Performance Analysis of QAM Signal Recognition Based on Theoretical Approach over Nakagami Fading Channel

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ABSTRACT

In this paper, we analyze the performance of a QAM signal recognizer based on the theoretical approach over Nakagami fading channel. Firstly, the amplitude probability density functions of M-ary QAM signal over flat and slowly Nakagami fading channel are derived, and then the recognition statistics are developed. In addition, an example for recognizing the 16/64 QAM signals is given to demonstrate the performance. Both the theoretical results and the Monte Carlo simulations indicate that the performance of recognizer depends heavily on the severity of channel fading. When the fading figure m approaches infinity (AWGN), the performance is the best. The performance, however, is degraded with the decrease of m, and the worst performance occurs when $m = 0.5$.

Keywords: modulation recognition, Nakagami fading, QAM

在中上衰落通道中基於理論法之 QAM 訊號調變型態識別性能分析之研究

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摘要

本文提出在中上衰落通道中基於理論法之 M 準位正交振幅調變(M-ary QAM)型態識別器之性能分析研究。首先推導在平坦慢速中上衰落通道中 M-ary QAM 訊號之振幅機率密度函數，接著據之發展識別統計。此外，並以 16/64 QAM 訊號識別為例，分析此識別器的性能。理論分析以及電腦模擬結果顯示，辨識器性能和通道之衰落程度息息相關，當衰落指數 m 為趨近於限大時(AWGN)，其性能表現最佳。當通道之衰落指數 $m = 0.5$ 時，其性能表現最差。

關鍵詞:調變型態識別，中上衰落，正交振幅調變

文稿收件日期 92.1.21；文稿修正後接受日期 93.6.30
Manuscript received Jan. 21, 2003; revised June 30, 2004

I. INTRODUCTION

Modulation recognition is a particular technique that recognizes the modulation type with the help of some parameters that are extracted from the received signals at the receiver. These parameters employed may be in the frequency domain or in the time domain, for examples, instantaneous frequency, instantaneous phase, envelope, histograms, zero-crossing, moments, cumulants, etc. Modulation recognition has already been playing an important role for various applications, such as spectrum management, network traffic administration, different data rate allocation, electronic surveillance, interception, interference identification and monitoring [1-8].

Some modulation recognition technologies were proposed in additive white Gaussian noise (AWGN) channel [1-3] or in fading channels [4-8]. From the point of view of practical use, the fading phenomenon always more or less exists in communication process. Usually, the fading channels can be classified as frequency nonselective or selective, and slow or fast. Frequency nonselective fading is also called flat fading, in which all the frequency components in transmitted signals are confronted by the same attenuation and phase shift in transmission through the channel. Flat or frequency selective fading channels can be characterized as channel time spreading, such as tapped-delay-line model, and slow or fast ones can be modeled as Doppler power spectral, such as Jakes' model [9]. In reality, both time spreading and Doppler spreading may appear simultaneously in fading channels. Therefore, the widespread discussions about fading channel models may include amplitude attenuation and spectral distortion separately or jointly.

Modulation recognition schemes can be usually categorized in the statistical pattern recognition and the decision theoretic (or likelihood function) approaches. For decision theoretic approach, it utilizes the probabilistic arguments to formulate the recognition problem. The solution is usually optimal in the sense that it minimizes the recognition error-rate. The drawback of this approach lies in that the characteristics of parameters must be known *a priori*. On the other hand, the statistical pattern recognition approach is functionally composed of two-stage (cascade) subsystems: firstly, the feature extraction subsystem that is used to extract the useful information from the received signal and secondly, the pattern recognition subsystem that is used to show the membership of modulation type.

In [1], Yang *et al.* classified the M-ary QAM signals by employing the maximum-likelihood decision theory in AWGN environments. The authors derived the log-likelihood functions based on the probability density function (PDF) of amplitudes, and then constructed the recognition rules. In [2], A. Polydoros *et al.* distinguished between BPSK and QPSK by decision theoretical approaches. In [3], Huse *et al.* proposed the recognizer using the statistical moment of the signals' phase to classify the CW and MPSK signals. In [4, 5], the authors utilized equalization algorithm to deal with the signals before recognition. In [4], Swami *et al.*

classified PAM, PSK, and QAM signals by the fourth-order cumulants in frequency-selective channel. In [5], Hatzichristos *et al*. employed higher-order statistics parameters as features to classify PSK, FSK, and QAM signals in multipath channel. El-Mahdy *et al*. [6] proposed the recognizer based on approximate likelihood function which was the Fourier transform of received signal and employed to classify the M-ary FSK signal over a frequency nonselective Rayleigh fading channel. In fading channels, the equalization technology was used to compensate for the distortion of the receiver. In [8], Nolan *et al.* proposed modulation recognition based on modified pattern and signal space approaches, and applied it to real-time software radio.

