中正嶺學報 第三十三卷 第二期 民國 94.5.
JOURNAL OF C.C.I.T., VOL.33, NO.2, MAY, 2005

短文 Short Note

EigenStructure Assignment in Loop Transfer Recovery Process

Huan-Liang Tsai*, Jium-Ming Lin, Tsang Chiang*, and Chen-Hung Lin***

**Department of Electrical Engineering, Chung Cheng Institute of Technology, National Defense University*
*** Department of Mechanical Engineering and Institute of Aeronautics and Astronautics, Chung Hua University*

ABSTRACT

In this paper, the well-developed EigenStructure Assignment (ESA) method applied to a Loop Transfer Recovery (LTR) process is originally proposed. An optimal controller of so-called LQG/LTR suffers from the high-gain problem. In fact, the weighting matrices in the LQG problems can be explicitly determined by the ESA design technique. The proposed method has the ability to efficiently improve the high-gain problem in the conventional LQG approach. In addition, the compensated system has better frequency-domain performance. With ESA technique in the LTR process, the control system can simultaneously provide prescribed stability and take into consideration both time-domain and frequency-domain specifications. Finally, the numerical results will demonstrate the advantage of our proposed method.

Keywords: eigenstructure assignment, linear quadratic gaussian, loop transfer recovery

運用特徵結構指定在迴路轉移函數迴歸法之設計

蔡渙良*、林君明、蔣璋*、林建宏***

*國防大學中正理工學院電機工程學系
**中華大學機械系與航空太空研究所

摘　要

本文運用發展成熟的特徵結構指定設計法在迴路轉移函數法的設計程序從事最佳化控制系統設計。傳統的線性二次高斯法具有高增益的問題，運用特徵結構指定從事迴路轉移函數迴歸程序之最佳化控制器設計，這種設計方法可有效改善解決線性二次高斯法高增益的問題，同時在性能響應上可提供較佳的結果，本文的設計方法同時考量時域、頻域需求。最後，我們可用數值結果來展示我們所提出的方法的優異性。

關鍵字：特徵結構指定、線性二次高斯法、迴路轉移函數迴歸法

文稿收件日期 92.6.5; 文稿修正後以短文接受日期 93.9.9.
Manuscript received June 5, 2003; revised as short note Sept. 9, 2004.

I. INTRODUCTION

A multivariable feedback control system can satisfy the performance specifications such as command tracking, disturbance rejection, bandwidth, and robust stability characteristics. Such requirements can be naturally cast in the frequency-domain requirements in term of sensitivity function and complementary sensitivity function with relation to the return ration, which is evaluated by breaking either the input or output point of the compensated plant. The so-called Linear Quadratic Gaussian (LQG) with Loop Transfer Recovery (LTR) process, originally proposed by Doyle and Stein [1], provides prominent “loop shaping” concept for the corresponding principal gains of the return ratio. In the two-step approach to design the LQG/LTR methodology, the first step is to design an optimal state feedback controller subject to Linear Quadratic (LQ) performance index and also “loop shaping” the above target loop transfer function at the plant input point to meet the satisfactory specifications. In the second step of design, the required observer is designed to recovery the return ratio at the input point of the LQG-compensated plant. Most of papers [1-11] use above design procedure and make much contribution to the improvement in the recovery quality of target loop transfer function for general systems. The literatures [3-11] pay much attention to the recoverable quality of the return ratio in frequency-domain requirements. However, the resulting gain of so-called LQG controller suffers from the high-gain problem. In addition, the better quality of loop transfer recovery cannot make sure that the compensated plant has better performance including time-domain and frequency-domain responses. There is a trade-off between the recoverable quality and the resulting performance even for general strictly and minimum phase systems.

It is often to design the return ratio at the output point of the plant rather than the input point. The design procedure is dual to one described above. In this paper, we first design a Kalman filter to provide an optimal estimate of the state vector and shape the principal gains of the return ratio at the output of the plant by meeting the required specifications. And then apply the Eigenstructure Assignment (ESA) [12-17] technique in the LTR process to design a better state-feedback controller subject to the same LQ performance index. Such design procedure is reasonable according to the use the separation principle and the two-step approach of LQG/LTR design. In fact, the weighting matrices Q and R in the LQG problems are free design parameters and manipulated to obtain better performance. On the other hand, the Eigenstructure Assignment (ESA) technique is characterized by using not only the eigenvalue assignment to achieve the required closed-loop damping ratio and bandwidth but also the eigenvector assignment to provide the necessary dynamic decoupling. For a multivariable system, the control system can provide suitable damping and natural frequencies with prescribed eigenvalues and eigenvectors assignment. In addition, the weighting matrices Q and R in the LQG problems are explicitly determined by the ESA design procedure. This motivates us to apply the ESA method for the LQG controller

短文 Short Note

design in the LTR. Here we propose a state-feedback controller design in the LTR procedure with the ESA technique. The proposed method can simultaneously preserve the guaranteed robust stability of a LQG method and take into account the performance of time- and frequency-domain responses.

By the derivation of this paper in Section II, we show that the eigenstructure assignment technique can be applied in the LTR design procedure. We shall illustrate the practical control system design with the proposed method in Section III, and comparisons with a conventional LQG one are also made. Finally, brief conclusions are drawn in Section IV.

