中正嶺學報 第三十二卷 第一期 民國 92.11
JOURNAL OF C.C.I.T. VOL.32 NO.1 Nov. 2003

短文 Short Paper

Performance Analysis of Carrier Frequency Offset on Up-link MC-CDMA System with Unbalanced EGC over Nakagami Fading Channels

Yawpo Yang*, Shu-Li Jong, Ji-Chyun Liu**, and Ching-Hwa Liu***

**Department of Electronics Engineering, China Institute of Technology*
***Department of Electrical Engineering, Chung Cheng Institute of Technology*
National Defense University

ABSTRACT

In this paper effects of carrier frequency offset on performance of an uplink MC-CDMA (multi-carrier code division multiple access) system with equal gain combining (EGC) receivers over Nakagami fading channel have been investigated, and the impacts of gain unbalance between branches in MC-CDMA system has also been examined. The performance of such an MC-CDMA system is compared with that of the conventional single-carrier DS-CDMA system. Numerical results indicate that the system performance is indeed deteriorated by both the carrier frequency offset and the gain unbalance between branches.

Keywords: frequency offset, MC-CDMA, unbalance EGC, Nakagami fading

上鏈多載波分碼多重進接系統具有載波頻率偏移及不平衡權重等增益合成於中上環境之效能分析

楊耀波*　鍾素麗　劉智群**　劉慶華***

*中華技術學院電子系
**國防大學中正理工學院電機系

摘 要

本論文在中上衰落環境下，探討完美的等增益合成之載波頻率偏移影響下的上鏈路多載波分碼多重進接系統(MC-CDMA)之性能及不平衡權重等增益合成(EGC)對效能的衝擊，同時與傳統的直序分碼多重進接(DS-CDMA)系統相比較。本研究結果已經證實載波頻率偏移確實降低系統性能，其中分支間不平衡權重造成系統性能退化更嚴重。

關鍵詞： 頻率偏移，多載波分碼多重進接系統，不平衡權重等增益合成，中上衰落

文稿收件日期 91.11.1；文稿修正後以短文接受日期 92.9.5.
Manuscript received Nov. 1, 2002; revised as short paper Sept. 5, 2003.

Ⅰ. INTRODUCTION

A large number of attentions in the field of mobile communications have been paid to the multicarrier code division multiple access (MC-CDMA) due to its capability of high transmitting data rate. MC-CDMA scheme is based on a combination of CDMA and multicarrier modulation [1-3]. It is demonstrated that signals in such scheme, using the fast Fourier transform (FFT) device, can be transmitted and received without significantly increasing the complexities. In addition, MC-CDMA is potentially robust to channel frequency selectivity.

In order to combat signal fading in mobile radio environments, the most commonly used techniques are diversity schemes, in which the maximal-ratio combining (MRC) and equal-gain combining (EGC) are two most popular and powerful schemes. MRC provides maximum performance improvement. MRC, however, has the highest complexity of all combining techniques since it requires the precise estimate of amplitude and phase in each signal branch; consequently, it is not often implemented in practice. EGC, as an alternative, is not necessary to estimate the faded amplitudes and is hence easier to implement [4].

So far, many papers have explored the bit error rate (BER) of system using EGC. Zhang [5,6] evaluated probability of error with EGC output over Rayleigh fading channel. In [5], the author utilized the characteristic function of the signal at the combiner output to analyze a predetection three-branch diversity system, and derived a closed-form solution for the probability of error with coherent and noncoherent detection. In ref. 6, the author presented an exact solution to the problem of system using EGC, which can be easily evaluated by using the Hermite method.

Kotsopoulos and Karagiannidis [7] derived a closed formula for bit error probability (BEP) of BPSK systems with EGC over Rayleigh fading channels. The formula is very simple and does not require complicated confluent hypergeometric function. Mallik et al. [8] analyzed BEP of binary signals with dual-diversity predetection EGC in correlated Rayleigh fading. Both equal branch signal-to-noise ratios and unequal branch signal-to-noise ratios were studied. The BEP was derived using an infinite series representation of the bivariate cumulative distribution function (CDF).

Although the Rayleigh statistical model has been long used for fading channels, the Nakagami-m distribution [9] is considered as one of the most versatile fading distributions that can model different fading environment. Suzuki [10] indicated that the path fading statistics, especially for the shortest paths, are more suitably described by Nakagami-m distribution; Rayleigh fading is a special case, where the fading figure fading is set $m=1$.

Alouini and Simon [11] evaluated the average symbol error rate (SER) of M-PSK signals with EGC over Nakagami fading channels. They also presented a final expression for average SER in the form of a single finite-range integral.

Annamalai et al. [12,13] analyzed SER of diversity system with EGC on Nakagami and Rician fading channels and they obtained new closed-form solutions for binary CPSK and CFSK over Nakagami fading channel for the number of branches $L \leq 3$ and for binary DPSK and NCFSK with EGC for $L < 3$.

Recently, Li and Latva-aho [14] analyzed bit error rate (BER) of EGC receiver for MC-CDMA over Nakagami fading channels. Both the uplink and downlink were considered, but the authors did not consider the effects of frequency offset and unbalanced gain of EGC on system performance.

Although MC-CDMA is generally better in performance than the traditional CDMA system, it suffers significant performance degradation from carrier frequency offset. Two main factors that cause carrier frequency offset are Doppler spread caused from the channel for a high speed mobile and the difference between the oscillators in the transmitter and the oscillators in the receiver. Tomba and Krzymien [15] proposed MC-CDMA system architecture in AWGN channel and explored the impact of both the carrier frequency offset and the phase noise on system performance. It was demonstrated that MC-CDMA system was very sensitive to the signal distortion generated by the imperfect frequency down conversion at receiver due to the local oscillator phase noise and frequency offset.