In this paper, we extend the result of [1] to classify M-ary QAM signals, but in flat, slowly Nakagami fading channel. The Nakagami fading model is the most widely used to describe the different fading environments that are either more or less severe than Rayleigh fading, and it is the best fit for received signals in urban radio multipath channel [9, 10]. The amplitude PDF of QAM signals will be derived and is employed to develop the required likelihood functions.

This paper begins with introduction in Section I, and is organized as follows. In Section II, the amplitude PDF of the received QAM signal is derived over Nakagami fading channel. In Section III, we derive the decision rule from the likelihood function based on the amplitude PDF of the received QAM signal and then we manifest the simple recognizer structure to classify 16/64 QAM signals. In Section IV, the performance of the 16/64 QAM recognizers are analyzed theoretically and proved by Monte Carlo simulations. In Section V, we give our conclusions.

II. SIGNAL REPRESENTATION OVER FADING CHANNEL AND AMPLITUDE PDF DERIVATION

In this section, we describe the QAM signal over Nakagami fading channel in terms of the equivalent lowpass representations, and derive its amplitude PDF. Assumed that $s_l(t)$, the equivalent low-pass signal of QAM signal $s(t)$, was transmitted over a flat, slowly fading channel with transfer function $\beta e^{-j\varphi}$, the received equivalent low-pass signal can be represented as [9]

$$r_l(t) = \beta e^{-j\varphi} s_l(t) + n_l(t) \tag{1}$$

where β and φ are the attenuation factor and the phase shift, respectively, of the equivalent low-pass channel, and $n_l(t)$ denotes the complex-valued white Gaussina noise with zero mean and covariance function $2\sigma^2\delta(\tau)$ and is assumed to be uncorrelated with $s_l(t)$.

For any fixed value of t, α and φ can be viewed as random variables; usually φ is uniformly distributed over $(-\pi,\pi)$, and β may be one of the distributions, such as Rayleigh, Rician and Nakagami distributions.

The equivalent low-pass signal $s_l(t)$ is given by

$$\begin{aligned} s_l(t) &= A_I g(t) + jA_Q g(t) \\ &= Ag(t)e^{j\theta} \end{aligned} \tag{2}$$

where A_I and A_Q denote the signal amplitudes of in-phase and the quadrature carriers, and

$g(t)=1, 0 \le t \le T$ is the signal pulse. $A=\sqrt{A_I^2+A_Q^2}$ and $\theta=\tan^{-1}[A_Q/A_I]$.

From Eqs.(1) and (2), the received equivalent low-pass signal can be expressed as [9]

$$\begin{aligned} r_l(t) &= \beta e^{-j\varphi} A g(t) e^{j\theta} + n_I(t) + j n_Q(t) \\ &= \beta A g(t)\cos(\theta-\varphi) + n_I(t) \\ &\quad + j[\beta A g(t)\sin(\theta-\varphi) + n_Q(t)] \end{aligned} \tag{3}$$

where $n_I(t)$ and $n_Q(t)$ denote in phase and the quadrature components of $n_l(t)$. Both $n_I(t)$ and $n_Q(t)$ are zero mean and covariance function $\sigma^2\delta(\tau)$.

In fading channel, the average signal-to-noise ratio, $\overline{SNR}$, is usually expressed in term of signal-to-noise ratio (SNR) as

$$\overline{SNR} = E[\beta^2]\cdot SNR \tag{4}$$

where $E[\cdot]$ denotes the mean of the random variable in the brackets following.

At the same time instant, the amplitude of $r_l(t)$ is then written as

$$z=\sqrt{z_I^2+z_Q^2} \tag{5}$$

where $z_I = \beta A\cos(\theta-\varphi)+n_I$ and $z_Q=\beta A\sin(\theta-\varphi)+n_Q$.