II. PROBLEM AND METHODOLOGY FORMULATION

2.1 Problem Formulation

Let the dynamic equations of a multivariable system shown in fig.1 be as follows

$$\dot{x}(t) = Ax(t) + Bu(t) + \Gamma w(t) \tag{1}$$

and

$$y(t) = Cx(t) + v(t) \tag{2}$$

where $x(t) \in \Re^n$ is the state vector, $u(t) \in \Re^m$ is the input vector, and $y(t) \in \Re^q$ is the measurement vectors. $A(t)$, $B(t)$, Γ, and $C(t)$ are respectively $n \times n$, $n \times m$, $n \times p$, and $q \times n$ matrices, $w(t)$ and $v(t)$ are p- and q-dimensional uncorrelated Gaussian white noise processes with zero-mean and covariances to be respectively as

$$E\{w(t)w^T(\tau)\} = W(t)\delta(t-\tau) \tag{3}$$

$$E\{v(t)v^T(\tau)\} = V(t)\delta(t-\tau) \tag{4}$$

and

$$E\{v(t)w^T(\tau)\} = 0 \tag{5}$$

where $E\{\cdot\}$ is an expectation function operator, $W(t)$ and $V(t)$ are respectively system disturbance and measurement noise covariance matrices.

The problem is to derive a feedback control law minimizing the following Linear Quadratic (LQ) performance index

$$J = E\{\frac{1}{2}\int_0^{t_f}[x^T(t)Qx(t) + u^T(t)Ru(t)]dt\} \tag{6}$$

where Q is a $n \times n$ positive semi-definite weighting matrix, R is a $m \times m$ positive definite control weighting matrix.

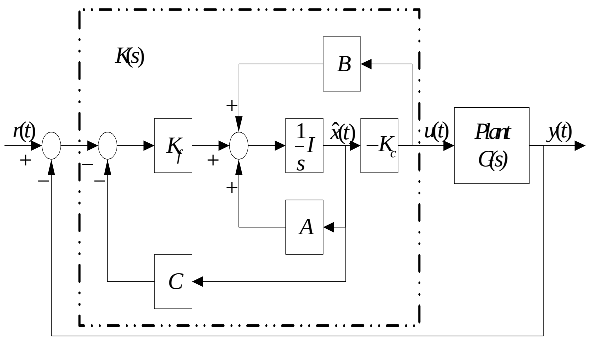

Fig. 1 LQG/LTR control system.

2.2 The Conventional LQG/LTR Methodology

Here the solution to the above LQG problem can be prescribed by the separation principle. The design results are achieved by adopting the following procedure.

Step1. Shaping the principal gains of target loop transfer function

To design a Kalman filter to obtain an optimal estimate $\hat{x}(t)$ of $x(t)$, which minimizes $E\{[x(t)-\hat{x}(t)]^T[x(t)-\hat{x}(t)]\}$ by Kalman filter theory. The optimal Kalman filter is used to shape the open-loop principal gains of the return ration $-C(sI-A)^{-1}K_f$ to meet the required specifications. This is also the so-called target loop transfer function. The gain matrix of the Kalman filter would be derived by the following state estimation equation

$$\dot{\hat{x}}(t) = A\hat{x}(t) + Bu(t) + K_f[y(t) - C\hat{x}(t)] \qquad (7)$$

where K_f is the Kalman filter gain matrix, defined as

$$K_f = P_f C^T V^{-1} \qquad (8)$$

and where P_f is the covariance of $x(t)-\hat{x}(t)$, defined as

$$P_f = E\{[x(t)-\hat{x}(t)]^T[x(t)-\hat{x}(t)]\} \qquad (9)$$

which can be obtained by the following Filter Algebraic Riccati Equation (FARE)

$$AP_f + P_f A^T + \Gamma W \Gamma^T - P_f C^T V^{-1} C P_f = 0 \qquad (10)$$

The gain matrix of a Kalman filter can be determined by manipulating the covariance matrices W and V. The target loop transfer function is then defined as

$$L_t(s) = -C(sI-A)^{-1}K_f \qquad (11)$$

and the sensitivity function and complementary sensitivity function are also defined as

$$S_f(s) = [I + C(sI-A)^{-1}K_f]^{-1} \qquad (12)$$

and

$$T_f(s) = I - S_f(s) \qquad (13)$$

The emphasis of a Kalman filter design is on approximately meeting the crossover frequencies for the principal gains of a return ration for the open-loop transfer function $-C(sI-A)^{-1}K_f$, balancing the principal gains as possible, and adjusting the low-frequency behavior.

