An Exact Solution of Pursuit Course with Lateral Acceleration Constraint for Non-maneuvering Target

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ABSTRACT

The method and discussion for derived an exact trajectory with lateral acceleration constraint to the pure pursuit course are presented. The solutions obtained before under pursuit scheme seems incomplete. Now in this paper, the exact and complete solutions are derived for non-maneuvering target, which are more general and comprehensive than those obtained before. The exact solution is based on scaling theory with the Buckingham Pi theorem to determine the relationship among the non-dimensional parameters. Some related important characteristics, such as lateral acceleration demand and energy cost, are investigated and discussed. The exact solution for graph the trajectory of pursuit guidance are validated and compared with the classical method derived by Howe etc.

Keywords: Pursuit guidance, Analytical solution, Lateral acceleration constraint

追逐彈道於側向加速度受限目標無閃躲時之確解

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摘　要

本文針對目標無閃躲且飛彈側向加速度受限制之條件下，推導並討論飛彈採純追逐式導引軌跡之正確解。在相同狀況下，文獻中之解法仍有不完整之處，本文將之補強，並求出正確解。正確解之求解係基於運用 Buckingham Pi 理論，從而獲得無因次參數，方得求出完整解。另相關重要性質如側向加速度需求、能量指標皆提出討論。最後將本文解法與傳統解法以模擬例證比較。

關鍵字：追逐導引，解析解，側向加速度限制

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I. INTRODUCTION

The pursuit guidance has been widely used as the guidance scheme in homing phase of flight for most missile systems. One of the most straightforward means to assure an intercept is to keep the missile, which must have velocity superiority, pointed at the target. This is the principle of pursuit course [1] which is an alternative terminal guidance law. In 1955, Locke presented a closed form solution of this, for pursuer range versus pursuer angular position. In addition, advanced closed-form solution of pure pursuit trajectory for non-maneuvering targets are derived by Howe in 1965. The derivations of the pure pursuit course are provided in the textbooks by Locke [2] and Howe [3]. Now, we try to derive an exact and complete solution of pursuit guidance with lateral acceleration constrain for non-maneuvering target, which is more general and comprehensive than those obtained before.

In pursuit guidance, some solution obtained before seems incomplete. The design of a guidance law for a given missile system is effected primarily from the prior knowledge of the missile lateral acceleration capability. The problem of limited lateral acceleration is then dealt with by a proper tuning of the guidance law parameters. In this paper, an exact solution of the differential equations describing the pursuit trajectory of the pursuer for non-maneuvering target is discussed. This paper presented a simple improvement to the well-known pursuit guidance law that assured that missile saturation of lateral acceleration was prevented.

II. HOWE'S CLOSED FORM SOLUTION

The purpose of homing guidance is to cause missile to hit or closed to a pre-selected target. Here we designate target and missile positions with respect to a two-dimensional inertial frame with coordinates $\boldsymbol{x}$, $\boldsymbol{y}$. The target has velocity of magnitude V_T and direction θ_T, as measured with respect to the $\boldsymbol{x}$ axis reference direction. Similarly, the missile has velocity of magnitude V_M and direction θ. From now on we shall refer to θ as the missile heading angle, although this is true in fact only if the missile has zero angle of attack, which for simplicity we shall assume. The LOS angle in Fig. 1 is β, and the distance separating target and missile is range R. Target and missile have velocity components $V_{\beta T}$ and $V_{\beta M}$, respectively, along the LOS and $V_{\alpha T}$ and $V_{\alpha M}$,respectively, perpendicular to the LOS. From Fig. 1, we have the fundamental equation of guidance:

$$\dot{R} = V_T \cos(\beta - \theta_T) - V_M \cos(\beta - \theta) \qquad (1)$$

$$\dot{\beta} = -\frac{V_T \sin(\beta - \theta_T) - V_M \sin(\beta - \theta)}{R} \qquad (2)$$

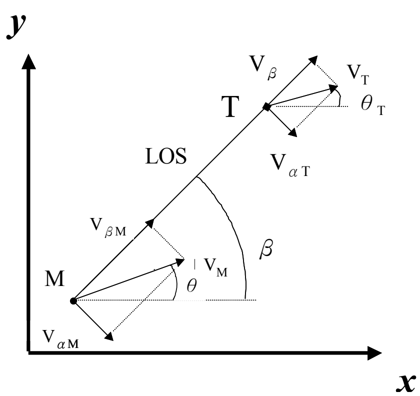

Fig. 1. Geometry of 2-D homing problem.

