the presence of correlated Rayleigh plus CCI (co-channel interference) was derived by authors in [9]. The linear scaling of throughput achieved in MIMO system for wireless networks was challenged. The authors described that linear scaling can be achieved in wireless networks using only receive antenna arrays, without using multiple transmit antenna [10].

All the previously mentioned results obtained from the assumption of that are with azimuth direction of the propagation channel. However, in fact the evaluation of the MIMO-beamforming signaling which should consider the direction of elevation for wave propagation, *i.e.*, the vertical direction has to be included in discussion the channel model [11]. In the previous work authors are to shed light on the current 3GPP activity around 3D (three dimension) beamforming (3D-BF) and FD-MIMO. They have enabled to enhance the understanding of the current industrial challenges as well as to deeply hint the standards' vision for 3D-BF. Initial implementations of the 3D-BF technology support the potential of this technique for yielding significant gains in real indoor and outdoor is deployed in [12]. There is one way to exploit the additional degree of freedom of 3D channels is to adapt for each user the beam pattern in the vertical direction, thereby improving the signal strength at the receiver and at the same time reducing the interference to other users. The authors encouraged by the preliminary results in [12], and extended the research activity on 3D channel modeling which carried out by both theoretical researchers and industrials [13]. In [14] the authors investigate interference coordination for 3D antenna array systems in multicell MIMO and OFDMA (orthogonal frequency division multiple-access) wireless networks. They take the specifications down tilts of subscribe into consideration for the interference coordination to maximize both the cell-edge user's and cell center users' throughput. An extension of the ITU2D channel model to 3D is proposed [15] in which based on observations from ray tracing a distance dependent elevation spread is added. Through system-level simulations we observe that the behavior of 3D MIMO is greatly impacted by the modeling of the 3D channel. After review the aforementioned contributions, the present authors focus on the consideration of the correlation effect in the transmitting and receiving branches, which were considered as having an arbitrary correlation coefficient and the AoA. It is known that the 3D channel models have frequently studied and there are many researches published recently [11-15, 20]. However, to the best of authors knowledge, there are still not the work of evaluation discussed about the MIMO with BF system over 3D channel model. This article is definitely a novelty publication. On the other hand, it is truely difficulty to compare the results from the deployment of this report with the others. For an MIMO system with a 3D-BF receiver combining with MRC (maximal ratio combining) operating over correlated fading channel is examined, in both different transmitter and receiver antennas environments. The impact of correlation on the performance of an MIMO system is analyzed with two well known statistical distributions, *i.e.*, correlated Indoor and Nakagami-*m* distributions. This report is organized as follows. In Section II, an MIMO system with 3D-BF combination and the statistical models of a fading channel are presented first; then, the CF (characteristic function) of the spatial correlated fading is derived. In Section III the channel capacity performance of the MIMO beamforming system with fading 3D channel is evaluated. Following the analytical results reported in Section III, a validation of the derived theoretical formulas is with numerical manifestation presented in Section IV. Finally, Section V concludes the report.

II. MODEL OF MIMO BEAMFORMING SYSTEM WITH 3D CORRELATED CHANNEL

In order to enable the optimization of a system performance for a next generation system, the technique with the consideration of channel between antenna elements rather than that between antenna ports is held. At the investigation of applying the utilization of 3D-BF is usually assumed that antennas are arranged in a 2D array where each column contains *M* antenna elements. There are exactly *K* antenna elements per antenna port with a pattern A_E is given by $A_E(\phi_e,\ \theta) = \mathrm{G}_{E,\max}\{-(A_H(\phi_e) + \mathrm{A}_V(\theta)),\ \mathrm{A}_m\}$, where $A_m = 30dB,\ G_{E,\max} = 8dBi$, and $A_H(\phi_e) = -\min\left[12(\phi_e/\phi_{3dB})^2,\ A_m\right]$, where $\phi_{3dB} = 65^0$, moreover, where $A_V(\theta) = -\min\left[12(\theta - \theta_{tilt}/\theta_{3dB})^2,\ \mathrm{SLAV}\right]$, in which $\mathrm{SLAV} = 30$ and $\theta_{3dB} = 65^0$. A simple deployment of 3D channel model is shown in Fig. 1 which is discussed in [11]. There are two situations either choosing $K = M$ or setting $K = 1$, in the former one the number of antenna ports per column is equivalent to *M* which obeys the rule of 3GPP group TSG-RANWG1. Each column in the situation corresponds to one port with the same polarization. Therefore, each column would correspond to two ports, (one port per polarization), if cross-polarized elements were used. A weighted sum of channel with *K* elements is assigned to antenna port. The channel with a given antenna port is more formally, the channel between the *t-th* receiving antenna and the *s-th* antenna port corresponding to the *n*-th path is given by