For any given value of β, θ and φ, both z_I and z_Q are Gaussian variables and may be shown to be uncorrelated, henceforth statistically independent [12]. Thus the joint PDF of z_I and z_Q conditioned on the β, θ and φ can be represented by

$$\begin{aligned} &p(z_I, z_Q \mid \beta,\theta,\varphi) \\ &= \frac{1}{2\pi\sigma^2} e^{-\frac{1}{2\sigma^2}[(z_I-\beta A\cos[\theta-\varphi])^2+(z_Q-\beta A\sin[\theta-\varphi])^2]} \end{aligned} \tag{6}$$

Let $z_I = z\cos\psi$ and $z_Q = z\sin\psi$, then Eq.(6) can be written

$$p(z,\psi|\beta,\theta,\varphi) = \frac{z}{2\pi\sigma^2} e^{-\frac{z^2+\beta^2A^2-2\beta Az\cos(\theta-\psi-\varphi)}{2\sigma^2}} \tag{7}$$

Eq.(7) is joint PDF of z and ψ. The marginal PDF of z can be obtained by integrating $p(z,\psi \mid \beta,\theta,\varphi)$ over the range of ψ, yields $p(z \mid \beta)$, that is

$$p(z|\beta) = \frac{z}{\sigma^2} e^{-\frac{1}{2\sigma^2}(z^2+\beta^2A^2)} I_0\left(\frac{\beta Az}{\sigma^2}\right) \tag{8}$$

where $I_0(\cdot)$ is the modified Bessel function of the first kind with zero order.

For $\beta>0$, the resulting PDF $p(z)$ is referred to Rician distribution.

2.1 Derivation of Amplitude PDF of Generic Signal over Nakagami Fading Channels

Suppose that β is random and β obeys the Nakagami distribution as [9-11]

$$p_\beta(\beta) = \frac{2}{\Gamma(m)}\left(\frac{m}{\Omega}\right)^m \beta^{2m-1} e^{-m\beta^2/\Omega},\ \beta \ge 0 \tag{9}$$

where $\Gamma(\cdot)$ is the Gamma function, $\Omega = E[\beta^2]$, $m=\Omega^2/E[(\beta^2-\Omega)^2]$ is called the fading figure which values from 0.5 to ∞. Note that the Rayleigh distribution is a special case of Nakagami distribution with $m=1$.

Eq.(8) is a conditional PDF of the random variable β, and the amplitude PDF of the received signal $s_l(t)$ over Nakagami fading channel can be calculated as [13]

$$\begin{aligned} & p_N(z) \\ & = \int_0^\infty p(z\,|\,\beta)p_\beta(\beta)d\alpha \\ & = \int_0^\infty \frac{z}{\sigma^2}e^{\frac{-(z^2+\beta^2A^2)}{2\sigma^2}} I_0(\frac{z\beta A}{\sigma^2})\frac{2}{\Gamma(m)}(\frac{m}{\Omega})^m \beta^{2m-1}e^{\frac{-m\beta^2}{\Omega}}d\beta \\ & = \frac{z}{\sigma^2}e^{\frac{-z^2}{2\sigma^2}}(\frac{2m\sigma^2}{A^2\Omega+2m\sigma^2})^m {}_1F_1[m;1;\frac{z^2A^2\Omega}{2\sigma^2(A^2\Omega+2m\sigma^2)}] \end{aligned} \quad (10)$$

where ${}_1F_1(a;b;x)$ is the confluent hypergeometric function, which is defined as

$${}_1F_1(a;b;x)=\sum_{k=0}^{\infty}\frac{\Gamma(a+k)\Gamma(b)x^k}{\Gamma(a)\Gamma(b+k)k!},\ \beta\neq 0,-1,\ldots \quad (11)$$

Fig. 1 shows Eq.(10) for various values of m under $A=10$, $\Omega=1$, and $\sigma^2=1$.

