

# Application of the Genetic Algorithms to a Specific Low-Pass Filter Prototype Synthesis

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## ABSTRACT

A synthesis method is addressed for the low-pass filter prototypes in a specific form of ladder-type circuits. These filter prototypes each includes two resonant circuits for producing a pair of attenuation poles. One pole is placed to reject the second harmonic and the other pole is shifted toward the cutoff frequency to enhance the cutoff sharpness. Genetic algorithms are applied in this method to increase the searching speed for the optimized value of each circuit element. Several prototypes are synthesized with steeper sharpness and better matching characteristic than that found at the 3-dB Chebyshev responses.

**Key words:** low-pass filter prototype, attenuation poles, genetic algorithms

## 基因演算法應用於特定低通濾波器原型之設計

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## 摘 要

本文藉集總元件構成之特殊梯形電路，提出一種低通濾波器原型的設計法則。這些低通濾波器原型均具有兩組共振電路，用以產生一對衰減極點。其中一個極點可抑制二次諧波，另一個則位於截止頻率附近以獲得陡峭裙邊。此法並運用基因演算法加速搜尋濾波器原型中最佳之電路元件值。最後將此法所設計之結果與柴比雪夫濾波器原型進行比較，顯現極佳的陡峭裙邊與匹配性能。

**關鍵詞：**低通濾波器原型，衰減極點，基因演算法

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## I. INTRODUCTION

Low-pass filter prototypes, which are normalized in terms of frequency and source impedance, simplify the design of filters at arbitrary frequency, impedance, and type (low-pass, high-pass, bandpass, or bandstop). The ladder-type circuits can make various filter prototypes. Element-values for such prototype filters were originally obtained by network synthesis method of Darlington and others [1-3]. Conventionally, reactive elements are chosen as the building blocks of the filter prototypes. Some common prototypes control the insertion loss to have the binomial responses or Chebyshev responses. Some of others, to avoid signal distortion, sacrifice the attenuation rate by obtaining a linear phase response in the passband. The element-values, which result in Chebyshev, binomial and linear phase response, have been tabulated [4, 5]. The sharpest cutoff is determined by Chebyshev response for such circuit configurations. There are many applications, however, require more stringent specifications on the passband matching, cutoff sharpness and harmonic rejection. In order to tackle this problem, resonant circuits are incorporated to modify the ladder-type scheme for making those improvements possible.

Genetic algorithms model evolution and genetic recombination in nature [6] and are employed to obtain the specific low-pass filter prototypes under specifications. The algorithms encode each interested parameter into binary sequences, called a gene, and a set of genes is a chromosome. The algorithm begins with a large list of random chromosomes. These chromosomes undergo natural selection, mating, and mutation, to arrive at the final optimal

solution. The selection is according to certain cost function tailored to reflect the requirements of specifications. The mutation attempts to prevent the final solution from getting stuck into local optima.

## II. SYNTHESIS METHOD

Consider the ladder-type circuits as shown in Fig. 1. The element values are numbered from  $g_0$  at the generator impedance to  $g_{N+1}$  at the load impedance. Two resonant circuits constructed by  $g_k$  and  $g'_k$  ( $k = 1$  and  $N$ ) and other reactive elements  $g_k$  ( $k = 2 \sim N-1$ ) alternating between series and shunt connections. Parallel and series resonant circuits are required respectively for series and shunt connections to preset the attenuation poles. The definitions of each circuit elements are as follows:

$g_0$  = generator resistance ( Fig. 1a ) or conductance ( Fig. 1b ).

$g_k, g'_k$  = inductance for inductors or capacitance for capacitors.

$g_{N+1}$  = load conductance ( Fig. 1a ) or resistance ( Fig. 1b ).