$$\left[\bar{H}_n\right]_{s,t} = \sum_{antenna \in port\ \ s} \omega_{antenna} \left[H_n\right]_{antenna.t} \tag{1}$$

where the sum presented above is performed over all antenna elements in port *s*. Accordingly, if the real 3D channel is considered, each antenna port adopts that deploy in [11], and which is rewritten as

$$\begin{aligned} h_{u,s,n}(t;\tau) = \sum_{m=1}^{M} & \begin{bmatrix} {}^{F}Rx,u,V(\varphi n,m) \\ {}^{F}Rx,u,H(\varphi n,m) \end{bmatrix} \begin{bmatrix} \alpha_{n,m}^{VV} & \alpha_{n,m}^{VH} \\ \alpha_{n,m}^{HV} & \alpha_{n,m}^{HH} \end{bmatrix} \begin{bmatrix} {}^{F}Tx,u,V(\phi n,m) \\ {}^{F}Tx,u,H(\phi n,m) \end{bmatrix} \\ & \times \exp(j2\pi\lambda_0^{-1}(\bar{\varphi} n,m \cdot {}^{\bar{r}}Rx,u)) \exp(j2\pi\lambda_0^{-1}(\bar{\phi} n,m \cdot {}^{\bar{r}}Tx,s)) \\ & \times \exp(j2\pi\upsilon_{n,m}t)\delta(\tau - \tau_{n,m}) \end{aligned} \tag{2}$$

However, the channel model will be assumed as the correlated-Nakagami-*m* fading, since there is the difficulty in implementation of 3D fading channel for MIMO beamforming scheme.

Consider MIMO beamforming scheme employing N_T transmitter and N_R receiver antennas is equipped with the receiver signal vector $Y_R \in \boldsymbol{C}^{N_R \times 1}$ and the transmit signal vector

$X_T \in C^{N_T \times 1}$, respectively. Normally, the radio MIMO channel is a $N_T \times N_R$ matrix and which is able to be re-written as

$$H(\mathrm{t},\ \tau) = \int \vec{g}_r(\Psi)^T h(\mathrm{t},\ \tau,\ \Omega,\ \Psi)\vec{\mathrm{g}}_T(\Omega) \times \vec{\mathrm{a}}_R(\Psi)(\vec{\mathrm{a}}_T(\Omega))^T d\Omega d\Psi \tag{3}$$

where $\vec{g}_r$ and $\vec{\mathrm{g}}_T$ are the patterns of the receiving and transmitting antennas, respectively. $\vec{g}_r(\Psi)$ and $\vec{\mathrm{g}}_T(\Omega)$ are 2×1 vectors whose entries represent the vertical and horizontal field patterns, when polarization is considered. Vectors $\vec{\mathrm{a}}_R(\Psi)$ and $\vec{\mathrm{a}}_T(\Omega)$ are the array responses of the transmitting and receiving antennas whose entries are given by $\left[\vec{a}_R(\Psi)\right]_i = \exp(j\,\vec{\mathrm{k}}_{R,\Psi} \cdot \vec{\mathrm{x}}_{R.i})$, $\left[\vec{a}_R(\Psi)\right]_i = \exp(j\,\vec{\mathrm{k}}_{\mathrm{T},\Psi} \cdot \vec{\mathrm{x}}_{\mathrm{T}.i})$, respectively, and where $\vec{\mathrm{x}}_{R.i}$ is the location vector of the *i-th* receiving antenna whereas $\vec{\mathrm{x}}_{\mathrm{T}.i}$ is that of the *i-th* transmitting antenna. The transmitted signal power is constricted as the value equivalent to the number of transmitter antennas, *i.e.*, $P_T = E\left[\|X_R\|^2\right] = N_T$, and the MIMO channel complex matrix $\mathbf{H} \in C^{N_T \times N_R}$ is characterized as the spatially corrected distributed. The no-coherent channel matrix can be written as $\bar{\mathbf{H}} = \mathbf{M}_R^{1/2}\mathbf{Z}\mathbf{M}_S^{1/2}$, where $\mathbf{Z} \in C^{N_R \times N_T}$ is including *i.i.d* Gaussian entries with zero mean and unit variance, *i.e.*, $\mathbf{Z} \sim CN_{M_R M_S}\left(0,\ \sigma_{M_R M_S}\right)$, where $\sigma_{M_R M_S}$ is the covariance matrix of the random matrix $\mathbf{Z}$, and $\mathbf{M}_R$ and $\mathbf{M}_S$ are corresponding to denote the matrices of the receiving and transmitting spatial correlation.