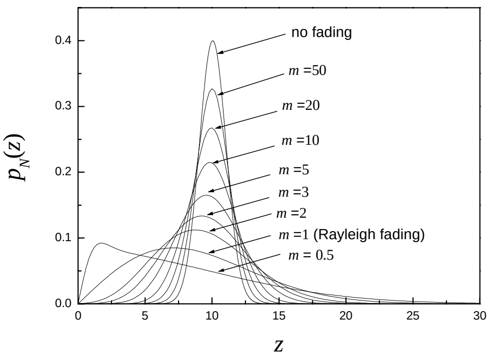

Fig.1. Plots of PDF $p_N(z)$ for various values of fading figure m.

2.2 Amplitude PDF Derivation of QAM Signals over Nakagami Fading Channels

Generally, an M-ary QAM signal in terms of its equivalent low-pass signal, $s_{ml}(t)$ can be written by

$$\begin{aligned} s_{ml}(t) &= A_{mI}\,g(t) + jA_{mQ}\,g(t) \\ &= A_m e^{j\theta_m} g(t),\ m=1,2,\ldots M,\ 0\le t\le T \end{aligned} \quad (12)$$

where A_{mI} and A_{mQ} are, respectively, the information-bearing signal amplitude of in-phase and the quadrature carriers. $g(t)=1, 0\le t\le T$ is the signal pulse, and $A_m=\sqrt{A_{mI}^2+A_{mQ}^2}$ and $\theta_m=\tan^{-1}(A_{mQ}/A_{mI})$ are the amplitude and phase of the $s_{ml}(t)$.

From Eq.(12), we can observe that the QAM signals are exactly the combination of amplitude and phase modulation. For a specific 'M-ary' QAM signal, the amplitudes $\{A_m, m=1,2\ldots,M\}$ consist of some subsets of same values, i.e., $\{A_{M,j}\}$, $j=1,2,\ldots,g$ where g represents the number of subsets. Fig. 2 illustrates that the constellations of 16QAM and 64QAM signal consist of 3 and 9 subsets, respectively.

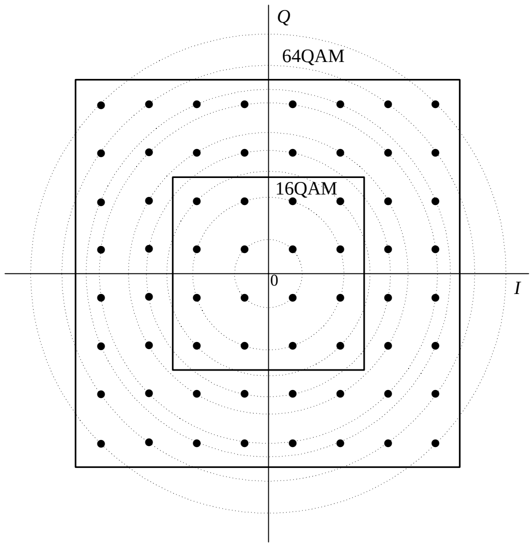

Fig.2. The constellation of 64QAM signals.

Besides, the average transmitted QAM signal power can be determined as

$$P_M = \frac{1}{2M}\sum_{m=1}^{M}A_m^2 = \frac{1}{2}\sum_{j=1}^{g}w_{M,j}A_{M,j}^2 \quad (13)$$

where $w_{M,j}$ represents the ratio of the number of elements in the jth subset to M , or equivalently, the probability of appearance of the same amplitudes.

To begin with, we describe the amplitude PDF of QAM signal in AWGN first. According to Eq.(8), the amplitude PDF of M-ary QAM perturbed by AWGN with variance σ^2 can be expressed as

$$p_M(z) = \sum_{j=1}^{g} w_{M,j} \frac{z}{\sigma^2} e^{\frac{-(z^2+A_{M,j}^2)}{2\sigma^2}} I_0(\frac{zA_{M,j}}{\sigma^2}) \quad (14)$$

For example, 16QAM signal possesses three distinct subsets that are $A_{16,j}$, $j=1,2,3$, and its amplitude PDF is

$$p_{16}(z) = \sum_{j=1}^{3} w_{16,j} \frac{z}{\sigma^2} e^{\frac{-(z^2+A_{16,j}^2)}{2\sigma^2}} I_0(\frac{zA_{16,j}}{\sigma^2}) \quad (15)$$

where $\{w_{16,j}, j=1,2,3\} = \{1/4, 1/2, 1/4\}$.

