Fatigue Life Prediction of Local-dented 6061-T6 Aluminum Alloy Tubes Subjected to Cyclic Bending

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ABSTRACT

In this paper, by using adequate stress-strain relationship, models, mesh elements, boundary conditions and loading conditions, the finite element ANSYS Workbench 15.0 was used to analyze the moment-curvature and ovalization-curvature relationships for local-dented 6061-T6 aluminum alloy tubes with dent depths of 0.0, 0.3, 0.6, 0.9 and 1.2 mm subjected to cyclic bending. Once the ANSYS simulations can accurately describe the experimental results, the maximum and minimum stresses for each cyclic bending case can also be accurately obtained. Because the failure type was the fatigue fracture, a simple fatigue model was used to estimate the fatigue life. In addition, due to different amounts between the maximum and minimum stresses, the Walker equation was used for considering the mean stress effect in the fatigue model. It has been shown that the fatigue model can properly predict the fatigue life of local-dented 6061-T6 aluminum alloy tubes with different dent depths subjected to cyclic bending.

Key words: Finite Element ANSYS Analysis, Local-dented 6061-T6 Aluminum Alloy Tubes, Cyclic Bending, Moment, Curvature, Ovalization, Stress Amplitude, Fatigue Model.

局部凹痕 6061-T6 鋁合金管承受循環彎曲之疲勞壽命估算

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摘　　要

本文係使用適當的應力-應變關係、模型、網格元素、邊界條件及負載條件，則有限元素ANSYS工作平台15.0可用來分析深度為0.0, 0.3, 0.6, 0.9 and 1.2 mm的局部凹痕6061-T6鋁合金管在循環彎曲負載下的彎矩-曲率與曲率-橢圓化關係。一旦確認ANSYS模擬能準確的描述實驗結果，則每一個循環彎曲負載事件中的最大應力及最小應力即可準確的求得。因為實驗試件的毀損型態是疲勞破壞，因此本文使用一個簡單的疲勞模式來估算疲勞壽命。此外，由於最大應力及最小應力數值上的差異，Walker方程式也利用來考慮疲勞模式中的平均應力影響。分析結果顯示，所提出的疲勞模式能適當的預測不同深度局部凹痕6061-T6鋁合金管在循環彎曲負載下的疲勞壽命。

關鍵詞：有限元素 ANSYS 分析、局部凹痕 6061-T6 鋁合金管、循環彎曲、彎矩、曲率、橢圓化、應力幅度、疲勞模式。

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I. INTRODUCTION

It is well known that bending circular tubes results in ovalization (change in the outer diameter divided by the original outer diameter) of the tube cross-section. This ovalization increases slowly during reverse bending and continuous cyclic bending, and in turn, results in the deterioration of the circular tube, which buckles or fractures when the ovalization reaches a critical value. Circular tubes are severely damaged during buckling and fracturing and cannot bear the load, which ultimately results in obstruction and leakage of the material being transported. As such, a complete understanding of the deterioration and failure of circular tubes under cyclic bending is essential for industrial application.

As part of the earliest research on this issue, Kyriakides' team began a series of experimental and theoretical studies on tubes subjected to monotonic and cyclic bending, as well as with and without external or internal pressure. Shaw and Kyriakides [1] designed and constructed a tube cyclic bending machine and conducted a series of experimental and theoretical investigations. Kyriakides and Shaw [2] later investigated the inelastic behavior of tubes subjected to cyclic bending and extended the analysis of tubes to stability conditions under cyclic bending. Corona and Kyriakides [3] investigated the stability of tubes subjected to combined bending and external pressure; they also later [4] studied the degradation and buckling of tubes under cyclic bending and external pressure. Corona and Vaze [5] studied the response, buckling, and collapse of long, thin-walled seamless steel square tubes under bending. Later they [6] experimentally investigated the elastic-plastic degradation and collapse of steel tubes with square cross-sections under cyclic bending. Corona and Kyriakides [7] also studied the asymmetric collapse modes of pipes under combined bending and pressure. Corona *et al.* [8] conducted a set of bending experiments on aluminum alloy tubes to investigate the yield anisotropy effects on buckling. Later, Kyriakides *et al.* [9] studied the plastic bending of steel tubes with a diameter-to-thickness ratio (D_o/t ratio) of 18.8, exhibiting Lüders bands through the experiment. Limam *et al.* [10] studied the inelastic bending and collapse of tubes in the presence of bending and internal pressure. Hallai *et al.* [11] experimentally studied the effect of Lüders bands on the bending of steel tubes. Limam *et al.* [12] later investigated the collapse of dented tubes under combined bending and internal pressure. Bechle and Kyriakides [13] later studied the localization of NiTi tubes subjected to bending.