In addition, the *n*-th eigenvalue of $\mathbf{M}_R$ and $\mathbf{M}_S$ are designated as $\lambda_i^{M_R},\ i = 0,\cdots,N_R - 1$ and $\lambda_i^{M_S},\ i = 0,\cdots,N_T - 1$, respectively. It is known that by using of eigenvalue decomposition method the spatial correlation matrices can be in advanced re-written as $\mathbf{M}_R = U_{M_R}\Lambda_{M_R}U_{M_R}^{\dagger}$ and $\mathbf{M}_S = U_{M_S}\Lambda_{M_S}U_{M_S}^{\dagger}$, respectively, where the symbol † denotes conjugate transpose where U_{M_R} and U_{M_S} are containing the corresponding entries of the eigenvectors with respect to the related eigenvalues, while Λ_{M_R} is a diagonal matrix with eigenvalues, which are obtained from spatial correlation matrices, in the main diagonal elements.

Thereafter, under the consideration of transmitting channel, $\mathbf{H}$, which is possible characterized as no-coherent and coherent components of the channel models' matrix and with spatial correlation. Once the propagating channel scenario is constructed completed, the received signal intensity, $\mathbf{Y}_R \in C^{N_T \times 1}$, for each spatial correlated channel experienced fading environment can be expressed as

$$\mathbf{Y}_R = \sqrt{E_s/N_T}\,\mathbf{H}\mathbf{X}_T + \mathbf{N} \tag{4}$$

where E_s is the symbol energy, $\mathbf{X}_T \in \mathbb{C}^{N_T \times 1}$, and $\mathbf{N} \in \mathbb{C}^{N_R \times 1}$ denote the transmit signal energy which is constrained in $E\left[\left|\mathbf{X}_T\mathbf{X}_T^{\dagger}\right|^2\right] = N_T\mathbf{I}_{N_R}$, and the additive Gaussian noise vector with zero-mean and covariance $E\left[\mathbf{N}\mathbf{N}^{\dagger}\right] = N_0\mathbf{I}_{N_R}$ respectively, where $N_0/2$ denotes as double-sided power spectral density of AWGN. The SNR (signal-to-noise ratio) at the receiver is defined as $\gamma_s = E_s / N_0$

III. Channel Capacity with 3D-BF System

In this section channel capacity of a MIMO system with 3D-BF transmission scheme considered over a double-sided (transmitter and receiver) correlated 3D channel is analyzed. Two scenarios of spatial correlation phenomena will be assumed for the (average) channel capacity evaluation which is following stochastic process way to calculate CF of SNR first. Generally, by means of the information capacity the channel capacity is easily able to be determined directly from the formula which is given as