Step2. Recovery the target loop transfer function at the plant output

Consider an observer-based controller shown in Fig.1. An optimal control law for the LQG problem is derived as follows. Let

$$u(t) = -K_c\hat{x}(t) \qquad (14)$$

where K_c is the optimal control gain matrix and is derived as

$$K_c = R^{-1}B^T P_c \qquad (15)$$

and where P_c is a positive definite symmetric matrix, which is defined by the following Controller Algebraic Riccati equation (CARE)

$$P_c A + A^T P_c - P_c B R^{-1} B^T P_c + Q = 0 \qquad (16)$$

The transfer function of an observer-based LQG controller is

$$K(s) = -K_c(sI - A + BK_c + K_f C)^{-1}K_f \qquad (17)$$

Therefore, the resulting return ration evaluated at the output of compensated plant is

$$G(s)K(s) = -C(sI - A)^{-1} BK_c (sI - A + BK_c + K_f C)^{-1} K_f \tag{18}$$

and the associated sensitivity function and complementary sensitivity function for the compensated plant are respectively as

$$S_{GK}(s) = [I + G(s)K(s)]^{-1} \tag{19}$$

and

$$T_{GK}(s) = S(s)G(s)K(s) \tag{20}$$

In the LQG/LTR methodology, the closed-loop eigenvalues of LOG-compensated plant are just the union of the eigenvalues of a Kalman filter and those of an optimal state-feedback. For the so-called LQG problem, the weighting matrices Q and R in the performance index are free design parameters and manipulated to recovery the principal gains of the return ration $G(s)K(s)$ at the output of compensated plant to the above target loop transfer function $L_t(s)$ as close as possible. In synthesis of the LQG state-feedback controller, we can solve the CARE with $Q = I$ and $R = \sigma I$ (or $Q = Q_0 + \rho I$ and $R = I$). As σ approaches to zero, it is obvious that $\lim_{\sigma \to 0} G(s)K(s,\sigma) = L_t(s)$. The higher gain matrix of state-feedback controller, the better quality of loop transfer recovery. However, the resulting controller suffers from the high-gain problem, and this leads to the reduction of controller band-width and the actuator saturation. Therefore, we proposed a better controller design in the LTR procedure with the ESA technique.

2.3 LQG controller design by Eigenstructure Assignment method

With the Kalman filter design, the estimate vector $\hat{x}(t)$ can be used as an exact measurement of the state vector to solve the LQ control problem. Given the available estimate state vector, the return ration at the input of the plant is then

$$H_o(s) = -K_c(sI - A)^{-1} B \tag{21}$$

and the associated return difference is

$$I - H_o(s) = I + K_c(sI - A)^{-1} B \tag{22}$$

With some manipulations of the CARE, the return difference identity can be expressed as

$$[I - H_o^T(s)]R[I - H_o(s)] = R + B^T(-sI - A^T)^{-1} Q(sI - A)^{-1} B \tag{23}$$

Let the state weighting matrix of LQG/LTR method be denoted as

$$Q = \rho D_\sigma D_\sigma^T \tag{24}$$

where ρ is a real number and D_σ is a $m \times n$ matrix. When the state weighting matrix approaches to infinity and $R = I$, there are $(n-m)$ eigenvalues of the closed-loop system s_i^ρ approaching the $(n-m)$ zeros of $\det[D_\sigma^T(-sI - A^T)^{-1} B]$ and the other m eigenvalues approaching infinity, and the associated closed-loop eigenvectors v_i^ρ should be the null space of $D_\sigma^T(-sI - A^T)^{-1} B$. Since Q approaches infinity as $\rho \to \infty$, one has

$$[I + K_c(sI - A)^{-1} B] \approx \rho^{1/2}[D_\sigma^T(sI - A)^{-1} B] \tag{25}$$

If v_i^ρ is the closed-loop eigenvector associated with the eigenvalue s_i^ρ, one has

$$[D_\sigma^T(s_i^\rho I - A)^{-1} B]v_i^\rho = 0 \tag{26}$$

Therefore, we can find a D_σ controlling over $(n-m)$ closed-loop eigenvalues and associated eigenvectors.

2.2.1 Finite Eigenvalues and Associated Eigenvectors

Given $(n-m)$ finite eigenvalues, there should be $(n-m)$ desired eigenvector meeting (26). However, the desired eigenvectors cannot be achieved perfectly and only a "best possible" choice for the eigenvectors can be determined. Given the desired eigenvectors x_i^ρ, the achievable eigenvectors v_i^ρ can be obtained by minimizing $\left\| x_i^\rho - A_i v_i^\rho \right\|^2$. Let the normalized finite eigenvector be

$$y_i^\rho = (s_i^\rho I - A)^{-1} B v_i^\rho \tag{27}$$

and we have

$$D_\sigma^T y_i^\rho = 0 \tag{28}$$

Taking (28) into consideration, one can form

$$Y^\rho = \begin{bmatrix} y_1^\rho & y_2^\rho & \cdots & y_{n-m}^\rho \end{bmatrix}_{n\times(n-m)} = T_{n\times n} \begin{bmatrix} I_{(n-m)\times(n-m)} & 0_{m\times(n-m)} \end{bmatrix}^T \tag{29}$$

where T is a nonsingular matrix. Since $D_\sigma^T y_i^\rho = 0$, we can arrange D_σ as

$$D_\sigma^T = \begin{bmatrix} 0_{m\times(n-m)} & *_{m\times m} \end{bmatrix} T_{n\times n}^{-1} \tag{30}$$

where * is an arbitrary $(m\times m)$ matrix.