Pan et al. [14] designed and set up a new measurement apparatus. The apparatus was used with a cyclic bending machine to study various types of tube under different cyclic bending conditions. For examples: Pan and Fan [15] studied the effect of the prior curvature-rate at the preloading stage on subsequent creep or relaxation behavior, Pan and Her [16] investigated the response and stability of 304 stainless steel tubes that were subjected to cyclic bending with different curvature-rates, Lee *et al.* [17] studied the influence of the D_o/t ratio on the response and stability of circular tubes that were subjected to symmetrical cyclic bending, Lee *et al.* [18] experimentally explored the effect of the D_o/t ratio and curvature-rate on the response and stability of circular tubes subjected to cyclic bending, Chang *et al.* [19] studied the mean moment effect on circular, thin-walled tubes under cyclic bending, and Chang and Pan [20] discussed the buckling life estimation of circular tubes subjected to cyclic bending.

In practical industrial applications, tubes are under the hostile environment, so the material in the environment may corrode the tube surface and produce notches. Additionally, a tube in the working condition often involves some notches. The mechanical behavior and buckling failure of a notched tube differs from that of a tube with a smooth surface. In 2010, Lee *et al.* [21] studied the variation in ovalization of sharp-notched circular tubes subjected to cyclic bending. Lee [22] investigated the mechanical behavior and buckling failure of sharp-notched circular tubes under cyclic bending. Lee *et al.* [23] experimentally discussed the viscoplastic response and collapse of sharp-notched circular tubes subjected to cyclic bending. Lee and Chang [24] investigated the response of

SUS304 stainless steel tubes subjected to pure bending creep and pure bending relaxation. Later, Lee *et al.* [25] experimentally investigated the mechanical behavior and buckling failure of 6061-T6 aluminum alloy tubes with different chop depths subjected to cyclic bending.

In 2012, Limam *et a*l. [12] experimentally and analytically inspected the collapse curvature of local-dented tubes submitted to pure bending with internal pressure. In their experiments, 321 stainless steel tubes with outer diameters of 1.5 in. and D_o/t of 52 were tested. The dented tubes were internally pressurized and then bent to failure. They found that such defect decreased the bending rigidity quite significantly. Lee *et al*. [26] experimentally investigated the response and failure of local-dented 6061-T6 aluminum alloy with dent depths of 0.0, 0.3, 0.6, 0.9 and 1.2 mm submitted to cyclic bending. They discovered that the moment-curvature curve showed symmetrical and steady loop for tubes without a dent and asymmetric and steady loops for dented tubes. The ovalization-curvature curve demonstrated symmetrical and ratcheting manner for tubes without a dent and unsymmetrical and ratcheting manner for dented tubes. A higher dent depth leads to a more unsymmetrical ovalization-curvature curve. The tube fractures when the ovalization of the tube cross-section reaches a critical amount. In addition, it is shown from the experimental controlled curvature-number of cycles required to failure relationship in a log-log scale that five nonparallel straight lines correspond to five different dent depths. Finally, a theoretical formulation was proposed in their study to simulate the relationship between the controlled curvature and the number of cycles required to failure.

It is well known in industrial applications that 6061-T6 aluminum alloy tubes are constantly used as transporting lines or load-carrying members. Such applications include the frames on bicycles, car suspensions, masts on sailboats, pipe lines on airplanes, … and others. Therefore, understanding the behavior of 6061-T6 aluminum alloy tubes undergoing cyclic bending is necessary for engineers in doing design or analysis.

In this study, by using adequate stress-strain relationship, models, mesh elements, boundary conditions and loading conditions, the finite element ANSYS Workbench 15.0 was used to analyze the moment-curvature and ovalization-curvature relationships for local-dented 6061-T6 aluminum alloy tubes with dent depths of 0.0, 0.3, 0.6, 0.9 and 1.2 mm subjected to cyclic bending. The experimental data for comparison was tested by Lee *et al*. [26]. Next, the maximum and minimum stresses were calculated for each cyclic bending case. Finally, a simple fatigue model with Walker [27] equation was employed for predicting the fatigue life of local-dented 6061-T6 aluminum alloy tubes with different dent depths submitted to cyclic bending.

II. FINITE ELEMENT ANALYSIS

In this study, the response of local-dented circular tubes subjected to cyclic bending was analyzed numerically using the finite element code ANSYS. The response is the correlation among the moment, curvature, and ovalization. The elastic-plastic stress-strain response, model, mesh, boundary conditions, and loading conditions are explained below.