Jang and Lee [16], Kim et al. [17] analyzed the effects due to frequency offset on the MC-CDMA system with EGC and MRC combining schemes in the downlink Rayleigh / Rician fading channels, but yet the authors did not touch the problem of unbalanced gain.

In implementation of combining techniques, it is assumed that the relative long-term mean signal level will be equal for all diversity branches in the receiver. If a common AGC (automatic gain control) is used to operate all IF amplifiers, then perfect gain tracking cannot be guaranteed, and the result is unequal mean signal levels into diversity circuit. Halpern [18] examined the effects of gain unbalance on performance of different diversity combiners. The author derived the SNR distribution for two-branch diversity systems with gain unbalances between branches for Rayleigh channel. Beaulieu [19] investigated the effects of gain unbalances between branches on the probability distribution of SNR and bit errors rates at the output of the L-branch EGC on Nakagami channel, in which both the coherent phase shift keying and differential coherent phase shift keying modulation schemes were discussed; the authors assumed the carriers were precisely estimated.

To the best of knowledge of authors, about the analysis of MC-CDMA system with carrier frequency offset and gain unbalances over Nakagami fading channel have not been reported yet. Our objective in this paper is to investigate the effects of gain unbalance and carrier frequency offset simultaneously on uplink MC-CDMA system over Nakagami fading channel. The parameters to be considered are carrier frequency offset, gain unbalance between branches of EGC, number of subcarriers, and number of users.

The reminder of this paper is organized as follows. In Section II, we first configure the

MC-CDMA system and the statistical model of fading channel. In Section III, we analyze the effect of gain unbalance between branches, and evaluate the average BER performance of the MC-CDMA with carrier frequency offset in an uplink Nakagami fading channel. We give numerical manifestation and brief discussion in Section IV. Finally, Section V is a conclusion.

II.MC-CDMA SYSTEM AND CHANNEL MODELS DESCRIPTION

Consider an uplink MC-CDMA system shown in Fig.1. $a_u[k] \in \{+1,-1\}$, which denotes the kth information bit of the uth user, is multiplied by a chip of the user's specific Walsh Hadamard code, $c_u[i]$ $(i = 0,\cdots,N-1)$. Walsh Hadamard orthogonal codes are used for spreading because of their good orthogonal cross-correlation property. Each branch is binary phase-shift keying modulated to a corresponding subcarrier. In this scenario, we assume that all users' signals experience different fading channel characteristics. Therefore, the signals between other users are independent and identically distributed (i.i.d.).

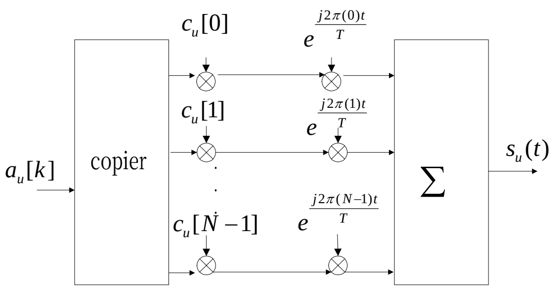

Fig.1. Block diagram of transmitter of the uth user in an MC-CDMA system.

The composite transmitted baseband signal $s(t)$ of such an MC-CDMA system is written as

$$s(t) = \sum_{u=0}^{U-1}\sum_{i=0}^{N-1} a_u[k]c_u[i]p(t-kT)e^{\frac{j2\pi it}{T}} \tag{1}$$

where N is the number of subcarriers, U is the number of users, and $c_u[i] \in \{+1,-1\}$ is the ith chip of the uth user's spreading code. The rectangular pulse $p(t)=1$ is defined on $[0,T)$, $\frac{i}{T}$ is the ith subcarrier frequency in the baseband, and T is the bit duration.

The fading channel is assumed frequency selective with Nakagami distribution. However, as shown in Fig.1, the narrowband signal, transmitted through each subcarrier which is assumed that the coherence bandwidth of the channel is larger than the signal bandwidth, experiences frequency non-selective Nakagami fading. Therefore, each amplitude and phase of the channel for narrowband signal is constant in each subcarrier. The transfer function of the channel of the uth user, the ith subcarriers in the frequency domain can be expressed as follows:

$$h_{ui} = \rho_{ui} e^{j\theta_{ui}} \tag{2}$$

where h_{ui} is modeled as a complex random variable. $\{\rho_{ui}\}$ and $\{\theta_{ui}\}$ indicate the amplitude and the phase distortion introduced by the channel, respectively. $\{\rho_{ui}\}$ and $\{\theta_{ui}\}$ are identical Nakagami random variables and uniform random variables over

$[0, 2\pi)$ respectively.