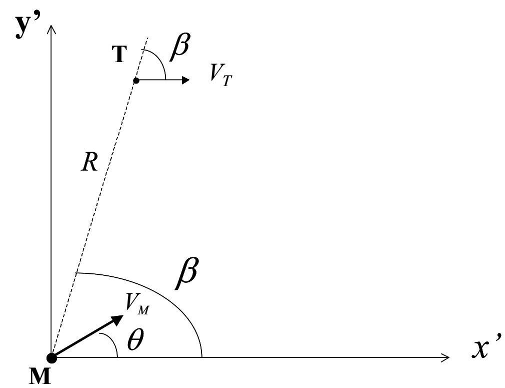

Fig. 2. Typical geometry of 2-D pursuit problem.

θ_T is a forced function. θ is dependent on guidance law. For simplicity assume a non-maneuvering target with V_T =constant. Furthermore, we let $\theta_T = 0$ since we can always reorient the x, y reference system of Fig. 1 so that this is true [4]. For ideal pursuit guidance $\theta = \beta$, and for $\theta_T = 0$, (1) and (2) become

$$\dot{R} = V_T \cos\beta - V_M \tag{3}$$

$$\dot{\beta} = -\frac{V_T \sin\beta}{R} \tag{4}$$

The geometry required for the derivation of the pursuit course equations of motion is given in Fig. 2. Note that $\dot{\beta}$ is not zero unless $\beta = 0$ or π, i.e., unless the attack is head on or tail chase. Since $\dot{\theta} = \dot{\beta}$ in pursuit guidance, the missile will always have to turn during the attack unless it is head on or tail chase. By eliminating time from (3) and (4) we can solve for β and hence the missile heading θ as a function of the range R.

Dividing Eq. (3) by (4), we obtain

$$\frac{dR}{R} = (-\cot\beta + \gamma\csc\beta)d\beta \tag{5}$$

Where $\gamma = V_M / V_T$, the ratio of missile to target velocity. After integration, (5) becomes

$$\ln R = -\ln|\sin\beta| + \gamma \ln\left|\tan\frac{\beta}{2}\right| + const$$

Assuming that $0 \le \beta \le \pi$, we can drop the absolute-value signs and write

$$\frac{R\sin\beta}{(\tan\frac{\beta}{2})^\gamma} = \frac{R_0 \sin\beta_0}{(\tan\frac{\beta_0}{2})^\gamma} = K = const \tag{6}$$

Where R_0 and β_0 are initial values of range and line-of-sight angle, respectively. As the missile approaches the kill, R approaches zero, and from (6) it is evident that β must also approach zero for the left-hand side of (6) to remain constant. Thus we are led to the important conclusion that in ideal pursuit guidance the trajectory always terminates in a tail chase with $\theta = \beta = 0$.

Substituting R from (6) into (4), we

obtain

$$\dot{\beta} = -\frac{V_T}{K}\frac{\sin^2\beta}{(\tan\frac{\beta}{2})^{\gamma}} \quad (7)$$

Near the trajectory termination, when $\beta << 1$ and $\sin\beta \cong \beta$, $\tan\frac{\beta}{2} \cong \frac{\beta}{2}$, Eq. (7) becomes

$$\dot{\beta} \cong -\frac{V_T 2^{\gamma}}{K}\beta^{2-\gamma} \quad (8)$$

III. THE PROPOSED EXACT SOLUTION

Physical model are often scale models. This means that the response of the model is in some way proportionate to the response of the prototype. If these models are to be useful, we must be able to determine the relationship between the augment and the prototype. This is generally referred to as a scaling factor. Scaling factors may be determined in a systematic manner using the Buckingham Pi theorem [5]. The traditional Buckingham Pi theorem is commonly used to derive a relationship between opening effectiveness and variables in terms of dimensionless parameters. Most importantly, it provides a method for computing sets of dimensionless parameters from the given variable, even if the form of the equation is still unknown. However, the choice of dimensionless parameters is not unique.