The circuits of Fig.1 can be considered as the dual of each other, and both will give the same response. The element values for some desirable specifications are determined via genetic algorithms. Much more detail of genetic algorithms will be found in [7] and a quick overview for our problem is as follows:

A gene, the basic building block, is a binary encoding of a parameter that manifests itself as an element value. A chromosome comprises genes of the  $N$  element values ( $g_1$  to  $g_N$ ). The values of the elements  $g'_k$  in the filter are

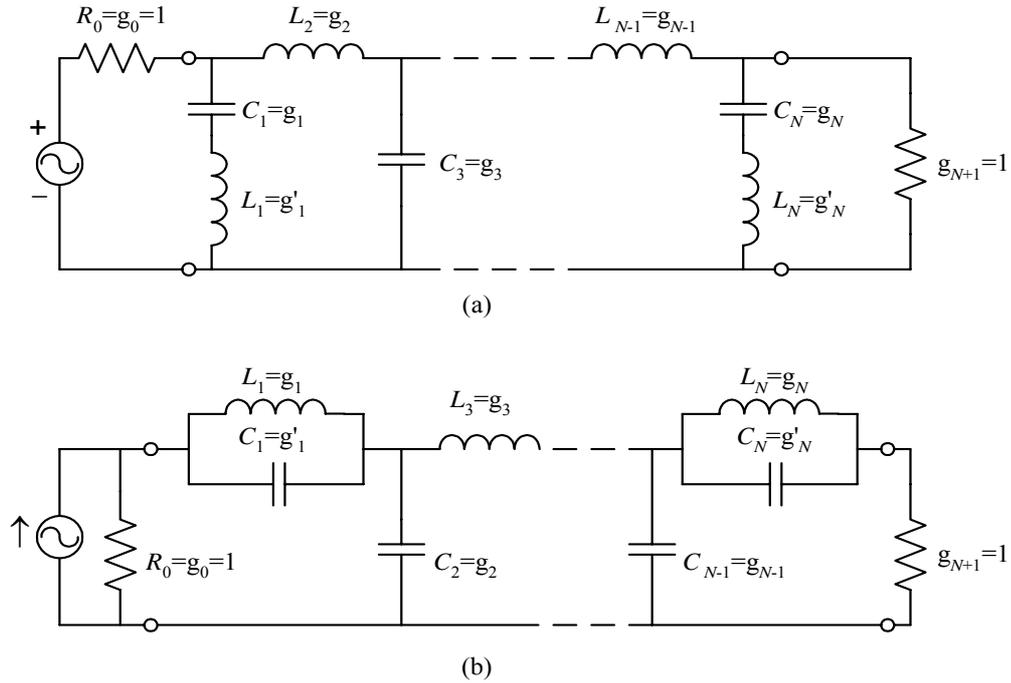


Fig. 1. Ladder circuits for low-pass filter prototypes and their elements definitions. (a) prototype with a parallel resonant element. (b) prototype with a series resonant element.

implicit in  $g_k$  and the pre-assigned resonance frequencies. Each chromosome has an associated cost function, assigning a relative merit to that chromosome. The algorithm begins with a large list of random chromosomes. Cost functions are evaluated for each chromosome. The chromosomes are ranked from the most-fit to the least-fit, according to their respective cost functions. Unacceptable chromosomes are discarded, leaving a superior species-subset of the original list. Chromosomes that survive become parents, by swapping some of their genetic material (some binary bits) to produce two new offspring. The parents reproduce enough to offset the discarded chromosomes. Thus, the total number of chromosomes remains constant after each iteration. Mutations cause small random changes in some chromosomes. Cost functions are evaluated for the offspring and the mutated chromosome, and the process is repeated. The algorithm stops after a set number

of iterations, or when an acceptable solution is obtained. The flow chart of genetic algorithm used in this work is shown in Fig.2.

The cost function according to general specifications may compromise a number of requirements. For each requirement, a specific function can be tailored ranging from a lower value ( the most-fit ) to a higher value ( the least-fit ). The cost function is the weighting sum of the specific functions and is employed to check the global fitness. A cost function  $f_{\text{cost}}$  due to  $n$  requirements can be expressed as

$$f_{\text{cost}} = \sum_{i=1}^n w_i f_i \quad (1)$$

where  $f_i$  is the  $i$ th specific function. The  $i$ th weighting factor,  $w_i$ , can be increased starting from zero to emphasize  $f_i$ . The cost function is preferable smaller in our case. The specific functions in the low-pass filter syntheses can be evaluated as follows:

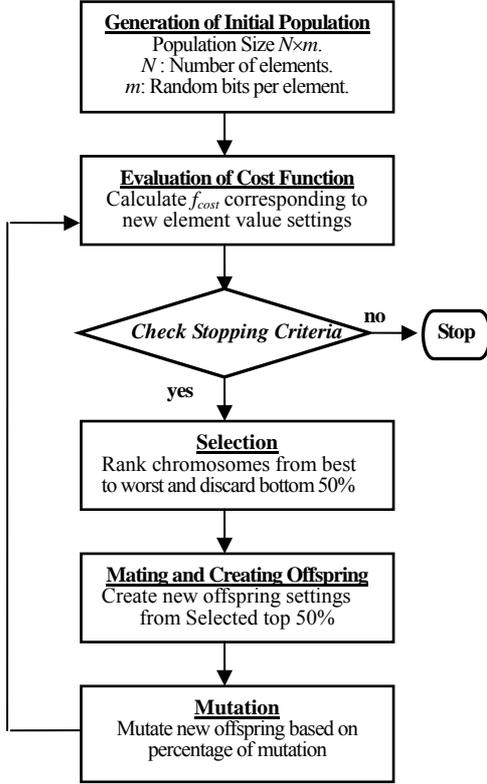


Fig. 2. Flow chart of genetic algorithm utilized in this paper.

1.  $f_1$ : The appreciation of the admissible attenuation on the passband.

$$f_1 = \exp\left(|20\log|\Gamma_M| - 20\log\Gamma_S + u(\Gamma_S)\right) \quad (2)$$

where  $|\Gamma_M|$  is the maximum reflection coefficient on the passband associated to each chromosome. The parameter  $\Gamma_S$  is the admissible attenuation specified on the passband. The step function  $u$  is defined as

$$u(\Gamma_S) = \begin{cases} 0, & |\Gamma_M| \leq \Gamma_S \\ 1, & |\Gamma_M| > \Gamma_S \end{cases} \quad (3)$$

The step function  $u$  makes  $f_1$  jump to a less-fit value if  $|\Gamma_M|$  increases over  $\Gamma_S$ . Meanwhile, it guarantees that  $f_2$  gains its minimum as  $|\Gamma_M| = \Gamma_S$ .

2.  $f_2$ : The partial sum of the leakage into the load on the stopband.

$$f_2 = \sum_{\omega_i} 10 \log(1 - |\Gamma(\omega_i)|^2) \quad (4)$$

The frequencies  $\omega_i$  avoiding the resonance points are selected on the stopband for checking the cutoff characteristic.

3.  $f_3$ : The worst mismatching in the return loss lobes over the passband.

$$f_3 = -\left(\min_{lobe-levels} (-20\log|\Gamma|)\right) \quad (5)$$

A better matching on the passband can be obtained as  $f_3$  is more negative (smaller).

### III. SIMULATION AND RESULTS

The encoded form,  $g_{encoded}$ , for a particular element  $g$ , can be established via the method of binary coding, i.e.,  $g_{encoded} = b_1 b_2 b_3 \cdots b_m$ , where  $m = 12$  is the number of bits used to encode the element. Hence,

$$g_{decoded} = ((b_{max} - b_{min}) / (2^m - 1)) * \sum_{i=1}^m (2^i b^i) + b_{min}$$

where  $b_{max}$  and  $b_{min}$  are the limits within which the value of a particular element may be allowed to vary. A chromosome consists of its parts, each of which is an encoded form of a particular element. Evolution occurs from one generation to the next through crossovers and mutations. The population size is taken to be the total number of bits in a chromosome ( $m \times N$ ). The probability of mutation is taken to be about 0.01. Multiple search is carried out a number of times separately instead of continuously, computing a large number of generations. Each search is terminated by a test of convergence or after 120 iterations are completed. Fig. 3 is the graph of the algorithm's performance of a typical search for this case. The computing time of each search is increasing with the filter-order  $N$  and it is usually less than 8min with a 667MHz Pentium-III processor for  $N = 8$ .