Elastic-plastic Stress-strain Curves

Fig. 1 shows the tested and ANSYS-constructed uniaxial stress (σ) - strain (ε) curves for the 6061-T6 aluminum alloy. The kinematic hardening rule was used as the hardening rule for cyclic loading. Table 1 shows the experimental stresses, ANSYS analysis stresses and error rates for different strains. The error rate range is from 0.57% to 3.63%.

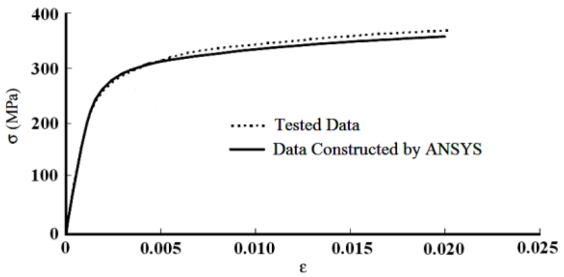


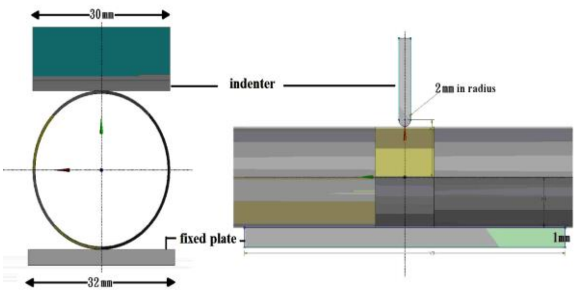
Fig. 1. Tested and ANSYS data of uniaxial stress-strain curves for 6061-T6 aluminum alloy.

Table 1. Experimental stresses, ANSYS analysis stresses and error rates for different strains.

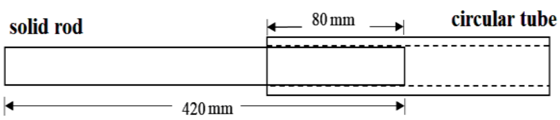
ε	0.0025	0.005	0.0075	0.010	0.0125	0.015	0.0175	0.020
σ_{exp} (MPa)	274.1	316.6	333.3	344.4	351.8	355.6	362.9	370.4
σ_{ANSYS} (MPa)	276.8	314.8	322.2	332.8	340.7	348.1	353.7	357.4
error rate	0.96%	0.57%	3.45%	3.49%	3.26%	2.15%	2.6%	3.63%

Models

The models include three parts, the local-dented tube, indenter and solid rod. The geometric shape and size of the local-dented tube are stated in Lee *et al*.'s experiments [26] and the schematic drawings of indenter and solid rod are shown in Figs. 2(a) and 2(b), respectively. It can be seen in Fig. 2(a) that the processing a dent includes the indenter and a fixed plate. The indenter with a rounded surface and 2 mm in radius exerts a pressure on the tube for creating a desired depth of the dent. The dent depths (*a*) were considered 0.0, 0.3, 0.6, 0.9 and 1.2 mm.



(a)



(b)

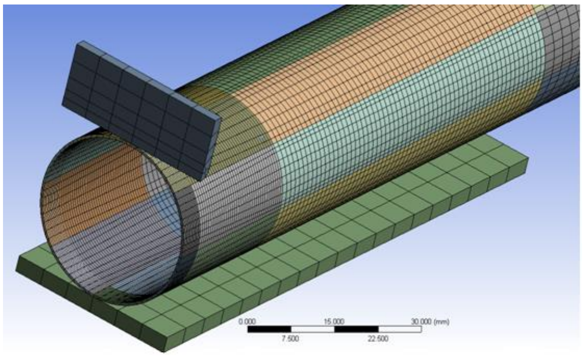
Fig. 2. Schematic drawings of (a) the indenter and (b) solid rod.

Mesh

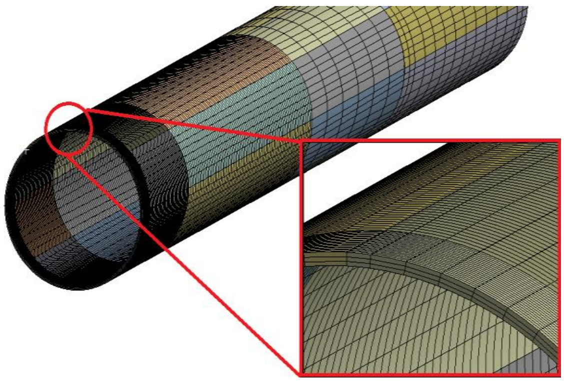
Due to the three-dimensional geometry and elastic-plastic deformation of the tube, the SOLID 185 element was used for relative analysis. This element is a tetrahedral element built in ANSYS and is suitable for analyzing the plastic or large deformation. In particular, this element is adequate to analyze a shell component under bending. Due to the symmetry of the right and left, only half of the tube's model was constructed. Fig. 3(a) shows the mesh constructed by ANSYS for the indenter and half tube and Fig. 3(b) shows the mesh constructed by ANSYS for half tube.