We want to account for miss distance due to limited missile maneuverability. Let missile centripetal acceleration be $a_c = \dot{\theta} V_M = V_M{}^2/\rho$ and $|\dot{\theta}| \le |\dot{\theta}|_{\max}$, we have $|\dot{\theta}| = |a_c|_{\max}/V_M$, $|a_c|_{\max}$: limited missile acceleration capability under these condition, the missile can no longer follow the guidance (i.e., $\theta = \beta$, $\dot{\theta} = \dot{\beta}$) but will have a flight path heading angle θ which lags behind β. Time for missile reaches the maximum turning rate, $t = t_{sat_0}$, it give $\dot{\theta} = -|\dot{\theta}|_{\max}$, $R = R_{sat_0}$ and $\beta = \beta_{sat_0} = \theta_{\max_0}$. For $t < t_{sat_0}$, $|\dot{\beta}| < |\dot{\theta}|_{\max}$ and $\theta = \beta$, missile able to pursuit perfectly. At $t = t_{sat_0}$, $|\dot{\beta}| = |\dot{\theta}|_{\max}$ and for $t > t_{sat_0}$ missile follows trajectory $\dot{\theta} = -|\dot{\theta}|_{\max}$ (right turn). When $t > t_{sat_0}$ let MD = transverse displacement of missile derivate from the ideal trajectory.

This paper is to construct linear time-varying dimensionless equation relating target moving to missile moving. We consider a constant bearing reference trajectory, then linearize about it. For linearized equation, define the dimensionless variable [6]:

$$r = R/R_{sat_0},\quad \tau = V_T t/R_{sat_0},\quad x_T = X_T/R_{sat_0},$$

$$y_T = Y_T/R_{sat_0},\; x_M = X_M/R_{sat_0},\quad y_M = Y_M/R_{sat_0},$$

$x = x_T - x_M$ and $y = y_T - y_M$. Where R is the range between missile and target, X_T, Y_T, X_M, Y_M is the position coordinate of target and missile respectively. τ is dimensionless time $\tau = V_T t/R_{sat_0}$, x, y, r are dimensionless variable of X, Y, R divided by R_0 respectively. This is shown simplistically in Fig. 3.

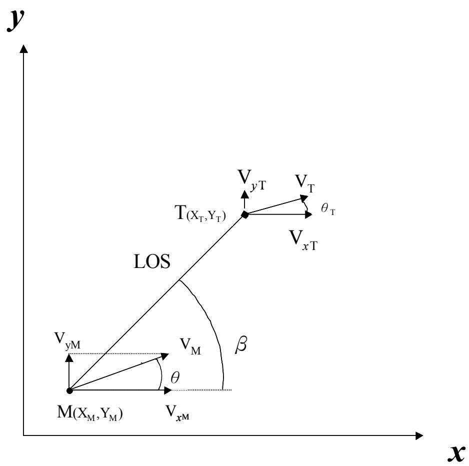

Fig. 3. Geometry of pursuit in Cartesian coordinate.

Next consider the homing problem of pursuit course, as shown in Fig. 4. We have

$$\dot{X}_T = V_{xT} = V_T \cos\theta_T,\quad \dot{Y}_T = V_{yT} = V_T \sin\theta_T,$$

$$\dot{X}_M = V_{xM} = V_T \cos\theta,\quad \dot{Y}_M = V_{yM} = V_M \sin\theta,$$

$$X = X_T - X_M,\quad Y = Y_T - Y_M,$$

$R = \sqrt{X^2 + Y^2}$ and $\beta = \tan^{-1}(X/Y)$. From the definition of dimensionless, we obtain

$$\frac{d(\bullet)}{dt} = \frac{d(\bullet)}{d\tau}\frac{d\tau}{dt} = \frac{V_T}{R_{sat_0}}\frac{d(\bullet)}{d\tau} \tag{9}$$

or $$\frac{d(\bullet)}{d\tau} = \frac{R_{sat_0}}{V_T}\frac{d(\bullet)}{dt} \tag{10}$$

The dimensionless relationships can be obtained as

$$\frac{dx}{d\tau} = \cos\theta_T - \gamma\cos\theta \tag{11}$$

$$\frac{dy}{d\tau} = \sin\theta_T - \gamma\sin\theta \tag{12}$$

$$r = \sqrt{x^2 + y^2} \tag{13}$$

$$\beta = \tan^{-1}(y/x) \tag{14}$$

From the Eqs. (1) and (2).