The $\{\rho_{ui}\}'s$ are assumed to be independent Nakagami random variables with probability density function (PDF)

$$p(\rho_{ui}) = \frac{2m^m \rho_{ui}^{2m-1}}{\Gamma(m)\Omega^m} \exp(-m\frac{\rho_{ui}^2}{\Omega}); \quad \rho_{ui} \ge 0 \quad (3)$$

where $\Gamma(\cdot)$ denotes the Gamma function, $\Omega = E[(\rho_{ui})^2]$ and m is called the Nakagami fading parameter $(m \ge 0.5)$ and is defined as the ratio of moments,

$$m = \frac{\Omega^2}{E\{[(\rho_{ui})^2 - \Omega]^2\}} \quad (4)$$

The structure of receiver is shown in Fig.2. The received signal $r(t)$ can be written as

$$r(t) = \sum_{u=0}^{U-1}\sum_{i=0}^{N-1} \rho_{ui} a_u[k] c_u[i] p(t-kT) e^{j(2\pi(i)t/T + \theta_{ui})} + n(t) \quad (5)$$

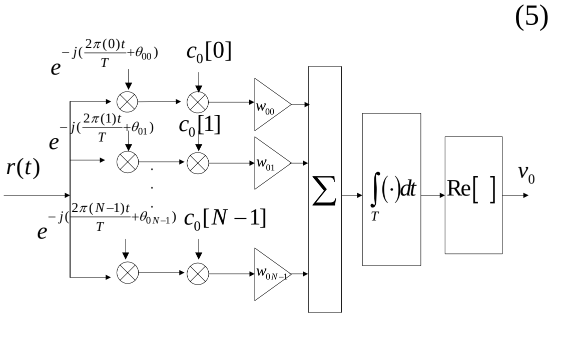

Fig.2. The structure of receiver of the $0th$ user in MC-CDMA system.

where $n(t)$ denotes the additive white Gaussian noise (AWGN) of zero mean with single sided power spectrum density of N_0. The local-mean power of one subcarrier is assumed to be $\overline{P}_{ui} = \frac{1}{2}E\{\rho_{ui}^2\}$. Suppose that the mean powers of all subcarrier signals are equal, then the total mean power of each user is $\overline{P}_u = N\overline{P}_{ui}$ and the bit energy is $E_b = \overline{P}_u T$.

III. THE AVERAGE BER PERFORMANCE ANALYSIS WITH FREQUENCY OFFSET FOR NAKAGAMI FADING CHANNEL

A. Bit Error Rate Analysis for EGC

The received signal carried by each subcarrier is extracted and multiplied by a chip of the user's assigned code. For EGC, the combining weight for the $0th$ user's sth subcarrier is denoted by w_{0s} and the value of w_{0s} is set to unity. The decision variable can be derived as follows.

$$\begin{aligned} v_0 &= \sum_{s=0}^{N-1}\sum_{u=0}^{U-1}\sum_{i=0}^{N-1} \rho_{ui} w_{0s} a_u[k] c_u[i] c_0[s] \frac{1}{T} \\ &\int_{kT}^{(k+1)T} p(t-kT)\,\mathrm{Re}\left\{e^{j2\pi(i-s-\varepsilon)t/T} e^{j(\theta_{ui}-\theta_{0s})}\right\} dt + \eta \\ &= D + MAI + SI_{fo} + MAI_{fo} + \eta \end{aligned} \quad (6)$$

where $\varepsilon\ (-0.5 \le \varepsilon \le 0.5)$ represents the frequency offset and is assumed to be constant during one symbol duration. D denotes the desired signal term and η denotes the noise term. MAI, SI_{fo}, and MAI_{fo} are the interference terms and represent, respectively, the multiple access interference from all the other users with the same carrier, the self-interference from the other carriers, and the multiple access interference from all the other users with the other carriers. Both SI_{fo} and MAI_{fo} item from the frequency offset.

The five terms in (6) are respectively described as follows. The desired signal term D for the reference user (i.e., $u=0$) is expressed as

$$D = a_0[k]\sum_{i=0}^{N-1}\rho_{0i}\frac{1}{T}\int_{kT}^{(k+1)T}\mathrm{Re}\left\{e^{-j2\pi\varepsilon t/T}\right\}dt = \frac{\sin\pi\varepsilon}{\pi\varepsilon}a_0[k]\cos(2\pi\varepsilon k+\pi\varepsilon)\sum_{i=0}^{N-1}\rho_{0i} \quad (7)$$

The MAI represents the condition $u \neq 0$ and is expressed by

$$MAI = a_u[k]\sum_{u=1}^{U-1}\sum_{i=0}^{N-1}\rho_{ui}c_u[i]c_0[i]\frac{1}{T}\int_{kT}^{(k+1)T}\mathrm{Re}\left\{e^{j(\frac{-2\pi\varepsilon t}{T}+\theta_{ui}-\theta_{0i})}\right\}dt = \frac{\sin\pi\varepsilon}{\pi\varepsilon}\sum_{u=1}^{U-1}\sum_{i=0}^{N-1}\rho_{ui}a_u[k]c_u[i]c_0[i]\cos\tilde{\theta} \quad (8)$$

where $\tilde{\theta} = \theta_{ui}-\theta_{0i}-2\pi\varepsilon k-\pi\varepsilon$. MAI can be reasonably approximated, by virtue of the central limit theorem (CLT), as a Gaussian random variable since it consists of a sum of statistically independent and identically distributed random variables $\rho_{ui}cos\tilde{\theta}$ and $a_u[k]c_u[i]c_0[i]\in\{-1,1\}$ if the total number of users and carriers, $N(U-1)$ is large enough [20]. The MAI can be shown to be of zero mean, and the variance, assuming the uth user's total mean power being known.

$$\sigma_{MAI}^2 = (\frac{\sin\pi\varepsilon}{\pi\varepsilon})^2 N(U-1)E[\rho_{ui}^2]c_u[i]c_0[i]E[\cos\tilde{\theta}^2] = (\frac{\sin\pi\varepsilon}{\pi\varepsilon})^2(U-1)\bar{P}_u \quad (9)$$