Similarly, the amplitude PDF of 64QAM perturbed by AWGN can be expressed as

$$p_{64}(z) = \sum_{j=1}^{9} w_{64,j} \frac{z}{\sigma^2} e^{\frac{-(z^2+A_{64,j}^2)}{2\sigma^2}} I_0(\frac{zA_{64,j}}{\sigma^2}) \quad (16)$$

where $\{w_{64,j}, j=1,2,...,9\} = \{1/16, 1/8, 1/16, 1/8, 1/8, 3/16, 1/8, 1/8, 1/16\}$.

By denoting a_M as the minimum amplitude of an M-ary QAM signal, for example, a_{16} represents the least value in all possible amplitudes of 16 QAM signal, then we have $\{A_{16,j}, j=1,2,3\} = \{a_{16}, \sqrt{5}a_{16}, 3a_{16}\}$. And the amplitudes of a 64 QAM signal is as $\{A_{64,j}, j=1,2,...,9\} = \{a_{64}, \sqrt{5}a_{64}, 3a_{64}, \sqrt{13}a_{64}, \sqrt{17}a_{64}, 5a_{64}, \sqrt{29}a_{64}, \sqrt{37}a_{64}, 7a_{64}\}$.

To show the amplitude PDF as a function of SNR, suppose that the QAM signal power is set to be 100 W. Figs. 3 and 4 illustrate the amplitude PDFs of 16QAM and 64QAM with $a_{16} = \sqrt{40}$ and $a_{64} = \sqrt{10}$, respectively.

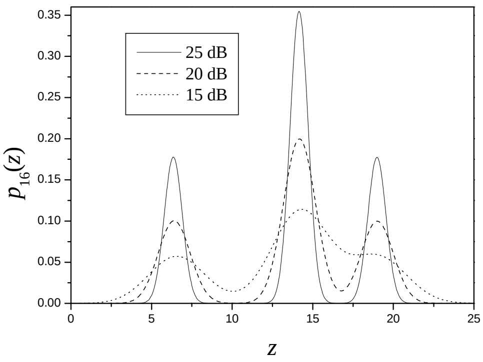

Fig. 3. Plot of the PDF $p_{16}(z)$ with various SNR.

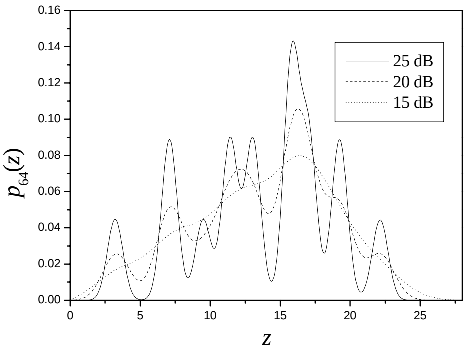

Fig. 4. Plot of the PDF $p_{64}(z)$ with various SNR.

The amplitude PDF of the M-ary QAM signals over a Nakagami channel, according to Eq.(10), can be expressed as

$$p_{N,M}(z) = \sum_{j=1}^{g} w_{M,j} \frac{z}{\sigma^2} e^{-\frac{z^2}{2\sigma^2}} (\frac{2m\sigma^2}{A_{M,j}^2\Omega + 2m\sigma^2})^m \cdot {}_1F_1(m;1;\frac{z^2 A_{k,j}^2 \Omega}{2\sigma^2(A_{k,j}^2\Omega + 2m\sigma^2)}) \quad (17)$$

The amplitude PDF of the 16QAM signals over a Nakagami channel can be expressed as

$$p_{N,16}(z) = \sum_{j=1}^{3} w_{16,j} \frac{z}{\sigma^2} e^{\frac{-z^2}{2\sigma^2}} (\frac{2m\sigma^2}{A_{16,j}^2\Omega + 2m\sigma^2})^m \cdot {}_1F_1[m;1;\frac{z^2 A_{16,j}^2 \Omega}{2\sigma^2(A_{16,j}^2\Omega + 2m\sigma^2)}] \quad (18)$$

In the same manner, the amplitude PDF of the 64QAM signals over a Nakagami channel can be expressed as

$$p_{N,64}(z)=\sum_{j=1}^{9} w_{64,j}\,\frac{z}{\sigma^2}e^{\frac{-z^2}{2\sigma^2}}\left(\frac{2m\sigma^2}{A_{64,j}^2\Omega+2m\sigma^2}\right)^m \cdot {}_1F_1\left[m;1;\frac{z^2A_{64,j}^2\Omega}{2\sigma^2(A_{64,j}^2\Omega+2m\sigma^2)}\right] \quad (19)$$

Fig.5 shows the plots of amplitude PDF of 16QAM signal over Nakagami fading channel under $\Omega=1$ and average SNR=20dB with various m.