2.2.2 Unbounded Eigenvalues and Associated Eigenvectors

As $\rho \to \infty$, there are m unbounded eigenvalues s_j^∞ to be as

$$s_j^\infty = \sqrt{\rho}\, s_j, \qquad j = 1,2,\cdots,m \tag{31}$$

where s_j is the jth normalized unbounded closed-loop eigenvalue, and the associated unbounded closed-loop eigenvector $y_j^\infty (= s_j^\infty y_j^\rho)$ is

$$y_j^\infty = B v_j^\infty, \qquad j = 1,2,\cdots,m \tag{32}$$

and where v_j^∞ is the jth infinite closed-loop associated eigenvector satisfying the following generalized eigenproblem

$$[(s_j)^2 R - B^T D_\sigma D_\sigma^T B] v_j^\infty = 0, \quad j = 1,2,\cdots,m \tag{33}$$

As D_σ^T has $(m\times m)$ degree-of-freedom to make $D_\sigma^T B$ being a diagonal matrix, R would also be a diagonal one for the desired infinite eigenstructure s_j^∞ and v_j^∞. In case all s_j are different, there is a unique positive definite R, iff

$$(v_j^\infty)^T B^T D_\sigma D_\sigma^T B v_j^\infty = 0, \quad i \neq j \tag{34}$$

and if all s_j are equal, one has

$$R = B^T D_\sigma D_\sigma^T B \tag{35}$$

As ρ approaches to infinity, the return difference identity in (23) can be expressed in the form of principal value operation

$$\sigma_i\{R^{1/2}[I + K_c^T (jwI - A)^{-1} B] R^{-1/2}\} \cong \sqrt{\rho}\,\sigma_i[D_\sigma^T (jwI - A)^{-1} B R^{-1/2}] \tag{36}$$

where $\sigma_i\{\cdot\}$ is an ith principal value operator. Given D_σ and R, ρ can be determined to meet the crossover frequency (or bandwidth) requirements and then the resulting state matrix Q can be determined by taking into consideration the frequency-domain specification. Having selected the weighting matrices Q and R, a gain matrix of optimal controller can also be determined with the CRAE in (16). Therefore, the closed-loop dynamic equation of compensated system can be arranged as

$$\begin{bmatrix} \dot{x}(t) \\ \dot{\hat{x}}(t) \end{bmatrix} = \begin{bmatrix} A & -BK_c \\ K_f C & A - BK_c - K_f C \end{bmatrix} \begin{bmatrix} x(t) \\ \hat{x}(t) \end{bmatrix} + \begin{bmatrix} 0 & \Gamma & 0 \\ -K_f & 0 & K_f \end{bmatrix} \tag{37}$$

and

$$y(t) = \begin{bmatrix} C & 0 \end{bmatrix} \begin{bmatrix} x(t) \\ \hat{x}(t) \end{bmatrix} + \begin{bmatrix} 0 & 0 & I \end{bmatrix} \begin{bmatrix} r(t) \\ w(t) \\ v(t) \end{bmatrix} \tag{38}$$

With the prescribed EigenStructure Assignment strategy, the state weighting and control weighting matrices in the performance index can be explicitly determined. This makes the proposed method more practical by taking into account both time-domain and frequency-domain requirements. In addition, an optimal controller with the ESA technique for an LQ problem can also provide some guaranteed robustness as the same as the LQG/LTR one, i.e., the gain margin and phase margin are at least -6dB and $\pm 60^\circ$, respectively. This can be easily verified with (23).

Ⅲ. Numerical Example and Simulation Results

Here a DGSR-CMG satellite attitude control system [15] is considered. The state, input, and output vectors of dynamic equations are respectively defined as follows

$$x = \begin{bmatrix} p & \varphi & r & \theta & \delta_i & \delta_o & \dot{\delta}_i & \dot{\delta}_o \end{bmatrix}^T \tag{39}$$

$$u = \begin{bmatrix} e_i & e_o \end{bmatrix}^T \tag{40}$$

and

$$y = \begin{bmatrix} \varphi & \theta \end{bmatrix}^T \tag{41}$$

where p and r are the angular rates of satellite in the X and Z axes vehicle coordinate frame, φ and θ are the Euler's angles, δ_i and $\dot{\delta}_i$ are the angle and angle rate of inner gimbal, δ_o and $\dot{\delta}_o$ are the angle and angle rate of outer gimbal, e_i and e_o are the electrical inputs of inner and outer gimbal motors. The matrices $A, B,$ and C are respectively as

$$A = \begin{bmatrix} 0 & 0 & -\frac{q_0(I_z - I_y) - H}{I_x} & 0 & -\frac{Hq_0}{I_x} & 0 & 0 & \frac{H}{I_x} \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -\frac{q_0(I_y - I_x) + H}{I_z} & 0 & 0 & 0 & 0 & -\frac{Hq_0}{I_z} & -\frac{H}{I_z} & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & -w_m & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -w_m \end{bmatrix}. \tag{42}$$

$$B = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & K_m & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & K_m \end{bmatrix}^T \tag{43}$$

and

$$C = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{44}$$

where H is the angular momentum of rotor, q_0 is the spinning rate of satellite, I_x, I_y, and I_z are the principal moments of inertia for the satellite in the vehicle coordinate frame, w_m and K_m are the pole and gain parameters of the gimbal motor. The parameters of the system are specified as