SI_{fo} represents the self interference due to frequency offset and can be obtained with the condition $u=0, i\neq s$,

$$SI_{fo} = \int_{kT}^{(k+1)T}\mathrm{Re}[e^{\frac{j2\pi(i-s-\varepsilon)t}{T}}e^{j(\theta_{0i}-\theta_{0s})}]dt = \frac{\sin\pi\varepsilon}{\pi\varepsilon}\sum_{s=0}^{N-1}\sum_{i=0,i\neq s}^{N-1}\rho_{0s}a_0[k]c_0[i]c_0[s]\cos\tilde{\theta}_1\frac{\varepsilon}{(\varepsilon+s-i)} \quad (10)$$

where $\tilde{\theta}_1 = \theta_{0i}-\theta_{0s}-2\pi\varepsilon k\text{-}\pi\varepsilon$

Also, MAI_{fo} represents the MAI from all the other users with the other carriers due to frequency offset and can be obtained with the condition $u\neq 0, i\neq s$,

$$MAI_{fo} = \frac{\sin\pi\varepsilon}{\pi\varepsilon}\sum_{s=0}^{N-1}\sum_{u=1}^{U-1}\sum_{i=0,i\neq s}^{N-1}\rho_{ui}a_u[k]c_u[i]c_0[s]\cos\tilde{\theta}_2\frac{\varepsilon}{(\varepsilon+s-i)} \quad (11)$$

where $\tilde{\theta}_2 = \theta_{ui}-\theta_{0s}-2\pi\varepsilon k\text{-}\pi\varepsilon$

Similar to the situation of MAI, SI_{fo} and MAI_{fo} are assumed to be zero mean Gaussian random variables. Variances of SI_{fo} and MAI_{fo} can be obtained as

$$\sigma^2_{SI_{fo}} = \frac{\sin\pi\varepsilon}{\pi\varepsilon}\sum_{s=0}^{N-1}\sum_{i=0,i\neq s}^{N-1}\rho_{0s}a_0[k]c_0[i]c_0[s]\cos\tilde{\theta}_1\frac{\varepsilon}{(\varepsilon+s-i)}$$
$$=(\frac{\sin\pi\varepsilon}{\pi\varepsilon})^2\frac{\bar{P}_0}{N}\sum_{s=0}^{N-1}\sum_{i=0,i\neq s}^{N-1}(\frac{\varepsilon}{(\varepsilon+s-i)})^2 \quad (12)$$

$$\sigma^2_{MAI_{fo}} = (\frac{\sin\pi\varepsilon}{\pi\varepsilon})^2(U-1)\sum_{s=0}^{N-1}\sum_{i=0,i\neq s}^{N-1}E[\rho_{0s}^2]c_u[i]c_0[s]E[\cos\tilde{\theta}_2](\frac{\varepsilon}{\varepsilon+s-i})^2$$
$$=(\frac{\sin\pi\varepsilon}{\pi\varepsilon})^2\frac{(U-1)\bar{P}_u}{N}\sum_{s=0}^{N-1}\sum_{i=0,i\neq s}^{N-1}(\frac{\varepsilon}{(\varepsilon+s-i)})^2 \quad (13)$$

The last term η, representing the interference due to the thermal noise, is Gaussian distributed with zero mean

$$\eta = \mathrm{Re}[\frac{1}{T}\int_{kT}^{(k+1)T}\sum_{i=0}^{N-1}c_0[i]n(t)e^{-j(\frac{2\pi[i]t}{T}+\theta_{0i})}dt]$$
$$=\frac{1}{T}\sum_{i=0}^{N-1}c_0[i]\int_{kT}^{(k+1)T}\left[x(t)\cos(\frac{2\pi[i]t}{T}+\theta_{0i})+y(t)\sin(\frac{2\pi[i]t}{T}+\theta_{0i})\right] \quad (14)$$

and variance as

$$\sigma_\eta^2 = E\left[\eta^2\right]-E^2[\eta]=\frac{N_0N}{T} \quad (15)$$

The total variance of interferences can be expressed as

$$\sigma_T^2=\sigma_{MAI}^2+\sigma_{SI_{fo}}^2+\sigma_{MAI_{fo}}^2+\sigma_\eta^2$$
$$=\frac{N_0}{T}\left(\frac{\sin\pi\varepsilon}{\pi\varepsilon}\right)^2\{\Delta_1\} \quad (16)$$

where

$$\Delta_1=(U-1)\frac{\bar{P}_uT}{N_0}+\frac{(U-1)\bar{P}_uT}{N_0N}\sum_{s=0}^{N-1}\sum_{\substack{i=0\\i\neq s}}^{N-1}\left(\frac{\varepsilon}{\varepsilon+s-i}\right)^2$$
$$+\frac{\bar{P}_0T}{N_0N}\sum_{s=0}^{N-1}\sum_{\substack{i=0\\i\neq s}}^{N-1}\left(\frac{\varepsilon}{\varepsilon+s-i}\right)^2+\left(\frac{\pi\varepsilon}{\sin\pi\varepsilon}\right)^2N \quad (17)$$