$$C_{pdf}^{3D-BF} = E\left[\Xi(S,\ y)\right] \tag{5}$$

where $\Xi(S,\ y)$ represents the instantaneous mutual information between input vector, *S*, and the output *y* of MIMO signaling scheme. Originally the fact of determination for information capacity in last equation should both depend on what kind the fading channels assumed and the amount of AOA. On the other hand, AOA is coming from the 3D channel consideration. However, due to the calculation of obtaining closed-form smoothly by just substituting the terms of mutual information into (5) is difficulty. Alternatively, the method of looking for CF of the random spatial correlation matrix in order to determine the channel capacity is a best way. Thus, according to the system model described in the previous section, the mutual information can be naturally expressed as $\Xi(S, y) = \log_2(\boldsymbol{I} + \mathbf{H}\mathbf{Q}\mathbf{H}^{\dagger} / N_0)$, where $\mathbf{H}$ has been defined in (1), and $\boldsymbol{Q}$ and $\boldsymbol{I}$ denote input covariance matrix and unit matrix of a MIMO system, respectively. Under the assumption of designating the all components are power-constrained for $\boldsymbol{Q}$, which is an important element to dominate the maximization of the system performance, since there are a lot of absolutely important information are included into it. For instance, if the distance between antennas is enough, the AOA of the in/out signals is suitable and so on. While the MIMO system employs 3D-BF with MRC (maximum ratio combining) receiver, the covariance matrix, which now it is replaced with, $\boldsymbol{Q}_{3D\text{-}BF}$, in the instantaneous mutual information could be implicitly decomposed as $\boldsymbol{Q}_{\text{3D-BF}} = U_{M_S}\Lambda_{\text{3D-BF}}U_{M_R}$, where U_{M_S} and U_{M_R} are defined in section II, and $\Lambda_{\text{3D-BF}} = Diag[E_s,\ 0,\cdots,0]$. Hence, by substituting the channel matrix into mutual information formula $\Xi(S,\lambda)$, and the new mutual information becomes as

$$\Xi(S,\lambda) = \log_2\left[(\boldsymbol{I}_{\text{3D-BF}} + (E_s/N_0)\mathbf{M}_R^{1/2}\mathbf{Z}\mathbf{M}_S^{1/2}\boldsymbol{Q}_{\text{3D-BF}}(\mathbf{M}_S\mathbf{Z}\mathbf{M}_R)^{1/2}\right] \tag{6}$$

where $\boldsymbol{I}_{\text{3D-BF}}$ is a unit matrix with same dimension of MRC, E_s/N_0 represents the *SNR*, $\mathbf{M}_R$, Z, and $\mathbf{M}_S$ have been defined in last section. By putting the covariance matrix $\boldsymbol{Q}_{\text{BF}}$ and the matrices combined with eigenvectors, (6) can be re-written as $\Xi(S,\lambda) = \log_2\left[\boldsymbol{I}_{\text{3D-BF}} + \left|\lambda_{\max}^{M_S}\gamma_s\zeta\zeta^\dagger\right|\right]$, where γ_s is defined in (6), $\lambda_{\max}^{M_S}$ indicts a maximum value selected from the eigenvalues found by the matrix $\mathbf{M}_S$, *i.e.*, $\lambda_{\max}^{\mathbf{M}_S} = \max(\lambda_i^{\mathbf{M}_S})$, $i = 0,\ 1,\ \cdots,\ N_R - 1$, ζ is defined as $\zeta = \left[(\lambda_1^{\mathbf{M}_R})^{0.5}z_1,\cdots,(\lambda_{N_R}^{\mathbf{M}_R})^{0.5}z_{N_R}\right]^T$, in which z_i, $i = 0,\ 1,\ \cdots,\ N_R - 1$, represents the *i*-th entry of first column generated from matrix $\mathbf{Z}$, and $z_i s'$ denotes *i.i.d.* zero-mean and unit variance complex Gaussian random variables. Note that by means of some simple operation in linear algebra, $|\mathbf{I}_u + \mathbf{XY}| = |\mathbf{I}_v + \mathbf{YX}|$, where $\mathbf{X} \in \boldsymbol{C}^{u\times v}$, and $\mathbf{Y} \in \boldsymbol{C}^{v\times u}$, after by the decomposition of matrices and substituting it back into (6) which becomes as $\Xi(S,\ y) = \log_2(1 + \eta\Omega)$, where $\eta = \sum_{i=0}^{N_R-1}\lambda_i^{\mathbf{M}_R}\varepsilon_i$, $\Omega = \gamma_S\lambda_{\max}^{\mathbf{M}_S}/2$, and where $\eta/2 = \zeta^\dagger\zeta$. Moreover, to get the pdf of η which is the combination of random variables ε_i, $i = 0,\ 1,\ \cdots,\ N_R - 1$. Thus, the evaluation of pdf for ε_i is necessary. As the fact aforementioned, it is difficult to calculate the channel capacity via the consideration of applying in substituting the pdf of receiving intensity directly to the mutual information formula. Usually, the most frequently to obtain the pdf of multiple random variables is applying the basic operation of random stochastic. However, it is adopted in the case with simple operations and sometime the closed-form is hard obtained. In this work an alternative way to use the procedure of CF definition is presented. Go to complete this, first the CF equation which can be determined as