$q_0 = 0.33$ rad/sec , $I_x = I_z = 9.1\times10^4$ slug − ft^2 , $I_y = 16.7\times10^4$ slug − ft^2 , $H = 5\times10^4$ slug − ft^2 , $w_m = 0.5$, and $K_m = 0.0025$, and the nominal system model is

$$A=\begin{bmatrix} 0 & 0 & 0.8251 & 0 & -0.1813 & 0 & 0 & 0.5495 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -0.8251 & 0 & 0 & 0 & 0 & -0.5495 & -0.1813 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & -0.5 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.5 \end{bmatrix} \tag{45}$$

$$B=\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0.0025 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.0025 \end{bmatrix}^T \tag{46}$$

The principal gains of $C(sI - A)^{-1}B$ for the nominal system are shown in Fig.2. The proposed method is applied as follows:

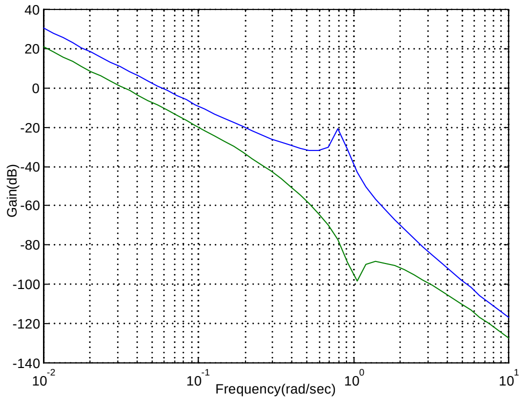

Fig. 2 Principal gains of $C(sI - A)^{-1}B$ for nominal system.

Step 1. Kalman filter Design

With the FARE in (10), the gain matrix K_f of a Kalman filter design by manipulating the covariance matrices W and V. In addition, this makes us "loop shaping" the principal gains of the target loop transfer function in (11). Since the disturbances would couple into the system through the inputs rather than directly on the states, Γ is chosen by setting $\Gamma = B$. Setting $W = I$ and $V = \rho_1 I$ (or setting $W = W_0 + \rho_2 I$ and $V = I$ alternative), we can manipulate ρ_1 to synthesize a Kalman filter by taking into account the frequency-domain requirements. Manipulating the largest principal gains $\bar{\sigma}\left[C(sI - A)^{-1}K_f\right]$ of target loop transfer function to meet the cross-over frequency w_c =0.1 rad/sec, one can let ρ_1 be a small value as $\rho_1 = 0.7$,and then the gain matrix of Kalman filter can be obtained as

$$K_f=\begin{bmatrix} 0.0043 & 0.0925 & -0.0029 & 0.0021 & -0.0006 & -0.0059 & -0.0000 & -0.0000 \\ 0.0032 & 0.0021 & 0.0015 & 0.0552 & 0.0059 & -0.0006 & 0.0000 & -0.0000 \end{bmatrix}^T \tag{47}$$

The principal gains of the return ration $L_t(s)$, sensitivity function $S_f(s)$ and complementary sensitivity function $T_f(s)$ are shown in Fig. 3-4.

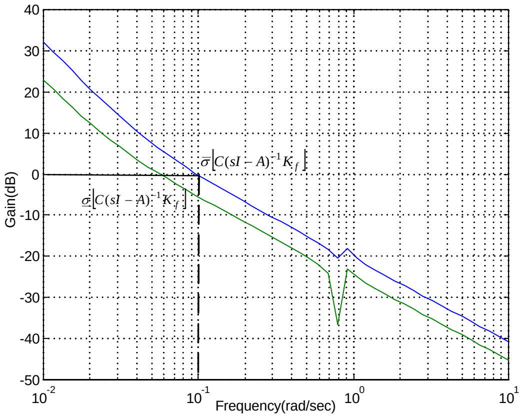

Fig. 3 Open-loop principal gains of return ratio $L_t(s) = -C(sI-A)^{-1}K_f$.

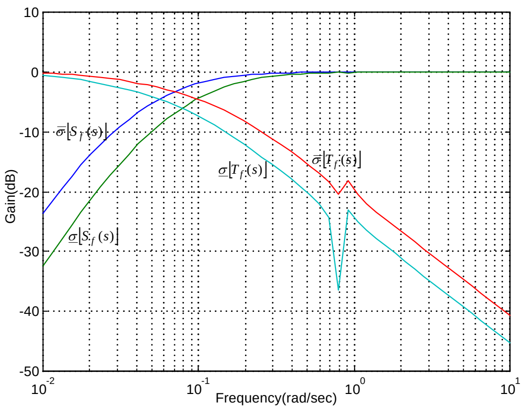

Fig. 4 Principal gains of $S_f(s)$ and $T_f(s)$ for $L_t(s) = -C(sI-A)^{-1}K_f$.