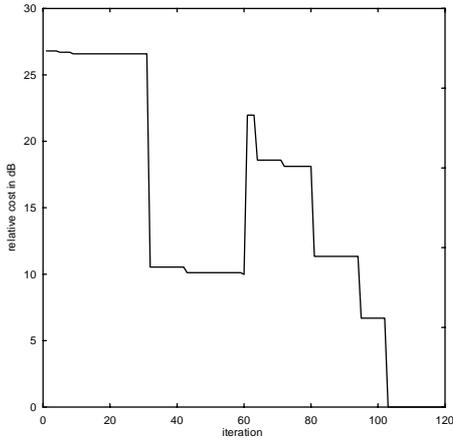


Fig. 3. Relative cost in dB as a function of iteration of the genetic algorithm. This is the cost that the genetic algorithm is minimizing.

The synthesized filter prototypes with order  $N = 3$  to 8 are shown in Table 1. In these prototypes, one of the normalized resonance frequencies of the two resonant circuits are distributed from 1.5 to 1.9 while the other is fixed at 2. The filters for making the two resonant circuits with the same type (series or parallel resonance) as  $N$  even are synthesized by adding  $g'_1$ , and  $g'_{N-1}$  in constructing the two attenuation poles. The prototypes have better cutoff, return loss and phase response than that of 3dB Chebyshev responses in the same order.

All the return loss lobe levels are better than 10 dB as shown in Table I. Even better property on passband matching can be obtained if the sharpness requirement is relaxed. The unity load resistance ( $g_{N+1}=1$ ) is more feasible than the non-unity resistance loaded in the Chebyshev even order prototypes. Part of the comparison results between the synthesized and the Chebyshev prototypes are shown in Figure 4 and 5. It is observed that the number of the return loss lobe for the synthesized prototype on the passband is equal to that of the Chebyshev prototype for odd  $N$  and less than one for even  $N$ . They also illustrate that the prototype with lower filter order generally gives more pronounced improvements than a higher order does for the cutoff and the passband matching property.

#### IV. CONCLUSION

A method being able to synthesize the low-pass filter prototypes with the application of genetic algorithms is addressed. These prototypes each have two resonant circuits for producing a pair of attenuation poles. One of the poles is fixed to reject the second harmonic while the other is used to enhance the cutoff

Table 1. Element Values of the Low-Pass Filter Prototypes Synthesized by GA.

| N | $g_0$  | $g_1$<br>$g'_1$  | $g_2$<br>$g'_2$ | $g_3$<br>$g'_3$  | $g_4$<br>$g'_4$ | $g_5$<br>$g'_5$  | $g_6$<br>$g'_6$ | $g_7$<br>$g'_7$  | $g_8$<br>$g'_8$ | $g_9$<br>$g'_9$ | Return loss lobe level |
|---|--------|------------------|-----------------|------------------|-----------------|------------------|-----------------|------------------|-----------------|-----------------|------------------------|
| 3 | 1.0000 | 1.2567<br>0.1989 | 1.3740<br>None  | 1.2567<br>0.2204 | 1.0000<br>None  |                  |                 |                  |                 |                 | 15.0 dB                |
| 4 | 1.0000 | 1.3553<br>0.1845 | 1.2030<br>None  | 1.5630<br>0.2214 | 0.6917<br>None  | 1.0000<br>None   |                 |                  |                 |                 | 11.2 dB                |
| 5 | 1.0000 | 1.3510<br>0.1850 | 1.3310<br>None  | 2.5450<br>None   | 1.3310<br>None  | 1.3510<br>0.2285 | 1.0000<br>None  |                  |                 |                 | 11.5 dB                |
| 6 | 1.0000 | 1.3610<br>0.1837 | 1.3623<br>None  | 2.5503<br>None   | 1.3417<br>None  | 1.4523<br>0.2690 | 0.6913<br>None  | 1.0000<br>None   |                 |                 | 10.9 dB                |
| 7 | 1.0000 | 1.3577<br>0.1841 | 1.2820<br>None  | 2.6430<br>None   | 1.4407<br>None  | 2.6430<br>None   | 1.2820<br>None  | 1.3577<br>0.2549 | 1.0000<br>None  |                 | 10.5 dB                |
| 8 | 1.0000 | 1.3743<br>0.1819 | 1.2793<br>None  | 2.6573<br>None   | 1.5630<br>None  | 2.3610<br>None   | 1.3710<br>None  | 1.3727<br>0.3238 | 0.5890<br>None  | 1.0000<br>None  | 10.4 dB                |