As a result, the conditional signal-to-noise ratio is expressed as

$$\Gamma=\frac{D^2}{\sigma_T^2}=\frac{2NT\cos^2\pi\varepsilon S_{egc}}{\{\Delta_1\}N_0} \quad (18)$$

where $S_{egc}=\frac{1}{2}\frac{\sum_{i=0}^{N-1}\rho_{0i}^2}{N}$. S_{egc} is Gamma distributed. The probability density function of S_{egc} is written as [21]

$$p(S_{egc})=\frac{1}{\Gamma(m_s)}(\frac{2m_s}{\Omega_s})^{m_s}(S_{egc})^{m_s-1}\exp(-\frac{2m_sS_{egc}}{\Omega_s}),\quad S_{egc}\geq 0 \quad (19)$$

where $m_s=Nm$, $\Omega_s=N\Omega$. The average bit error probability can be expressed as

$$\bar{P}_{e,egc}=\int_0^\infty P\{S_{egc}\left|\bar{P}_0,\bar{P}_u,\rho_{0i}\right.\}p(S_{egc})dS_{egc} \quad (20)$$

where $P\{S_{egc}\left|\bar{P}_0,\bar{P}_u,\rho_{0i}\right.\}=Q(\sqrt{\Gamma})$ and

$Q(x)=\frac{1}{\sqrt{2\pi}}\int_x^\infty e^{-\frac{1}{2}t^2}dt$. Substituting Eq. (19) into Eq. (20), we have

$$\bar{P}_{e,egc}=\int_0^\infty\frac{1}{\Gamma(m_s)}\left(\frac{2m_s}{\Omega_s}\right)^{m_s}\left(S_{egc}\right)^{m_s-1}\cdot\exp(\frac{-2m_sS_{egc}}{\Omega_s})$$
$$\left[\frac{1}{\sqrt{2\pi}}\int_{\sqrt{\frac{2NT\cos^2\pi\varepsilon S_{egc}}{\{\Delta_1\}N_0}}}^\infty\exp(-\frac{t^2}{2})dt\right]dS_{egc}$$
$$=\int_0^\infty\frac{1}{\Gamma(Nm)}(a)^{Nm}\left(S_{egc}\right)^{Nm-1}\cdot\exp(-aS_{egc})$$
$$\left[\frac{1}{\sqrt{2\pi}}\int_{\sqrt{cS_{egc}}}^\infty\exp(-\frac{t^2}{2})dt\right]dS_{egc} \quad (21)$$

where

$$a=\frac{2m_s}{\Omega_s}=\frac{2Nm}{2\bar{P}_0(\frac{5m-1}{5m})} \quad \text{and} \quad c=\frac{2NT\cos^2\pi\varepsilon}{N_0\{\Delta_1\}}$$

(21) can also expressed by the Gauss hypergeometric function as

$$\bar{P}_{e,egc} = \sqrt{\frac{\gamma_s}{1+\gamma_s}} \frac{(1+\gamma_s)^{-mN}\,\Gamma(mN+\frac{1}{2})}{2\sqrt{\pi}\,\Gamma(mN+1)}\; {}_2F_1\left(1; mN+\frac{1}{2}; mN+1; (1+\gamma_s)^{-1}\right)$$
$$= \frac{1}{2}\left[1-\sqrt{\frac{\gamma_s}{1+\gamma_s}}\sum_{k=0}^{mN-1}\binom{2k}{k}\left(\frac{1}{4(1+\gamma_s)}\right)^k\right] \quad (22)$$

with

$$\gamma_s = \frac{c}{2a} = \frac{\bar{P}_0 T(\frac{5m-1}{5m})\cos^2 \pi\varepsilon}{mN_0\{\Delta_1\}} \quad (23)$$

B. Bit Error Rate Analysis for Unbalanced EGC

Consider an L-branch EGC system where the power gain of the ith branch ($i=1,\cdots,L$) is some factor β_i ($0<\beta_i\le 1$) less than the gain in the Lth branch. The amplitude of the resultant signal at the output of the EGC is written as

$$\rho = \rho_1+\rho_2+\cdots+\rho_L \quad (24)$$

where ρ_i is the amplitude of the signal received by the ith channel. Assume that the ρ_i are independent Nakagami random variables [19] with the same fading parameter and define

$$\Omega_i = E[\rho_i^2] = \beta_i E[\rho_L^2] = \beta_i\Omega, \quad i=1,2,\ldots,L-1 \quad (25)$$

Let $n = n_1+n_2+\cdots+n_L$ be the resultant noise with

$$E[n_i^2] = \beta_i\Psi, \quad i=1,2,\ldots,L-1 \quad (26)$$

and

$$E[n_L^2] = \Psi \quad (27)$$

Note that, in spite of the gain unbalance, all the branches have equal average SNR. The resultant SNR, Γ, is

$$\Gamma = \frac{\rho^2}{2E[n^2]} = \frac{\rho^2}{2(\beta_1+\beta_2+\cdots+\beta_{L-1}+1)\Psi} \quad (28)$$

Where ρ is the sum of L independent Nakagami distributed RV's having the same fading parameter m and different Ω_i (due to unbalanced gain).