Step 2. LTR process with Eigenstructure Assignment:

Having a satisfactory return ration for the Kalman filter design, the following LTR process is aimed to recover the target loop transfer function at the output of the compensated plant. At first, the weighting matrices Q and R in the performance index can be determined by the prescribed eigenvalues and eigenvector assignment. The desired finite asymptotic eigenvalues[18] and the associated desired eigenvectors are assigned as follows:

$$s_1^\rho, s_2^\rho = -0.64 \pm 0.64j$$

$$s_3^\rho = s_4^\rho = -0.64$$

$$s_5^\rho = s_6^\rho = -0.5$$

$$x_{1,2}^\rho = [* \; 1 \; 0 \; 0 \; * \; *]^T \pm j[1 \; * \; 0 \; 0 \; * \; *]^T$$

$$x_3^\rho = [0 \; 0 \; 1 \; * \; * \; *]^T, \; x_4^\rho = [0 \; 0 \; * \; 1 \; * \; *]^T$$

$$x_5^\rho = [* \; * \; * \; * \; 1 \; 0]^T, \; x_6^\rho = [* \; * \; * \; * \; 0 \; 1]^T$$

where $*$ denotes "do not care". Thus, a set of the achievable closed-loop eigenvectors V_a, which minimizing $\|x_i^\rho - A_i v_i^\rho\|^2$, can be obtained as

$$V_a = \begin{bmatrix} -0.6404 & 0.9996 & 0 & 0 & 0.1778 & -0.3395 \\ 1.0004 & -0.7806 & 0 & 0 & -0.3555 & 0.6790 \\ -0.0695 & -0.0002 & 0.9999 & -0.6400 & 0.1120 & 0.5387 \\ -0.0596 & 0.0002 & -1.5624 & 1 & -0.224 & -1.0774 \\ -4.1138 & 7.0101 & 1.6252 & -1.0402 & 1 & 0 \\ 0.9902 & -0.1674 & 1.5080 & -0.9652 & 0 & 1 \end{bmatrix} \quad (48)$$

The D_σ^T, which satisfying prescribed eigenstructure s_i^ρ and v_i^ρ, is obtained as

$$D_\sigma^T = \begin{bmatrix} 4.7751 & 0.1446 & -5.9468 & -1.7356 & -0.0202 & 2.8566 & 1 & 0 \\ 3.3182 & 0.4689 & -0.1794 & -0.7472 & -0.5705 & 0.5998 & 0 & 1 \end{bmatrix} \quad (49)$$

The 2×2 degrees-of-freedom is embedded in (49) by forcing the last two columns as the identity matrix, which can make $D_\sigma^T B$ diagonal.

The unbounded eigenvalues and the associated eigenvectors are obtained as follows. Since the bandwidth of the inner and outer gimbals motors are 0.5 rad/sec, it should make sense to choose

$s_1 = s_2 = 0.01$. Since ρ is determined by the crossover frequency requirement, the associated modes approach $\sqrt{\rho}s_1$ and $\sqrt{\rho}s_2$, and the magnitude ratio of the closed-loop poles are remained. It also makes sense to choose the associated eigenvectors of the actuators respectively as

$$y_1^{\infty} = \begin{bmatrix} * & * & * & * & * & 0 & 1 & 0 \end{bmatrix}^T$$
$$y_2^{\infty} = \begin{bmatrix} * & * & * & * & 0 & * & 0 & 1 \end{bmatrix}^T$$

and the associated closed-loop eigenvectors are as

$$\begin{bmatrix} v_1^{\infty} & v_2^{\infty} \end{bmatrix} = (B^T B)^{-1} B^T \begin{bmatrix} y_1^{\infty} & y_2^{\infty} \end{bmatrix} = \begin{bmatrix} 400 & 0 \\ 0 & 400 \end{bmatrix} \quad (50)$$

Since all s_j are equal in this case, the control-weighting matrix can be obtained by using (35)

$$R = \begin{bmatrix} 0.0625 & 0 \\ 0 & 0.0625 \end{bmatrix} \quad (51)$$

Once D_{σ}^T and R are determined to satisfy the eigenstructure requirements, we can adjust ρ to meet the cross-over frequency w_c =0.1 rad/sec. The result is $\rho = 100$, and the state-weighting matrix Q is obtained as

$$Q = \begin{bmatrix} 3.3812 & 0.2246 & -2.8992 & -1.0767 & -0.1989 & 1.5631 & 0.4775 & 0.3318 \\ 0.2246 & 0.0241 & -0.0944 & -0.0601 & -0.0270 & 0.0694 & 0.0145 & 0.0469 \\ -2.8992 & -0.0944 & 3.5396 & 1.0455 & 0.0222 & -1.7095 & -0.5947 & -0.0179 \\ -1.0767 & -0.0601 & 1.0455 & 0.3571 & 0.0461 & -0.5406 & -0.1736 & -0.0747 \\ -0.1989 & -0.0270 & 0.0222 & 0.0461 & 0.0326 & -0.0400 & -0.0020 & -0.0570 \\ 1.5631 & 0.0694 & -1.7095 & -0.5406 & -0.0400 & 0.8520 & 0.2857 & 0.6000 \\ 0.4775 & 0.0145 & -0.5947 & -0.1736 & -0.0020 & 0.2857 & 0.1000 & 0 \\ 0.3318 & 0.0469 & -0.0179 & -0.0747 & -0.0570 & 0.6000 & 0 & 0.1000 \end{bmatrix} \times 10^3$$

(52)

Finally, the gain matrix of an optimal controller can be obtained by (15) and (16)

$$K_c = \begin{bmatrix} -34.0853 & -19.6083 & -114.6444 & 51.4441 & 119.4052 & 125.5002 & 191.8943 & 13.1413 \\ 335.0783 & -0.8444 & 26.0260 & -55.3758 & -68.6215 & 253.4987 & 13.1413 & 425.7103 \end{bmatrix}$$