$$\frac{dr}{d\tau} = \cos(\beta - \theta_T) - \gamma\cos(\beta - \theta) \tag{15}$$

$$\frac{d\beta}{d\tau} = -\frac{\sin(\beta - \theta_T) - \gamma\sin(\beta - \theta)}{r} \tag{16}$$

where we have assumed $\tau(t_{sat}) = \tau_{sat_0}$, $r(t_{sat}) = r_{sat_0}$, $\beta(t_{sat}) = \beta_{sat_0}$ and $\theta_T = 0$, yield

$$\frac{d\theta}{d\tau} = \left.\frac{d\beta}{d\tau}\right|_{\tau=\tau_{sat_0}} = -\Omega_{\max} = const \tag{17}$$

where $\Omega_{\max} = R_{sat_0}\dot{\theta}_{\max}/V_T$ is dimensionless maximum turning rate. Integrating Eq. (17), we obtain

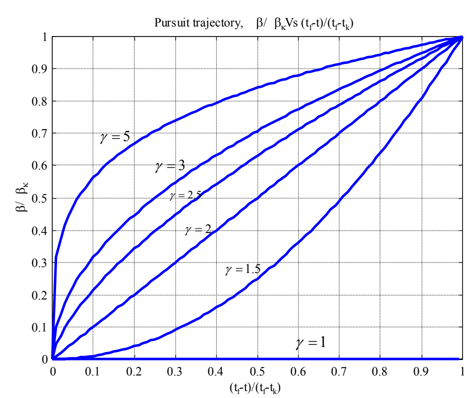

Fig. 4. Missile normalized turning angle for pursuit guidance.

$$\theta = \beta_{sat_0} - \Omega_{\max}(\tau - \tau_{sat_0}) \tag{18}$$

$$\theta(\tau_{sat_0}) = \beta_{sat_0}$$

Substitute Eq. (18) into Eqs. (11) and (12), we have

$$\frac{dx}{d\tau} = 1 - \gamma \cos[\beta_{sat_0} - \Omega(\tau - \tau_{sat_0})] \tag{19}$$

$$\frac{dy}{d\tau} = -\gamma \sin[\beta_{sat_0} - \Omega(\tau - \tau_{sat_0})] \tag{20}$$

Integrating Eqs. (19) and (20) with respect to $(\tau - \tau_{sat_0})$, we obtain

$$x = \cos\beta_{sat_0} + (\tau - \tau_{sat_0}) + \frac{\gamma}{\Omega_{\max}}\{\sin[\beta_{sat_0} - \Omega_{\max}(\tau - \tau_{sat_0})] - \sin\beta_{sat_0}\} \tag{21}$$

$$y = \sin\beta_{sat_0} - \frac{\gamma}{\Omega_{\max}} * \{\cos[\beta_{sat_0} - \Omega_{\max}(\tau - \tau_{sat_0})] - \cos\beta_{sat_0}\} \tag{22}$$

where $x(\tau_{sat_0}) = \cos\beta_{sat_0}$ and $y(\tau_{sat_0}) = \sin\beta_{sat_0}$.

According to the definition of dimensionless equation, the miss distance is

$$MD = R_{sat_0} r_{\min} \tag{23}$$

IV. QUANTITATIVE STUDY

Reduction of the complexities is by simplifying assumption. Here one simply hopes that the behavior of the reduced solution will be sufficiently similar to the full solution for useful conclusions to be drawn from the simplified solution. A terminal phase analysis of pursuit guidance has been performed by Kuo [7,8] and the approximate solution are derived.

We have seen $\beta \to 0$ as missile approaches target, suppose $\beta << 1 \Rightarrow \cos\beta \approx 1$ and $\sin\beta \approx \beta$. Eqs. (3) and (4) become

$$\dot{R} = V_T - V_M \tag{24}$$

$$\dot{\beta} = -\frac{V_T \beta}{R} \tag{25}$$

If we assume $R = (V_T - V_M)(t - t_f)$ and $R(t_f) = 0$, Eq. (25) then becomes

$$\dot{\beta} = -\frac{V_T \beta}{(V_T - V_M)(t - t_f)} \tag{26}$$

Solving for β, we have

$$\frac{d\beta}{\beta} = \frac{dt}{(\gamma - 1)(t - t_f)} \tag{27}$$

$$\ln\beta = \frac{1}{\gamma - 1}\ln(t - t_f) + \text{constant} \tag{28}$$

$$\ln\frac{\beta}{(t - t_f)^{\frac{1}{\gamma-1}}} = \ln\frac{\beta_k}{(t_k - t_f)^{\frac{1}{\gamma-1}}} = \text{constant} \tag{29}$$

Where $\beta_k = \beta(t_k)$, t_k=some time when $\beta << 1$.

$$\beta = \beta_k \left(\frac{t_f - t}{t_f - t_k}\right)^{\frac{1}{\gamma-1}} \tag{30}$$

From Eq. (30), we can plot β / β_k against non-dimensional time-to-go impact

$(t_f - t)/(t_f - t_k)$ for different γ. For γ=1, $\dot{\beta} \to 0$ as $t \to t_f$. For γ >2, $\dot{\beta} \to \infty$ as $t \to t_f$. For γ=2, $\dot{\beta} \propto t$, β is linear.