The total variance can be expressed as

$$\sigma_T^2 = \sigma_{MAI}^2+\sigma_{SI_{fo}}^2+\sigma_{MAI_{fo}}^2+\sigma_\eta^2 = \frac{N_0}{T}\left(\frac{\sin\pi\varepsilon}{\pi\varepsilon}\right)^2\{\Delta_2\} \quad (29)$$

and

$$\Delta_2 = (U-1)\frac{\bar{P}_u T}{N_0} + \frac{(U-1)\bar{P}_u T}{N_0(1+\sum_{i=1}^{L-1}\beta_i)}\sum_{s=0}^{N-1}\sum_{\substack{i=0\\ i\ne s}}^{N-1}\left(\frac{\varepsilon}{\varepsilon+s-i}\right)^2 + \frac{\bar{P}_0 T}{N_0(1+\sum_{i=1}^{L-1}\beta_i)}\sum_{s=0}^{N-1}\sum_{\substack{i=0\\ i\ne s}}^{N-1}\left(\frac{\varepsilon}{\varepsilon+s-i}\right)^2 + \left(\frac{\pi\varepsilon}{\sin\pi\varepsilon}\right)^2(1+\sum_{i=1}^{L-1}\beta_i) \quad (30)$$

The conditional signal-to-noise ratio is represented as

$$\Gamma = \frac{X^2}{\sigma_T^2} = \frac{T\cos^2\pi\varepsilon(1+\sum_{i=1}^{L-1}\beta_i)E[\rho_0^2]}{N_0\{\Delta_2\}} \quad (31)$$

and the conditional error rate in terms of Γ and ρ can be written as

$$P_e(\rho) = Q(\sqrt{\Gamma}\rho) \quad (32)$$

where the conditioning is on the signal amplitude. Equation (32) can be averaged over all values of ρ to obtain the average bit error rate in fading conditions. That is,

$$\bar{P}_e = \int_0^\infty P_e(\rho)P(\rho)d\rho \quad (33)$$

where $P(\rho)$ is the pdf of a sum of independent Nakagami distributed RVs. To evaluate $\overline{P}_e$, a convergent infinite series derived in [23] is employed. Defining ρ as in (24), the pdf of ρ can be written as [23]

$$P(\rho)=\frac{2}{T_1}\sum_{\substack{n=1\\ n\ odd}}^{\infty}\left\{e^{-jnw\rho}\Phi(n\omega)+e^{jnw\rho}\Phi(-n\omega)\right\} \tag{34}$$

where $\omega=\frac{2\pi}{T_1}$, and T_1 is the period of the square wave.

$$\Phi(n\omega)=\prod_{i=1}^{L}\Phi_i(n\omega) \tag{35}$$

$$\begin{aligned}\Phi_i(n\omega)&=E[e^{jn\omega\rho_i}]\\ &=A_{in}e^{j\tau_i}\end{aligned} \tag{36}$$

and

$$A_{in}=\sqrt{E^2[\cos(n\omega\rho_i)]+E^2[\sin(n\omega\rho_i)]} \tag{37}$$

$$\tau_i=\tan^{-1}\left\{\frac{E[\sin(n\omega\rho_i)]}{E[\cos(n\omega\rho_i)]}\right\} \tag{38}$$

Substituting (36) into (35) to obtain

$$\Phi(n\omega)=A_ne^{j\tau_n} \tag{39}$$

where $A_n=\prod_{i=1}^{L}A_{in}$, and

$$\tau_n=\sum_{i=1}^{L}\tau_i \tag{40}$$

It can also be shown

$$\Phi(-n\omega)=A_ne^{-j\tau_n} \tag{41}$$

By substituting (34) and (32) into (33), interchanging the integration and summation and using [24: 6.311, P.680 and 6.315.2, P680], (33) can be written as

$$\overline{P}_e=\frac{2}{T_1}\sum_{\substack{n=1\\ n\ odd}}^{\infty}A_nB_n\cos(\tau_n-\alpha_n) \tag{42}$$

where

$$B_n=\sqrt{\frac{1}{(n\omega)^2}[1-\exp(\frac{-n^2\omega^2}{4b^2})]^2+\frac{1}{\pi b^2}[{}_1F_1(1;\frac{3}{2};\frac{-n^2\omega^2}{4b^2})]^2} \tag{43}$$

$$\alpha_n=\tan^{-1}\left\{\frac{\sqrt{\pi}b[1-\exp(\frac{-n^2\omega^2}{4b^2})]}{n\omega[{}_1F_1(1;\frac{3}{2};\frac{-n^2\omega^2}{4b^2})]}\right\} \tag{44}$$

$$b=\sqrt{\frac{T\cos^2\pi\varepsilon(1+\sum_{i=1}^{L-1}\beta_i)}{2N_0\{\Delta_2\}}} \tag{45}$$

Equation (42) will be used to compute the BER.

IV. NUMERICAL RESULTS

In this section, the effect of both frequency offset and gain unbalance EGC on BER performance are illustrated and discussed. Figure 3 shows the BER of an MC-CDMA with 8 subcarriers and 6 users at carrier frequency offsets of 0, 20% for the Nakagami fading parameter m =1. In this figure, the gain is set to be unity for each branch in the balanced case, and the branch gains are assumed 0.07, 0.06, 0.05, 0.04, 0.03, 0.02, 0.01 and 1, respectively, in the unbalanced case. It can be observed that the performance with unbalanced gain is significantly degraded. Furthermore, the system with frequency offset is deteriorated worse.

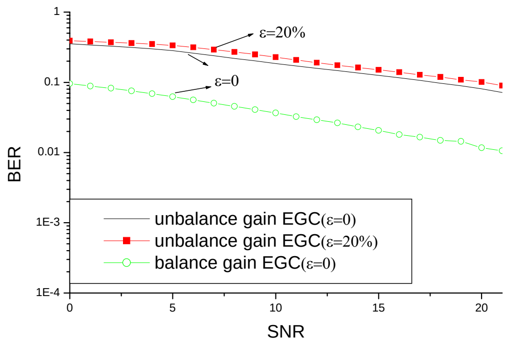

Fig.3. BER of an MC-CDMA with 8 subcarriers and 6 users at different carrier frequency offset and effect of unbalances EGC for the Nakagami fading parameter m =1.