(53)

For the purpose of comparing with the so-called LQG/LTR method, the one of LQG controller can be obtained by setting $Q = I$ and $R = 10^{-9} \times I$, and the result is

$$K_c = \begin{bmatrix} -5.7395 & -2.4000 & -4.0034 & 2.0591 & 3.7213 & 3.8161 & 3.1981 & 0.0080 \\ 2.1111 & -2.0591 & -8.5806 & -2.4000 & -0.9298 & 6.6690 & 0.0080 & 3.2399 \end{bmatrix} \times 10^4$$

(54)

It is obvious that the gain matrix of optimal controller for the proposed method can efficiently improve the high-gain problem in the LQG controller. In addition, the limitations in actuators are well solved.

The principal gains of the return ration, sensitivity function and complementary sensitivity function for the compensated system are shown in Figs. 5-6. The unit step responses of φ and the actuator inputs are also shown in Figs. 7-8. For comparison purpose, the results obtained with a conventional LQG/LTR method (by using (61)) are also shown in Figs. 5-8. The largest principal gains of return ratio $\bar{\sigma}[G(s)K(s)]$ and its condition numbers $(\bar{\sigma}/\underline{\sigma})$ of the proposed method at low frequencies are decreased (about –2dB), and the disturbance rejection capability is increased. At high frequencies, the principal gains of return ratio $G(s)K(s)$ for the proposed method are approximately decreased $-30 \sim -40\text{dB}$. This makes the complementary sensitivity function $T_{GK}(s)$ have better performance in the rejection of measurement noises. Of course, the penalty is the quality of loop transfer recovery. We consider that better performance is more important than the recoverable quality. In addition, the largest principal gains $\overline{\sigma}(S_{GK})$ of the proposed method at low frequencies is a very little increased (about –0.1dB), but its condition numbers $(\overline{\sigma}/\underline{\sigma})$ are also decreased.

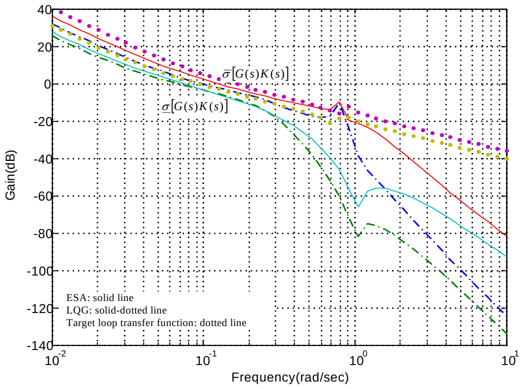

Fig. 5 Principal gains of return ratio $G(s)K(s)$ for compensated systems.

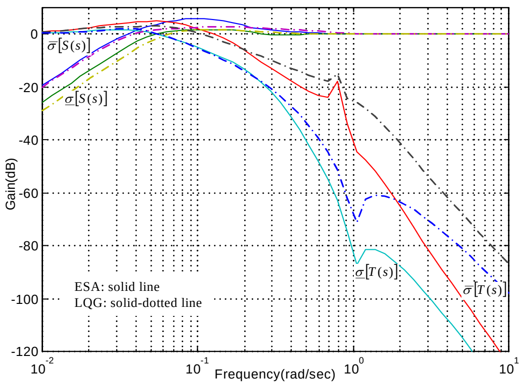

Fig. 6 Principal gains of $S(s)$ and $T(s)$ for compensated systems.

In time-domain responses, the overshoot of unit step responses for our proposed method is increased about 0.1deg. However, the cross-coupling effect, and the amplitudes of the actuators inputs are all reduced by the proposed method. The root-mean-square of the angular rate of actuator for the inner and outer gimbal is well reduced about 0.0357deg/sec. This makes the power consumption of satellite well reduced.

Ⅳ. Conclusions

From the previous derivation, the proposed method can take into consideration all the time- and frequency- domain specifications, and robust decoupling design techniques. The high-gain problem can be solved by our proposed approach. This makes the optimal controller more practical. By the results of numerical simulation, the proposed method has better time-domain and frequency-domain responses, comparing with ones of the conventional LQG/LTR method.

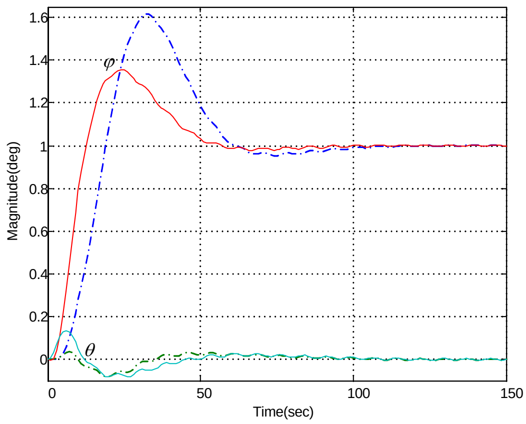

Fig. 7 Unit-step responses of φ for compensated system.