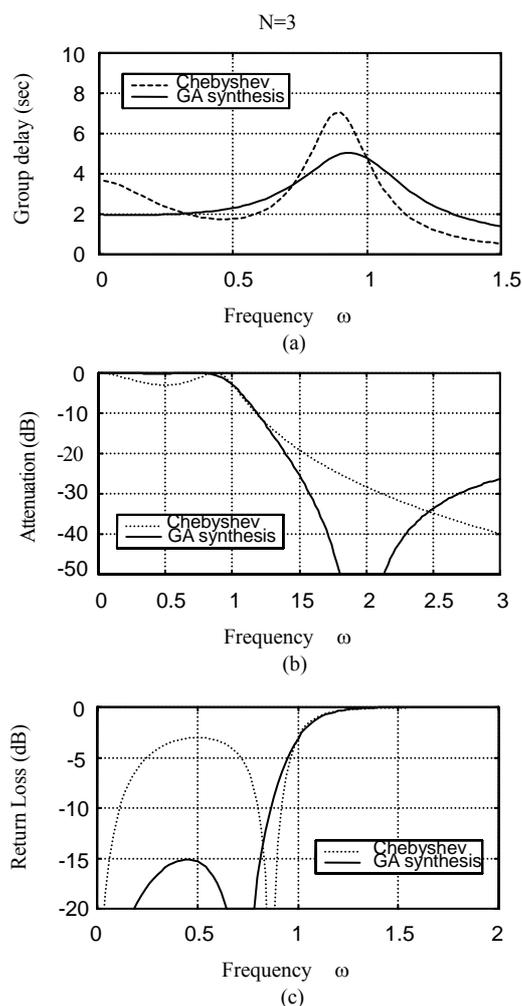


Fig. 4. The comparison of filter prototype responses of  $N=3$  for (a) group delay, (b) attenuation and (c) return loss.

sharpness. This method is applied successfully to the synthesis of low-pass prototypes that are better than the corresponding Chebyshev prototypes on a sensible frequency range although it companions inevitably an inferior cutoff characteristic as the frequencies beyond this range. For the synthesized filter of even order, the unity load is advantageous over the Chebyshev non-unity load in accordance with the standard termination. In principle, the synthesized prototypes can also be implemented with distributed elements at microwave frequencies if Richard's transformation and Kuroda's identities are applied.

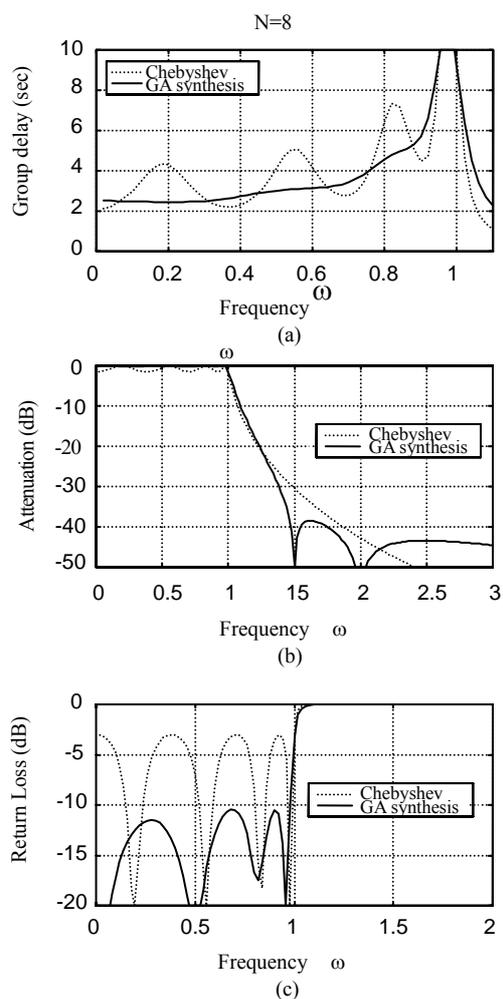


Fig. 5. The comparison of filter prototype responses of  $N=8$  for (a) group delay, (b) attenuation and (c) return loss.

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