Figure 4 shows the BER comparison between MC-CDMA and DS-CDMA scheme [25] with 10 users. The MC-CDMA with unbalanced gain EGC is supposed to be 8 subcarriers with 10 % carrier frequency offset, and the branch gains are assumed 0.9, 0.8, 0.7, 0.6, 0.5, 0.4, 0.3, and 1, respectively. The DS-CDMA system with three-finger Rake receiver has the decaying rate $\lambda = 0.4$ of multipath delay profile and the processing gain is eight with $m = 1$. We assume that the processing gain is the same as that in MC-CDMA. It can be observed that the performance of an MC-CDMA system with balanced branch gain (setting to be is one) is superior to that of DS-CDMA. All the subcarriers in the MC-CDMA have equal SNR. In the DS-CDMA, the multipath profile is assumed exponential decaying. Therefore, the overall performance of the DS-CDMA is worse than that of the MC-CDMA. When SNR is below 7 dB, however, due to unbalanced gain EGC, DS-CDMA system outperforms MC-CDMA system.

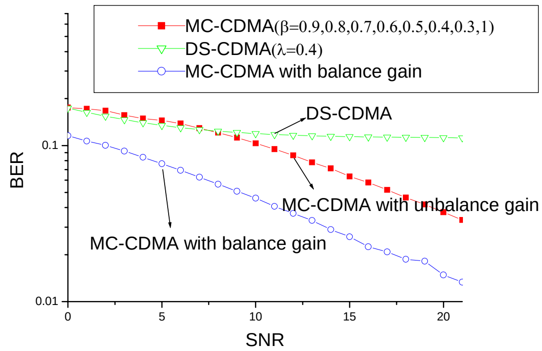

Fig.4. BER of an MC-CDMA using unbalances gain of EGC with 8 subcarriers and 10 users at carrier frequency offsets 10 % versus DS-CDMA with 10 users and three branches and exponential MIP (decaying value $\lambda = 0.4$) for the Nakagami fading parameter m =1.

Figure 5 shows BER of an MC-CDMA with 8 subcarriers and six users at 20% carrier frequency offset and different branch gains with EGC and m =1. The branch gains are arbitrarily assumed 0.07, 0.06, 0.05, 0.04, 0.03, 0.02, 0.01, and 1, respectively. Another possible branch gains are arbitrarily assumed 0.7, 0.6, 0.5, 0.4, 0.3, 0.2, 0.1, 1, respectively. It is apparent that the first situation performs worse than the second one.

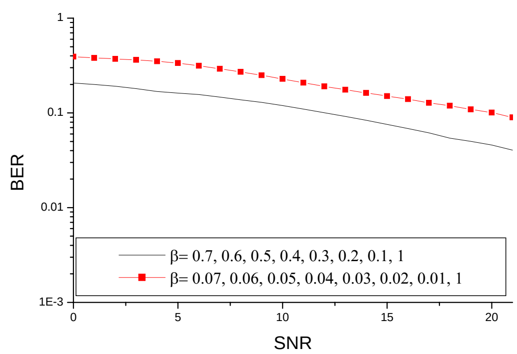

Fig.5. BER of an MC-CDMA with 8 subcarriers and 6 users at carrier frequency offset 20% and different unbalanced gains of EGC and m =1.

Figure 6 shows BER of an MC-CDMA

with 8 subcarriers and 6 users at 10% carrier frequency offset and $m=1$. It is shown that if only the order of branch gain interchanged, the performance of system is unchanged.

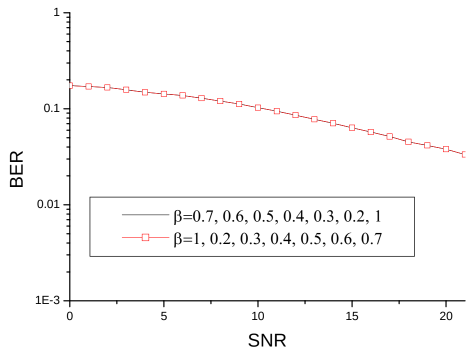

Fig.6. BER of an MC-CDMA with 8 subcarriers and 6 users at carrier frequency offset 10% and change order of unbalances gain of EGC m =1.

V. CONCLUSION

In this paper, the effects of carrier frequency offset and gain unbalance between branches of EGC have been theoretically analyzed on uplink MC-CDMA system over Nakagami fading channel. Results show that the extent of degradation in MC-CDMA depends not only carrier frequency offset and unbalanced gains of branches, but also the number of subcarriers and users. The performance of MC-CDMA system is also compared with that of the conventional single-carrier DS-CDMA system. Numerical results indicate that the MC-CDMA system with unbalanced branch gains and frequency offset suffers more performance degradation. Other observations that can be made include the following. The performance of an MC-CDMA system with balanced gain is superior to that of DS-CDMA. This is because the MC-CDMA can effectively combine all the received signal energy scattered. The performance of DS-CDMA depends on the number of fingers in the Rake receiver. Usually, a three- or four-finger Rake receiver is used due to hardware complexity. Consequently, DS-CDMA misses a part of the received signal energy scattered. However, when SNR is below 7 dB, due to unbalanced gain in EGC, DS-CDMA system outperforms MC-CDMA system. Note from the graphs that the performance of MC-CDMA system is sensitive to gain unbalance between branches of EGC. On other hand, if just the order of gain unbalance between branches is changed, there is no effect on the performance of system. We conclude that the accuracy in carrier frequency offset and unbalanced gain are the two factors that have significant impact on system performance. It should be taken into account in designing MC-CDMA systems.