$$\begin{aligned} CF(\phi) &= E\left[\exp\left(\phi\cdot\Xi(S,y)/\log_2 e\right)\right] \\ &= E\left[\log_2\left|\boldsymbol{I}_t + \boldsymbol{Q}_t\right|\right] \end{aligned} \tag{7}$$

Since the channel information is considered as previously unknown and it is necessary to search out a conditional pdf for calculating the channel capacity results, *i.e.* the CSI (channel side information) has not been involved in the scenario of this study. In this article two types of statistical distributions include indoor and Nakagami-*m* distributions, which have been proof enough to stand up formerly the fading channel models, are explored in the implementation of evaluation of channel capacity for MIMO system over correlated transmission environments.

Due to calculate the equation of mutual information, it is necessary to obtain the pdf of ε_i and η first. Consider 3D-BF with MRC applied in the MIMO system, the MIMO system operating over indoor environment is considered in this subsection. To take the parameter of AOA (angle of arrival) into the account of performance evaluation is a critical point, since the deployment is consider under indoor environment. Thus, the vertical angle will become an important variable to affect the channel capacity of MIMO beamforming system. Now the discussion of situation with spatial correlated Nakagami-*m* channel, and in which the intensity of transmitted signals is expressed as, $\varepsilon_i,\ i = 0,\ldots.N_R - 1$, are assumed as arbitrarily correlated Nakagmi-*m* random variable [17]. Then let the transmitted average signals' power are normalized with the fading parameters of Nakagmi-*m* fading and represented as $\sigma_{N_R} = \mathrm{E}[Z_{N_R}^2]/m_i$, $i = 0,\ldots.N_R - 1$, where $m_0 \le m_2 \le \cdots \le m_{N_R-1}$ represent the fading parameters of the corresponding correlated fading channel, and $\mathrm{E}[\cdot]$ is the expectation operator. The squared value of the intensity z_i is distributed in Gamma distribution, *i.e.*, $\zeta_i = \varepsilon_i^2$ and $\zeta_i \sim Gamma(z; m_i, \sigma_i)$. Moreover, ζ_i becomes as a special case with Erlang distribution when all fading parameters, m_i, are designated as integer numbers, that is, the pdf of ζ_i is given as

$$f_{\zeta_i}\left(z,\ m_i,\ \sigma_i,\theta\right)=\frac{z^{m_i-1}\cdot A_E(\phi_e,\ \theta)\cdot\exp(-z/\sigma_i)}{\Gamma(m_i)\sigma_i^{m_i}}u(z) \quad (8)$$

where $u(z)$ is the unit step function, with the same assumption when takes the parameter of AOA that made in the last sub-section. After the MIMO beamforming signal is gathering at the referred subscriber. Since the 3D-BF with MRC is adopted as the reception scheme, then the summation of all multiple input at the MIMO receiver is written as $\eta=\sum_{i=0}^{N_R-1}\zeta_i$ which pdf can be obtained as [6]

$$\begin{aligned}f\left(\eta,\theta\right)=&\sum_{i=0}^{N_R-1}A_E(\phi_e,\ \theta)\sum_{k=1}^{m_i}\Delta_{N_R}\left(i,k,m_u,\sigma_u,l_q\right)\\&\times f_{\zeta_i}\left(z,\ m_i,\ \sigma_i\right)\end{aligned} \quad (9)$$

where $A_E(\phi,\ \theta)$ which involves the AoA component has been aforementioned, $u=0,\cdots,N_R-1$, $q=1,\cdots,N_R-1$, and $\Delta_{N_R}\left(i,k,m_j,\sigma_j,l_q\right)$ is a recursive formula given as

$$\begin{aligned}\Delta_{N_R}\left(n,m_n-k,m_u,\sigma_u,l_q\right)=&\frac{1}{k}\sum_{\substack{j,u,q=0\\n\neq u}}^{N_R-1}\frac{m_u}{\sigma_n^j}\left(\frac{1}{\sigma_n}-\frac{1}{\sigma_u}\right)^{-j}\\&\times\Delta_{N_R}\left(n,m_u-k+j,m_u,\sigma_u,l_q\right)\end{aligned} \quad (10)$$