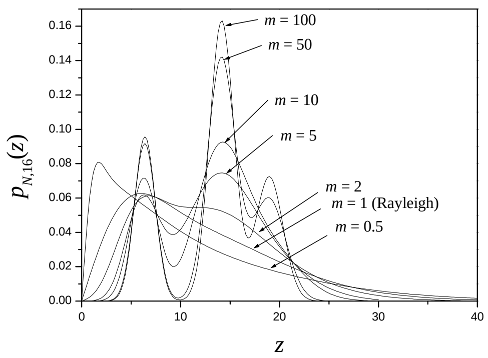

Fig. 5. Plots of the amplitude PDF of 16QAM signal over Nakagami fading channel with various m.

III. DERIVATIONS OF RECOGNITION STATISTICS FOR QAM SIGNALS

A vector $\mathbf{r}_l=[r_{l1}\ r_{l2}\ ...\ r_{lN}]$ is often employed to represent the equivalent low-pass received signals over a Nakagami fading channel, where N is referred to the number of symbol interval. In this section, we develop the recognition statistics by virtue of the amplitude $\mathbf{z}$ extracted from the vector $\mathbf{r}_l$ and demonstrate a recognizer to indicate the member from the set of all the possible signal waveforms being transmitted.

The recognition problem can be viewed as an L-hypothesis detection problem as follows

$$H_\alpha : r_{li}=\beta s_{mli}+n_{li}=z_i e^{j\psi_i},\ \alpha=1,2,...,L, \quad i=1,2,...,N,\quad m=1,2,...,M \quad (20)$$

where $M=2^{\alpha+3}$ and r_{li}, s_{mli}, and n_{li} are, respectively, the sampled data from the ith symbol interval $iT\le t\le(i+1)T$ of $r_l(t)$, $s_{ml}(t)$ and $n_l(t)$. z_i and ψ_i are the amplitude and the phase of r_{li}, respectively. The samples n_{li} are assumed to be statistically independent.

The posterior probabilities can be expressed as $P(H_\alpha\,|\,\mathbf{z})$. Two criteria can be used to derive the recognition statistics. One is called the maximum a posteriori probability (MAP) and the other is called the maximum-likelihood (ML). By MAP, one uses all the information of the posterior probabilities to decide the recognition statistic, and the result is corresponding to selecting the maximum from the set of posterior probabilities. That is, the recognition statistic is to choose H_α if $P(H_\alpha\,|\,\mathbf{z})>P(H_\beta\,|\,\mathbf{z})$, for all $\alpha\ne\beta$

On the other hand, the ML criterion only uses $p(\mathbf{z}\,|\,H_\alpha)$, the conditional PDF of the observed vector under hypothesis H_α. Consequently, the recognition statistic based on the maximum of $P(H_\alpha\,|\,\mathbf{z})$ is corresponding to finding the signal having the maximum of $p(\mathbf{z}\,|\,H_\alpha)$.

Because the noise samples n_{li} are assumed to be i.i.d., the conditional PDF of the random vector $\mathbf{z}$ is the product of N individual PDF's as

$$p(\mathbf{z}|H_\alpha) = p(z_1, z_2, ..., z_N \mid H_\alpha) = \prod_{i=1}^{N} p(z_i \mid H_\alpha) \quad (21)$$

It can take the natural logarithm of $p(\mathbf{z}\mid H_\alpha)$ to simplify the computations. The resulting recognition statistic of hypothesis H_α, x_α, is expressed by

$$x_\alpha = \ln[p(\mathbf{z}\mid H_\alpha)] = \sum_{i=1}^{N} \ln[p(z_i \mid H_\alpha)], \quad \alpha = 1, 2, ..., L \quad (22)$$