Boundary Conditions

For processing a local dent on the tube, the indenter moved in y-direction only. Therefore, the friction supports were set to prevent any displacement in x- or z-direction as shown in Fig. 4. The indenter was set to exert a pressure on the tube for processing a dent and back to its original position. The contact between the indenter and tube was set to be frictionless as shown in Fig. 5. The plate was fixed, thus, a fixed support was set on the plate as shown in Fig. 6. Because there wasn't any related displacement between the tube and fixed plate, the bonded contact between them was used.



(a)



(b)

Fig. 3. Mesh constructed by ANSYS for (a) the indenter and (b) half tube.

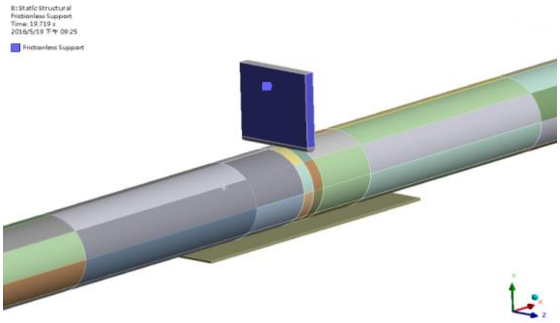


Fig. 4. Boundary conditions for the indenter.

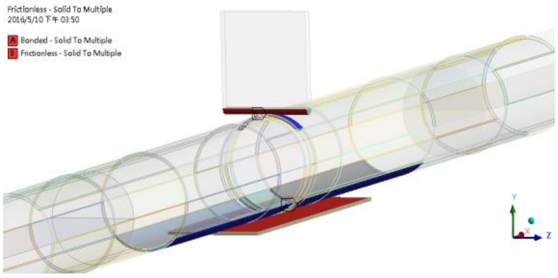


Fig. 5. Boundary conditions for the contact between the indenter and tube.

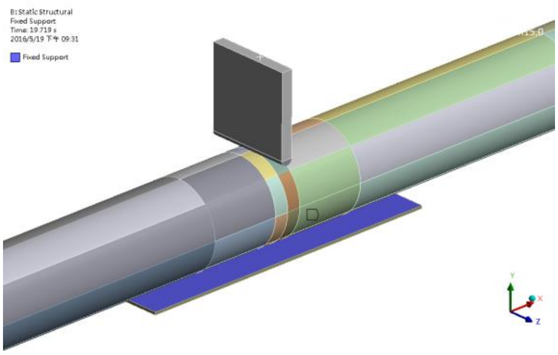


Fig. 6. Boundary conditions for the contact between the tube and fixed plate.

Loading Conditions

For the stage of processing a dent, the indenter was set to move downward to create a desire dent depth and back to its original position. Fig. 7 shows the loading condition of the indenter constructed by ANSYS. Fig. 8 shows the loading condition constructed by ANSYS on the basis of the tube bending device. As the figure shows, the remote displacement of the solid rod in the z-direction was unrestricted, i.e., the rotation was free to move in the z-direction. In addition, the bending moment was applied only in the z-direction and hence, the rotations in the x- and y-directions were set to zero.

Fig. 9 depicts a tube is subjected to pure bending. The rotating angle θ was employed as the input data for curvature-controlled cyclic bending. The curvature κ is

$$\kappa = 1/\rho = 2\theta/L_o \tag{1}$$

where ρ is the radius of curvature and L_o is the original tube length.

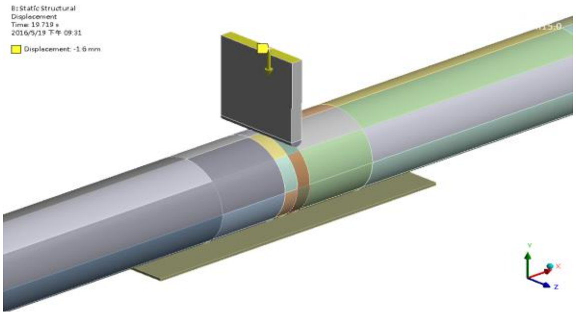


Fig. 7. Loading conditions for the indenter.