Assigning D and R as the distance between a mobile user and a BS, and radius of the BS disk, respectively. As the 3D channel shown in Fig. 1, the joint pdf of AoA and AoD (arrival of departure) can be rewritten as [20]

$$p_{\theta_{n,m},\phi_{n,m}}(\theta_{n,m},\phi_{n,m})=\left|J\right|^{-1}p_{\mathrm{r},\phi_{n,m}}\left(\frac{Dsin(\theta_{n,m})}{\sin(\phi_{n,m}-\theta_{n,m})},\phi_{n,m}\right) \quad (11)$$

where $\left|J\right|$ is the Jacobian of the transformation, which denotes as

$$\left|J\right|=\begin{vmatrix}\frac{\partial r}{\partial\tau'} & \frac{\partial r}{\partial\phi_{n,m}}\\ \frac{\partial\phi_{n,m}}{\partial\tau'} & \frac{\partial\phi_{n,m}}{\partial\phi_{n,m}}\end{vmatrix}^{-1}=c_0^{-1}\frac{2(\tau'\mathrm{c}_0+\mathrm{D}\cos(\phi_{n,m}))^2}{(\tau'\mathrm{c}_0)^2+2\tau'\mathrm{c}_0\mathrm{D}\cos(\phi_{n,m})+D^2} \quad (12)$$

, $-\pi\leq\phi_{n,m}\leq\pi$, and $-\sin^{-1}(R/D)\leq\theta_{n,m}\leq\sin^{-1}(R/D)$ are the degree of the AOA and AOD, respectively, n and m indicate the cluster number.

Since the general form of the pdf of squared correlated-Nakagami-*m* distributed is with the results from [17], the detail is not rewritten here. Thus, by substituting the pdf of received intensity shown in (9) into the definition of CF shown in (7), the CF of Nakagami-*m* spatial correlation channel of MIMO system becomes as

$$\begin{aligned}CF\left(\phi\right)=&\sum_{i=1}^{N_R}A_E(\phi,\ \theta)\sum_{k=1}^{m_i}\Delta_{N_R}\left(i,k,m_u,\sigma_u,l_q\right)\\&\times\frac{1}{\sigma_i^{m_i}\left(m_i-1\right)!}\int_0^\infty\left(1+\Omega\eta\right)^{\phi}\cdot\eta^{(m_i-1)}\cdot\exp\left(-\eta/\sigma_i\right)d\eta\end{aligned} \quad (13)$$

The completion of the integral in previous equation that can be solved by using of the equivalent formulas given as [16]

$$\int_0^\infty e^{-gx}\cdot x^{h-1}\cdot\left(1+ax\right)^{-\nu}dx=a^{-h}\Gamma\left(h\right)\psi\left(h,h+1-\nu;g/a\right) \quad (14)$$

where $\psi\left(i,\ j;k\right)=\int_0^\infty e^{-kt}t^{\alpha-1}\left(1+t\right)^{j-i-1}/\Gamma\left(i\right)dt$ is the confluent Hypergeometric function [19], thus, by variables changing and let $g=\left(1/\sigma_i\right)$, $h=m_i$, $a=\Omega$, and $\nu=-\phi$, then equation (14) becomes as

$$\begin{aligned}CF\left(\phi\right)=&\sum_{i=0}^{N_R-1}A_E(\phi_e,\ \theta)\sum_{k=1}^{m_i}\Delta_{N_R}\left(i,k,m_u,\sigma_u,l_q\right)\\&\times\frac{1}{\sigma_i^{m_i}\left(m_i-1\right)!}\cdot\Omega^{-m_i}\cdot\Gamma\left(m_i\right)\cdot\psi\left(m_i,m_i+1-\phi;\frac{1}{\Omega\sigma_i}\right)\end{aligned} \quad (15)$$