As a result, the recognition statistics for M-ary QAM signal over Nakagami fading channel can be expressed as

$$\begin{aligned} x_\alpha &= \ln\left[p_N(\mathbf{z}\mid H_\alpha)\right] \\ &= \sum_{i=1}^{N} \ln\left[p_{N,M}(z_i)\right] \\ &= \sum_{i=1}^{N} \ln\{\sum_{j=1}^{g} w_{M,j} \frac{z_i}{\sigma^2} e^{\frac{-z_i^2}{2\sigma^2}} \left(\frac{2m\sigma^2}{A_{M,j}^2\Omega + 2m\sigma^2}\right)^m \\ &\quad \cdot {}_1F_1[m;1;\frac{z_i^2 A_{M,j}^2 \Omega}{2\sigma^2(A_{M,j}^2\Omega + 2m\sigma^2)}]\}, \\ &\alpha = 1, 2, \cdots, L, \quad M = 16, 32, ..., 2^{L+3} \end{aligned} \quad (23)$$

Eq.(23) is the required recognition statistic for the QAM signal recognizer over Nakagami fading channels.

The M-ary QAM recognizer can be structured in Fig. 6.

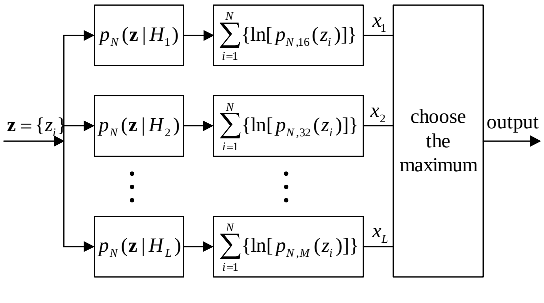

Fig.6. The structure of M-ary QAM recognizer in Nakagami fading channel.

IV. PERFORMANCE ANALYSIS AND COMPUTER SIMULATION

Based on the structure of the recognizer in the previous section, in this section, we will evaluate the performance of recognizer over the Nakagami fading channel. The recognition statistics, consisting of N samples, are denoted as $x_\alpha, \alpha = 1, 2, ..., L$. According to the central limit theorem (CLT), when N is large enough, x_α can be treated as Gaussian distributions with means m_α and variances σ_α^2. Furthermore, there may exist correlation between the statistic branches. The covariances function between statistics is denoted as $\mu_{\alpha\alpha'}$, $\alpha, \alpha' = 1, 2, ...L$.

Referring to Fig. 6, x_α can be modeled as joint Gaussian PDF as [9]

$$p(x_1, x_2, ..., x_L) = \frac{1}{(2\pi)^{L/2}(\det \mathbf{M})^{1/2}} e^{-(\mathbf{x}-\mathbf{m})'\mathbf{M}^{-1}(\mathbf{x}-\mathbf{m})/2} \quad (24)$$

where $\mathbf{M}$ denotes $L \times L$ covariance matrix with elements $\{\mu_{\alpha\alpha'}\}$ and $\mathbf{x}$ denote the column vector $[x_1, x_2, ..., x_L]'$ and $\mathbf{m}$ denote the column vector $[m_1, m_2, ..., m_L]'$. $\mathbf{M}^{-1}$ is the inverse of $\mathbf{M}$ and $\mathbf{x}'$ is the transpose of $\mathbf{x}$.

The probability of success recognition can be defined as

$$P_{S,H_\alpha} = P(x_\alpha > x_1, x_\alpha > x_2, ... \mid H_\alpha) \quad (25)$$

where $P(x_\alpha > x_1, x_\alpha > x_2, ... \mid H_\alpha)$ denotes the joint probability that $x_1, x_2, ...$ are all less than x_α, when H_α is true. Thus Eq.(25) can be represented as

$$P_{S,H_\alpha} = \int_{-\infty}^{\infty} \underbrace{\int_{-\infty}^{x_\alpha} ... \int_{-\infty}^{x_\alpha}}_{\alpha'=1,...,L,\alpha'\neq\alpha} p(x_1, ..., x_L \mid H_\alpha) dx_1 ... dx_{\alpha'} dx_\alpha \quad (26)$$