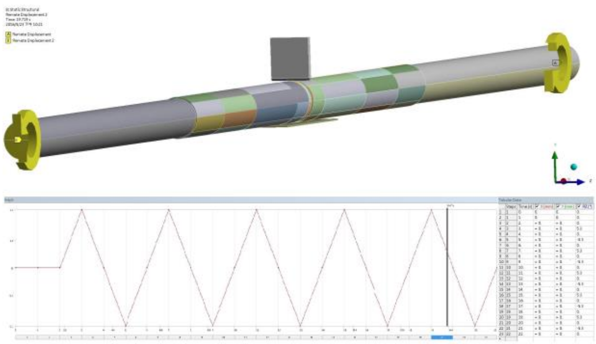


Fig. 8. Loading conditions for the solid rod.

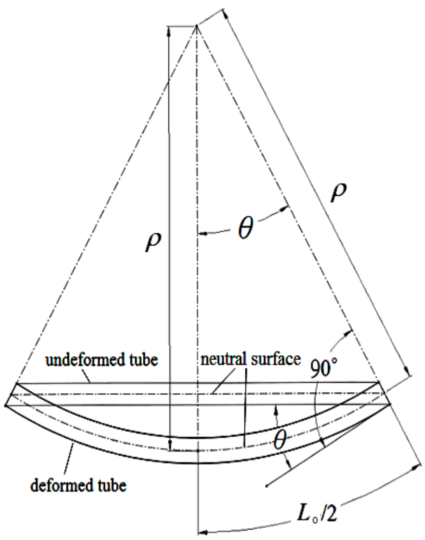
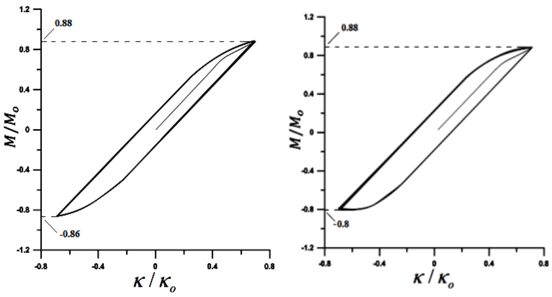


Fig. 9. Relationship between θ and κ for a tube under pure bending.

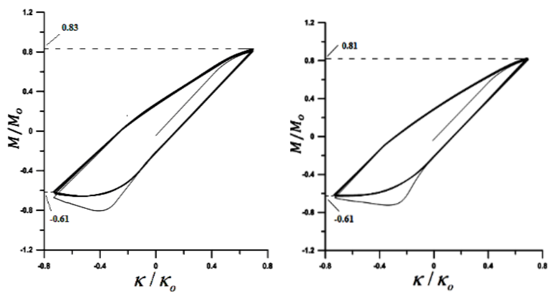
III. FAILURE ANALYSIS

Moment, Curvature and Ovalization

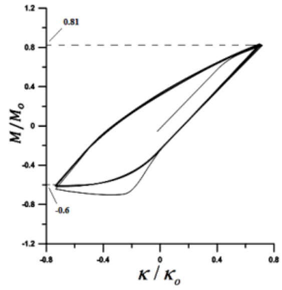
A typical set of experimental relationships between moment (M/M_o) and curvature (κ/κ_o) for local-dented 6061-T6 aluminum alloy tubes with different *a* subjected to cyclic bending is shown in Figs. 10(a)-(e) [26]. The bending moment and tube's curvature were normalized by $M_o = \sigma_o D_o^2 t$ and $\kappa_o = t/D_o^2$, respectively (Corona and Kyriakides [3]), in which σ_o is the yield stress, D_o is the outer diameter and t is the wall thickness. The tube's geometry and mechanical properties can be found in Lee *et al.*'s experiments [26]. The tubes were cycled between $\kappa/\kappa_o = \pm 0.71$. It can be observed that the M/M_o-κ/κ_o relationships for different *a* are a nearly closed and steady hysteresis loop. The magnitudes of M/M_o are almost equal to each other at $\kappa/\kappa_o = +0.71$ and $\kappa/\kappa_o = -0.71$ for smooth tubes ($a = 0.0$ mm). As for dented tubes ($a \neq 0.0$ mm), due to the contact of the two sides of the dent for the reverse bending, the magnitude of M/M_o at $\kappa/\kappa_o = -0.71$ is smaller than that at $\kappa/\kappa_o = +0.71$. But, the M/M_o-κ/κ_o curves are almost the same when $a \geq 0.6$ mm. Figs. 11(a)-(e) show the corresponding simulated M/M_o-κ/κ_o curves by ANSYS.