By means of applying the well known formula for differentiation of confluent Hypergeometric function [18],

$$\frac{d^n}{dz^n}\psi\left(\alpha,r;z\right)=\frac{(\alpha)_n}{(r)_n}\cdot\psi\left(\alpha+1,r+1;z\right) \quad (16)$$

where $(\alpha)_n$ is the Pochhammer symbol. Consequently, the derivative of CF can be obtained by putting (14) into (13) and which is expressed as

$$\begin{aligned}\frac{\partial CF\left(\phi\right)}{\partial\phi}=&\sum_{i=1}^{N_R}A_E(\phi_e,\ \theta)\sum_{k=1}^{m_i}\Delta_{N_R}\left(i,k,m_u,\sigma_u,l_q\right)\\&\times\frac{1}{\sigma_i^{m_i}\left(m_i-1\right)!}\cdot\Omega^{-m_i}\cdot\Gamma\left(m_i\right)\cdot\frac{m_i}{m_i+1-\phi}\cdot\psi\left(m_{i+1},m_i+2-\phi;\frac{1}{\Omega\sigma_i}\right)\end{aligned} \quad (17)$$

where $u=0,\cdots,N_R-1$, $q=1,\cdots,N_R-1$. Once the CF of correlated Nakagami-*m* fading for received antenna is obtained, by following up the similarly analyzed steps that of the Indoor case's, and then the channel capacity of MIMO system over the spatial correlated Nakagami-*m* channel, C_{Nak}^{3D-BF}, can be evaluated as

$$C_{Nak}^{3D-BF} = \log_2 e \cdot \sum_{i=0}^{N_R-1} A_E(\phi_e,\ \theta) \sum_{k=1}^{m_i} \Delta_{N_R}\left(i,k,m_u,\sigma_u,l_q\right) \times \frac{1}{\sigma_i^{m_i}\left(m_i-1\right)!} \cdot \Omega^{-m_i} \cdot \Gamma\left(m_i\right) \cdot \frac{m_i}{m_i+1} \cdot \psi\left(m_i+1, m_i+2; \frac{1}{\Omega\sigma_i}\right) \quad (18)$$

where $\Delta_{N_R}\left(\cdot\right)$ has been defined in (13), and $A_E(\phi,\ \theta)$ which takes account into the calculation of C_{Nak}^{3D-BF}. Accordingly, the channel capacity of MIMO beamforming system with both the parameters of correlated coefficient and AOA can be finally determined via the combination of (7) and (15). The previous formula is the final result of channel capacity for MIMO beamforming system. There is an example for examining the result, hence, C_{Nak}^{3D-BF} becomes

$$C_{Nak}^{3D-BF} = \log_2 e \cdot 8(dBi) \cdot \sum_{i=0}^{N_R-1} \sum_{k=1}^{m_i} \Delta_{N_R}\left(i,k,m_u,\sigma_u,l_q\right) \times \frac{1}{\sigma_i^{m_i}\left(m_i-1\right)!} \cdot \Omega^{-m_i} \cdot \Gamma\left(m_i\right) \cdot \frac{m_i}{m_i+1} \cdot \psi\left(m_i+1, m_i+2; \frac{1}{\Omega\sigma_i}\right) \quad (19)$$

where with the values assigned as $\theta_{3dB} = 65^0$ and then $A_m = 30dB$.

IV. Numerical Results

The validation of the previous derived theoretical formulas is illustrated in this section, first the indoor factor is always set as a value $K = 3.6$. For simplicity, however, without loss the generality, the correlation coefficients generated by the Gaussian correlation model of an equally spaced linear array with an arbitrary correlation coefficient is adopted. It is of interest to note that the correlation matrix followed by the linear array has a Toeplitz form constructed by correlation elements, $\rho_{ij},\ i, j = 0,\cdots,N_R-1$ [18]. The simulated plots are marked with circle symbols for comparison, and it is significant to see that both the curves obtained theoretically and numerically are matched each other pretty well. The results from assigning $N_T \times N_R = 2\times 4$ and different fading parameters, $m=3$ and $m=4$, are shown in Fig. 2. It is easy to understand that the performance is becoming much better when the fading parameter increase. On the other hand, the capacity performance of curve group belong to $m=4$ is superior to that belongs to $m=3$. Finally, the phenomena of AoA parameter impacts the MIMO beamforming system is illustrated in Fig. 3. The curves are corresponding to different $N_T \times N_R$ values. It is valuable to see the correlation coefficient that generally relates to the outcome of d/λ, and which changes the size of AoA. Hence, the channel capacity definitely becomes degradation when the correlation coefficient is promoted. It is known that the correlation coefficient is negative proportion to the size of d/λ. The verified fact is in accordance with those results from the claims of [19].