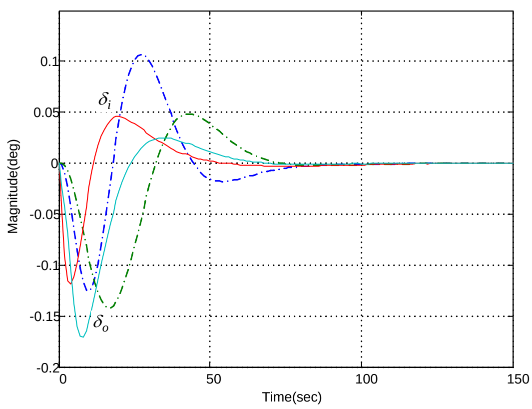

Fig. 8 Actuator inputs for unit-step responses of φ for compensated system.

REFERENCE

[1] Doyle, J. C. and Stein, G., “Multivariable Feedback Design: Concepts for a Classical / Modern Synthesis,” IEEE Transaction on Automatic Control, AC-26, pp. 4-16, 1981.

[2] Stein, G. and Athan, M., “The LQG/LTR Procedure for Multivariable Feedback Control Design,” IEEE Transaction on Automatic Control, AC-32, pp. 105-114, 1987.

[3] Sogaard-Andersen, P., “Loop Transfer Recovery–An Eigen-Structure Interpretation, ”Control-Theory and Advanced Technology, Vol. 5, No. 3, pp. 351-365, 1989.

[4] Zhang, Z. and Freudenberg, J. S., “Loop Transfer Recovery for Non-Minimum Phase Plants,” IEEE Transaction on Automatic Control, Vol. 35, No. 5, pp. 547-553, 1990.

[5] Saberi, A., Chen, B. M., and Sannuti P., “Theory of LTR for Non-Minimum Phase Systems, Recoverable Target Loops Recovery in a Subspace – Part 1: Analysis,” International Journal of Control, Vol. 53, No. 5, pp. 1067-1115, 1991.

[6] Saberi, A., Chen, B. M., and Sannuti P., “Theory of LTR for Non-Minimum Phase Systems, Recoverable Target Loops Recovery in a Subspace – Part 2: Design,” International Journal of Control, Vol. 53, No. 5, pp. 1117-1160, 1991.

[7] Niemann, H. H., Sogaard-Andersen, P., and Stopstrup, J., “Loop Transfer Recovery: Analysis and Design for General observer architecture,” International Journal of Control, Vol. 53, No. 5, pp. 1177-1203, 1991.

[8] Chen, B. M., Saberi, A., and Sannuti P., “A New Stable Compensator Design for Exact and Approximate Loops Transfer Recovery,” Automatica, Vol. 27, No. 2, pp. 257-280, 1991.

[9] Chen, B. M., Saberi, A., and Sannuti P., “Necessary and Sufficient Conditions for a Non-Minimum Plant to Have a Recoverable Target Loop – A Stable Compensator design for LTR,” Automatica, Vol. 28, No. 3, pp. 493-507, 1991.

[10] Chen, B. M., Saberi, A., and Sannuti P., “Loop Transfer Recovery for General Non-Minimum Phase Non-Strictly Proper Systems – Part 1: Analysis,” Control-Theory and Advanced Technology, Vol. 8, No. 1, pp. 59-100, 1992.

[11] Chen, B. M., Saberi, A., and Sannuti P., “Loop Transfer Recovery for General Non-Minimum Phase Non-Strictly Proper Systems – Part 2: Design,” Control-Theory and Advanced Technology, Vol. 8, No. 1, pp. 101-144, 1992.

[12] Klein, G. and Moore, B. C., “Eigenvalue - Generalized Eigenvector Assignment with State Feedback,” IEEE Transactions on Automatic Control, AC-22, pp. 140-141, 1977.

[13] Harvey, C. A. and Stein, G., “Quadratic Weights for Asymptotic Regulator Properties,” IEEE Transactions on Automatic Control, AC-23, pp. 378-387, 1978.

[14] Stein, G., “Generalized Quadratic Weights for Asymptotic Regulator Properties,” IEEE

Transactions on Automatic Control, AC-24, pp. 559-566, 1979.

[15] Fahmy, M. M. and O'Reilly, J., “On Eigenstructure Assignment in Linear Multivariable System,” IEEE Transactions on Automatic Control, AC-27, pp.690-693, 1982.

[16] Andry, A. N., Shapiro, E. Y., and Chung, J. C., “Eigenstructure Assignment for Linear system,” IEEE Transactions on Aerospace and Electronic Systems, AES-19, pp. 711-729, 1983.

[17] Sobel, K. M. and Shapiro, E. Y., “Eigenstructure Assignment: A Tutorial - Part I Theory and Part II Applications,” Proceedings American Control Conference, pp.456-467, 1985.

[18] Iyer, A. and Singh, S. N., “MFDs of Spinning Satellite and Attitude Control Using Gyrotorquers,” IEEE Transactions Aerospace and Electronic Systems, AES-25, pp. 611-620, 1989.

[19] Maciejowski, J. M., Multivariable Feedback Design, Addition-Wesley Publishing Co., Chap.5, pp. 222-264, 1989.

[20] Anderson, B. D. O. and Moore, J. B., Optimal Control :Linear Quadratic Methods, Prentice-Hall International, Inc., pp. 139-163, 1990.