(a) (b)

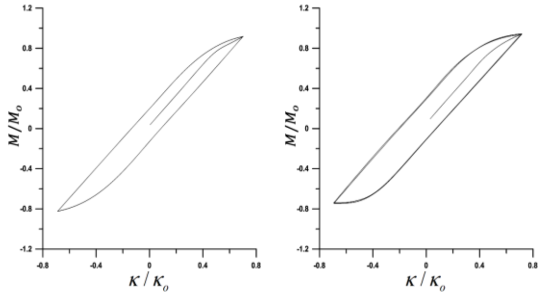


(c) (d)

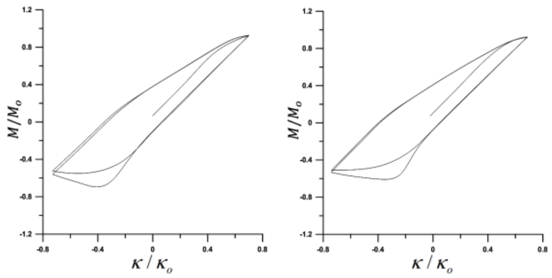


(e)

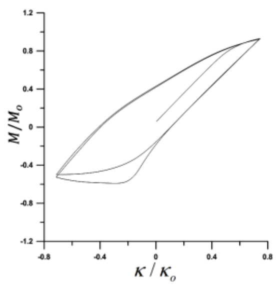
Fig. 10. Experimental moment (M/M_o) - curvature (κ/κ_o) curves for local-dented 6061-T6 aluminum alloy tubes with a = (a) 0.0, (b) 0.3, (c) 0.6, (d) 0.9 and (e) 1.2 mm under cyclic bending [26].



(a) (b)



(c) (d)



(e)

Fig. 11. ANSYS simulated moment (M/M_o) - curvature (κ/κ_o) curves for local-dented 6061-T6 aluminum alloy tubes with a = (a) 0.0, (b) 0.3, (c) 0.6, (d) 0.9 and (e) 1.2 mm under cyclic bending.

The corresponding set of ovalization ($\Delta D_o/D_o$) - curvature (κ/κ_o) relationships are demonstrated in Figs. 12(a)-(e) [26]. It is noticed that the $\Delta D_o/D_o$-κ/κ_o curves exhibit a ratcheting trend and increasing with the number of bending cycles. For smooth tube (a = 0.0 mm), the $\Delta D_o/D_o$-κ/κ_o curve is symmetrical. However, for dented tubes ($a \neq$ 0.0 mm), the $\Delta D_o/D_o$-κ/κ_o curves are unsymmetrical. Larger a of dented tubes causes a more unsymmetrical look of the $\Delta D_o/D_o$-κ/κ_o curve. Moreover, larger a of dented tubes leads to larger ovalization. Figs. 13(a)-(e) demonstrate the corresponding ANSYS simulated $\Delta D_o/D_o$-κ/κ_o curves for Figs. 12(a)-(e), respectively.

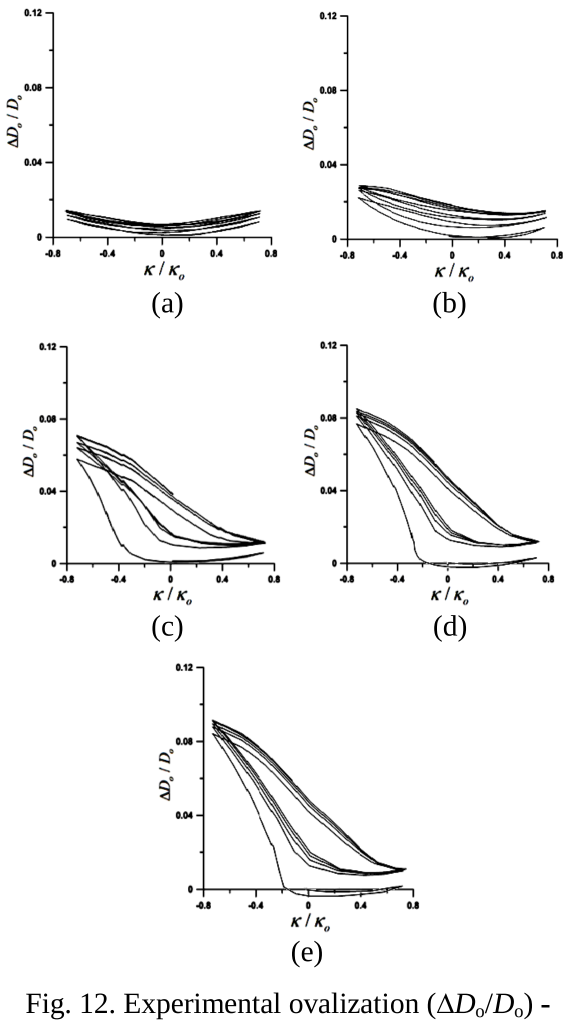


Fig. 12. Experimental ovalization ($\Delta D_o/D_o$) - curvature (κ/κ_o) curves for local-dented 6061 -T6 aluminum alloy tubes with a = (a) 0.0, (b) 0.3, (c) 0.6, (d) 0.9 and (e) 1.2 mm under cyclic bending [26].