The average probability of successful recognition can be expressed as

$$P_S = \sum_{\alpha=1}^{L} P_{S,H_\alpha} P(H_\alpha) \quad (27)$$

To demonstrate the performance of the developed recognizer, for instance, we apply the structure in Fig. 6 to classify 16/64QAM signals. Suppose that the input signal is a 16QAM signal, the conditional successful recognition probability can be calculated as

$$\begin{aligned} P_{S,H_1} &= \int_0^\infty \int_{-\infty}^{x_1} p(x_1 > x_2 \mid H_1) dx_2 dx_1 \\ &= \int_0^\infty \int_{-\infty}^{x_1} \frac{1}{2\pi\sigma_1\sigma_2\sqrt{1-\rho_{16}^2}} \\ &\cdot e^{-\frac{\frac{(x_1-m_{1,16})^2}{\sigma_{1,16}^2} - \frac{2\rho_{16}(x_1-m_{1,16})(x_2-m_{2,16})}{\sigma_{1,16}\sigma_{2,16}} + \frac{(x_2-m_{2,16})^2}{\sigma_{2,16}^2}}{2(1-\rho_{16}^2)}} dx_2 dx_1 \\ &= \frac{1}{2} erfc\left(\frac{m_{2,16} - m_{1,16}}{\sqrt{2(\sigma_{1,16}^2 + \sigma_{2,16}^2 - 2\rho_{16}\sigma_{1,16}\sigma_{2,16})}}\right) \end{aligned} \quad (28)$$

where $m_{1,16}$, $m_{2,16}$, $\sigma_{1,16}^2$ and $\sigma_{2,16}^2$ are the means and the variances of x_1 and x_2 under input signal being 16QAM. ρ_{16} is the correlation coefficient between x_1 and x_2, and it can be defined as $(\mu_{12} - m_{1,16}m_{2,16})/\sigma_{1,16}\sigma_{2,16}$, and $erfc(\cdot)$ denotes complementary error function, defined as $erfc(x) = 2\int_x^\infty e^{-y^2/2} dy/\sqrt{\pi}$.

Similarly, when the input signal is a 64QAM signal, the conditional successful recognition probability is given by

$$\begin{aligned} P_{S,H_2} &= \int_0^\infty \int_{-\infty}^{x_2} p(x_2 > x_1 \mid H_2) dx_1 dx_2 \\ &= \frac{1}{2} erfc\left(\frac{m_{1,64} - m_{2,64}}{\sqrt{2(\sigma_{1,64}^2 + \sigma_{2,64}^2 - 2\rho_{64}\sigma_{1,64}\sigma_{2,64})}}\right) \end{aligned} \quad (29)$$

The average probability of successful recognition with prior equal probability is then written as

$$P_S = \frac{P_{S,H_1} + P_{S,H_2}}{2} \quad (30)$$

In addition to the theoretical analysis, the Monte Carlo computer simulation is another alternative and is often used for the verification purpose. In simulations, the number of samples is set to be 1024. Fig. 7 illustrates the theoretical results in Eq.(30) and the Monte Carlo simulations. Both the theoretical analysis and the Monte Carlo computer simulations are consistent and indicate that the recognition performance is heavily affected by the severity of channel fading.

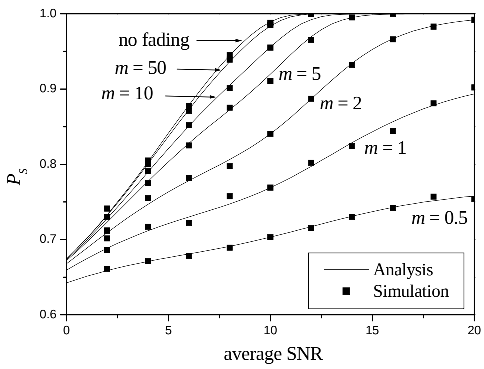

Fig.7. The probability of successful recognition versus average SNR for several values of m

V.CONCLUSIONS

In this paper, we derived the amplitude PDF of QAM signals over flat, slowly Nakagami fading channel, developed the required recognition statistics, and demonstrated a 16/64 QAM recognizer as an example. The results indicate that the performance is heavily influenced by the severity of channel fading.

When the channel is AWGN (no fading), which means that the fading figure m, approaches infinity in this case, henceforth, the performance is the best. However, the performance is degraded with the decrease of m, and the worst performance occurs when $m = 0.5$.

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