REFERENCES

[1] Prasad, R., CDMA for wireless personal communications. Artech House, Boston: London, 1996.

[2] Hara, S., and Prasad, R., "Overview of Multicarrier CDMA," IEEE Communications Magazine, pp. 126-133, 1997.

[3] Yee, N., Linnartz, J. P., and Fettweis, G., "Multi-carrier CDMA in Indoor Wireless Radio Networks," IEICE Trans. Communiction, Vol. E77-B, pp. 900-904, 1994.

[4] Stuber, G. L., Principles of Mobile

Communications. Norwell, MA: Kluwer Academic Publishers, 1996.

[5] Zhang, Q. T., "Probability of Error for Equal-Gain Combiners over Rayleigh Channels: Some Closed-Form Solutions," IEEE Transactions on Communications, Vol. 45, No. 3, pp. 270-273, 1997.

[6] Zhang, Q. T., "A Simple Approach to Probability of Error for Equal Gain Combiners over Rayleigh Channels," IEEE Transactions on Vehicular Technology, Vol. 48, No. 4, pp. 1151-1154, 1999.

[7] Kotsopoulos, S. A., and Karagiannidis, G. K., "Error Performance for Equal-Gain Combiners over Rayleigh fading channels," Electronics Letters, Vol. 36, No. 10, pp. 892-894, 2000.

[8] Mallik, R. K., Win, M.Z., and Winters, J. H., "Performance of Dual-Diversity Predetection EGC in Correlated Rayleigh Fading with Unequal Branch SNRs," IEEE Transactions on Communications, Vol. 50, No. 7, pp. 1041-1044, 2002.

[9] Nakagami, M., "The m -distribution-A General formula of intensity distribution of fading," in Statistical Methods in Radio Wave Propagation, W. C. Hoffman, Ed. Oxford U. K.: Pergamon, 1960.

[10] Suzuki, H., "A Statistical Model for Urban Radio Propagation," IEEE Transactions on Communications, Vol. COM-25, pp. 673-680, No. 7, 1977.

[11] Alouini, M. S., and Simon, M. K., "Error Rate Analysis of M-PSK with Equal Gain Combining over Nakagami Fading Channels," IEEE vehicular technology conference 49th Vol. 3, pp. 2378-2382, 1999.

[12] Annamalai, A., Tellambura, C., and Bhargava, V. K., "Unified Analysis of Equal-Gain Diversity on Rician and Nakagami Fading Channels," Wireless Communications and networking conference, Vol. 1, pp. 10-14, 1999.

[13] Annamalai, A., Tellambura, C., and Bhargava, V. K., "Equal-Gain Diversity Receiver Performance in Wireless Channels," IEEE Transactions on Communications Vol. 48, No. 10, pp. 1732-1745, 2000.

[14] Li, Z., and Latva-aho, M., "Error Probability for MC-CDMA in Nakagami-m Fading Channels Equal Gain Combining," IEEE International Conference on Communications Vol. 1, pp. 227-231, 2002.

[15] Tomba, L., and Krzymien, W. A., "Sensitivity of the MC-CDMA Access Scheme to Carrier Phase Noise and Frequency Offset," IEEE Transactions on Vehicular Technology, Vol. 48, No. 5, pp. 1657-1665, 1999.

[16] Jang, J., and Lee, K. B., "Effects of frequency offset on MC/CDMA system performance," Vehicular Technology Conference, IEEE 49th Vol. 1, pp. 566-570, 1999.

[17] Kim, Y., Bang, K., Choi, S., You, C., and Hong, D., "Ber computation of an MC-CDMA systems with carrier frequency offset," IEEE International Conference, Vol. 5, pp. 2555-2558, 1999.

[18] Halpern, S., "The Effect of Having

Unequal Branch Gains in Practical Predetection Diversity Systems for Mobile Radio," IEEE Transactions on Vehicular Technology, Vol. VT-26, No. 2, pp. 94-105, 1977.

[19] Beaulieu, N., and Abu-Dayya, A., "Analysis of Equal gain diversity on Nakagami Fading Channels," IEEE Transactions on Communications, Vol. 39, No. 2, pp. 225-234, 1991.

[20] Papoulis, A., Probability, Random Variables and Stochastic Processes, 3rd ed. New York: McGraw-Hill, 1991.

[21] Abdel-Hafez, M., and Safak, M., "Simplified analysis of equal gain combining in Nakagami fading channels," Electronics Letters Vol. 33, No. 23, pp. 1926-1927, 1997.

[22] Alouini, M. S., Kim, S. W., and Goldsmith, A., "Rake Reception with Maximal-Ratio Equal-Gain Combining for DS-CDMA systems in Nakagami Fading," IEEE 6th international Conference on communications, Vol. 2, pp. 708-712, 1997

[23] Beaulieu, N., "An infinite series for the computation of the complementary probability distribution function of a sum of independent random variables and its application to the sum of rayleigh random variables," IEEE Transactions on Communications, Vol. 38, No. 9, pp. 1463-1474, 1990.

[24] Gradshteyn, I., and Ryzhik, I., Tables of Integrals, Series, and Products. New York: Academic 1980.

[25] Eng, T., and Milstein, L. B., "Coherent DS-CDMA performance in Nakagami multipath fading," IEEE Transactions on Communications, Vol. 43, No. 2-4, pp. 1134-1143, 1995.