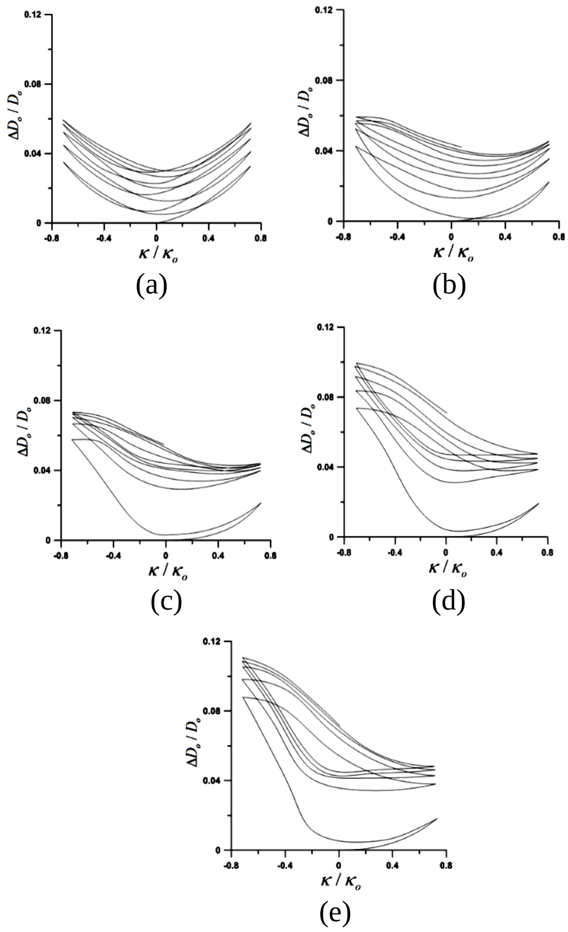


Fig. 13. ANSYS simulated ovalization ($\Delta D_o/D_o$) - curvature (κ/κ_o) curves for local-dented 6061-T6 aluminum alloy tubes with a = (a) 0.0, (b) 0.3, (c) 0.6, (d) 0.9 and (e) 1.2 mm under cyclic bending.

Fatigue Life Estimation

Fig. 14 shows a picture of the fatigue failure for local-dented 6061-T6 aluminum alloy circular tubes with a = 0, 0.3, 0.6, 0.9 and 1.2 mm under cyclic bending. It can be seen that the crack initiates at the one or both sides of the dent. In addition, once the crack initiation is observed, the tube rapidly breaks.

Table 2 shows the maximum and minimum stresses for local-dented 6061-T6 aluminum alloy circular tubes subjected to cyclic bending. The magnitudes of stress were determined by ANSYS discussed in Sec. II. It can be observed that for a given dent depth, a tube with a higher value of control curvature yields a higher maximum stress and minimum stress. In addition, for a given control curvature, a tube with a higher value of dent depth yields a higher maximum and minimum stresses. Although the test was a symmetrical curvature-controlled cyclic bending test (the maximum control curvature equals to the minimum control curvature), the amounts of the maximum and minimum stresses were different. Therefore, the mean stress effect was considered in the failure analysis.

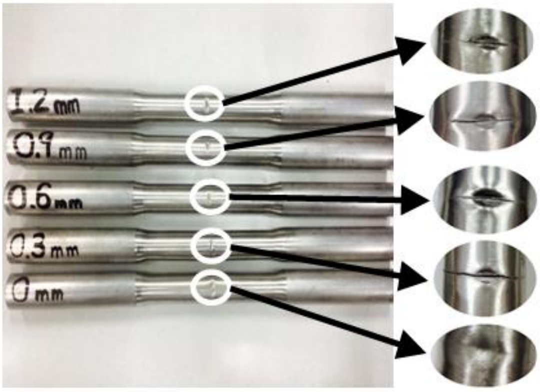


Fig. 14. A picture of the failure of local-dented 6061-T6 aluminum alloy with different dent depths under cyclic bending.

Table 2. Maximum and minimum stresses for local-dented 6061-T6 aluminum alloy circular tubes under cyclic bending.