V. Conclusion

The impact parameters of both AoA and AoD for the MIMO (multi-input multi-output) 3D-BF system over the spatial correlated-fading channel are explored in the report. The channel correlation is alternative to present the AoA for numerical analysis. There are some cases of spatially correlated fading channels assumed in this report for calculating the channel capacity of MIMO beamforming system. The numerical results are gained from different scenarios with distinct number of antennas at transmission and receipt ends. Specifically, discussing and saying that the results are always outperform for the situations which the larger number in receiver antennas when spatially correlated channel exists in it. Moreover, it is worthy to note that the performance of a MIMO beamforming scheme is still mostly dominated by fading parameter when the correlated-Nakagami-*m* environment (considering the AoA and AoD parameters) is established as the channel model for MIMO system.

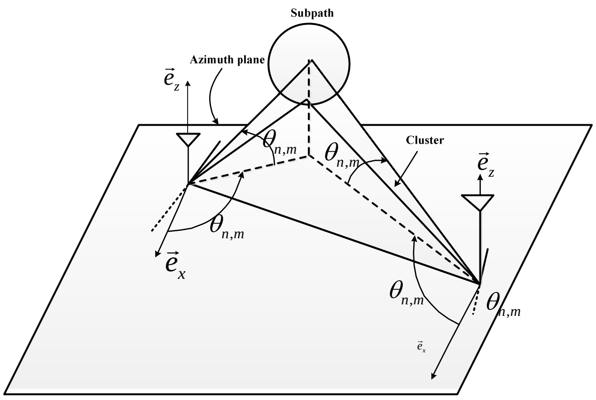

Fig. 1. A simple deployment of 3D channel model [11]

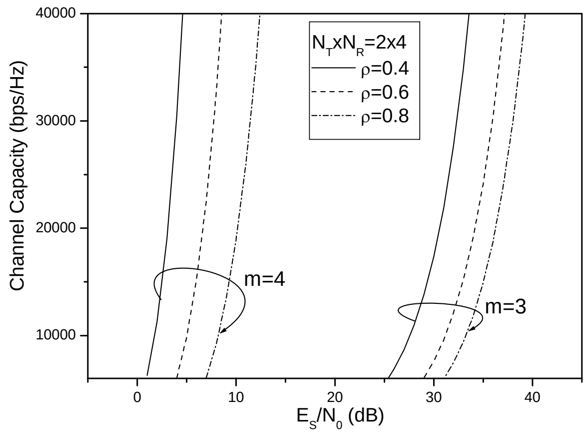

Fig. 2 SNR vs channel capacity for MIMO system with $N_T \times N_R = 2 \times 4$, $\rho = 0.4,\ 0.6,\ 0.8$.

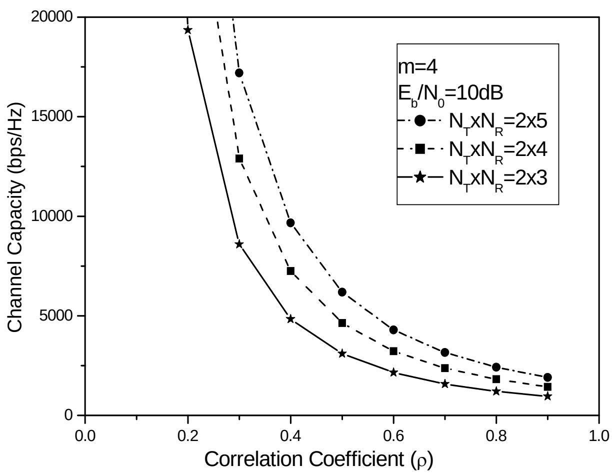

Fig. 3 Channel capacity for MIMO system over correlated-Nakagami-*m* fading with $m = 4$, $E_b / N_0 = 10dB$.

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