Dent Depth (mm)	Control Curvature (m^{-1})	Maximum stress (σ_{max}) (MPa)	Minimum stress (σ_{min}) (MPa)
0	0.50	204.98	-205.66
0	0.55	230.57	-231.47
0	0.60	240.77	-247.65
0	0.65	250.01	-250.38
0	0.70	266.05	-269.10
0	0.75	289.87	-290.47
0.3	0.45	344.53	-347.18
0.3	0.50	350.83	-366.15
0.3	0.55	380.44	-391.23
0.3	0.70	398.21	-397.71
0.3	0.80	412.11	-411.02
0.3	0.85	420.18	-421.26
0.6	0.40	381.67	-410.63
0.6	0.45	420.11	-450.8
0.6	0.50	424.83	-466.87
0.6	0.55	480.33	-501.32
0.6	0.70	499.41	-510.28
0.6	0.75	512.08	-520.33
0.9	0.30	336.76	-401.63
0.9	0.35	338.52	-440.52
0.9	0.40	377.42	-451.73
0.9	0.45	414.89	-483.66
0.9	0.50	490.74	-525.31
0.9	0.55	499.13	-530.66
1.2	0.30	375.71	-430.23
1.2	0.35	378.29	-450.66
1.2	0.40	402.46	-453.21
1.2	0.45	460.51	-490.00
1.2	0.55	530.86	-570.41
1.2	0.65	580.65	-630.44

A simple fatigue model was used to be

$$\sigma_{ar} = \sigma_f^{'}(2N_f)^b \qquad (2)$$

or

$$\log\sigma_{ar} = \log\sigma_f^{'} + b\log 2N_f \qquad (3)$$

where σ_{ar} is the completely reversed stress amplitude, N_f is the fatigue life, $\sigma_f^{'}$ and b are the fatigue strength coefficient and exponent, respectively. For considering the mean stress effect, the Walker's equation was used as [27]

$$\sigma_{ar} = \sigma_{max}^{1-r}\sigma_a^r \qquad (\sigma_{max} > 0) \qquad (4)$$

where σ_a is the stress amplitude and r is the material parameter. Note that σ_a = (σ_{max} – σ_{min}) / 2. When the magnitude of r was used to be 0.51, the relationship between σ_{ar} and $2N_f$ was found to be a straight line on a log-log scale. The straight line in Fig. 15 was determined by least-square fit. The magnitudes of $\sigma_f^{'}$ and b were determined to be 692.33 MPa and -0.15, respectively.

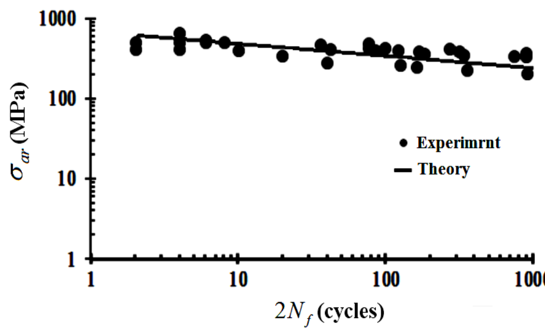


Fig.15. Experimental and simulated completely reversed stress amplitude (σ_{ar}) versus fatigue life ($2N_f$) for local-dented 6061-T6 aluminum alloy circular tubes with different dent depths under cyclic bending in a log-log scale

V. CONCLUSIONS

The fatigue failure of local-dented 6061-T6 aluminum alloy circular tubes subjected to cyclic bending was investigated in this study. On the basis of the experimental and theoretical results, the following conclusions can be drawn:

(1) For symmetrical cyclic bending, the experimental M/M_o-κ/κ_o relationship for local-dented 6061-T6 aluminum alloy tubes with any a exhibits a closed and stable hysteresis loop. In addition, the $\Delta D_o/D_o$-κ/κ_o curves are symmetrical for a = 0.0 mm. However, the $\Delta D_o/D_o$-κ/κ_o curves are unsymmetrical for $a \neq$ 0.0 mm. Moreover, larger a causes more unsymmetrical trend and larger ovalization.

(2) By employing proper stress-strain relationship, models, mesh elements, boundary conditions, and loading conditions, the ANSYS can describe the behavior of local-dented circular tubes under cyclic bending. The experimental moment-curvature and ovalization-curvature relationships [26] were compared with the ANSYS simulation. Although there are still a little differences in the value, but the trends are very similar.

(3) According to the ANSYS calculated σ_{max} and σ_{min} for local-dented 6061-T6 aluminum alloy circular tubes under cyclic bending. A simple fatigue model (Eq. (2)) and the Walker's equation (Eq. (4)) were used for estimate the fatigue life. Material parameters $\sigma_f^{'}$, b and r were determined to be 316.43 MPa, -0.12 and 0.51, respectively. It can be seen that the simulation is in good agreement with the experimental result as shown in Fig. 15.

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