This turns out to be excellent approximation for $\beta << 1$. Evaluations of terminal trajectory for the outgoing target are straightforward. For $\dot{y}_M = V_M(\theta - \beta)$, where $|\theta - \beta| << 1$ and $\dot{\theta} = -|\dot{\theta}|_{\max}$ = constant are assumed, upon integrating and let $MD = y_M(t_f)$ = Miss distance, yield

$$\theta(t) = \beta_k - |\dot{\theta}|_{\max}(t - t_k) \tag{31}$$

ie, when $t = t_k$ and $\theta = \theta_k = \beta_k$ =LOS angle at which turning rate limitation is encountered. Till now, from Eqs. (30) and (31), we have seen

$$\dot{y}_M = V_M\left[\beta_k - |\dot{\theta}|_{\max}(t - t_k) - \beta_k\left(\frac{t_f - t}{t_f - t_k}\right)^{\frac{1}{\gamma-1}}\right] \tag{32}$$

Integrating Eq. (32) with $y_M(t_k) = 0$, gives

$$y_M = V_M[\beta_k(t - t_k) - |\dot{\theta}|_{\max}\frac{(t - t_k)^2}{2} \tag{33}$$

$$+\frac{\gamma - 1}{\gamma}\beta_k(t_f - t_k)\left(1 - \frac{t - t_k}{t_f - t_k}\right)^{\frac{\gamma-1}{\gamma}} - \frac{\gamma - 1}{\gamma}\beta_k(t_f - t_k)]$$

Assume ideal and actual trajectories are close enough so that the miss distance is

$$MD \cong y_M(t_f) = V_M(t_f - t_k)\left(\frac{\beta_k}{\gamma} - \frac{t_f - t_k}{2}|\dot{\theta}|_{\max}\right) \tag{34}$$

Here we would like to eliminate $(t_f - t_k)$ and $|\dot{\theta}|_{\max}$. Recall that we known, range at which missile encounters turning rate limitation, so

$$t_f - t_k = \frac{R_k}{V_M - V_T} = \frac{\gamma R_k}{(\gamma - 1)V_M} \tag{35}$$

We had also seen

$$\dot{\beta} = \frac{\beta_k}{(1-\gamma)(t_f - t_k)}\left(\frac{t_f - t}{t_f - t_k}\right)^{\frac{2-\gamma}{\gamma-1}} \tag{36}$$

Hence, from Eq. (34), the approximate miss distance for pursuit guidance is

$$MD = V_M\frac{\gamma R_k}{(\gamma - 1)V_M}\left[\frac{\beta_k}{\gamma} + \frac{\beta_k}{2(1-\gamma)}\right] = \frac{R_k\beta_k(\gamma - 2)}{2(\gamma - 1)^2} \tag{37}$$

Equation (37) is valid when $\beta_k << 1$ and $\gamma > 2$, i.e. the missile turning rate $\dot{\theta}$ continuously remain at the limiting value after t_k. For $\gamma < 2$, the missile could not get back on to the ideal pursuit trajectory and Eq. (37) would not give a valid result.

If the only disturbance is a target small perturbation the appropriate second-order differential equation becomes

$$\ddot{y}_M = -V_M\dot{\theta} + a_t \tag{38}$$

The target can maneuver slowly with acceleration magnitude a_t. Zarchan [9] have already shown that closed-form solution for the required missile acceleration exists for the zero-lag guidance system. The solution, which is repeated here for convenience, is given by

$$a_c = -\left[1 - \left(1 - \frac{t}{t_f}\right)^{-1}\right]a_t \tag{39}$$

The solution, from Eq. (39), for the

missile acceleration response due to target maneuver is displayed in normalized form in Fig. 5. From a system sizing point of view, the designer must ensure that the acceleration capability of the missile is adequate at the end of flight so that saturation can be avoided so that the missile can hit the target.

The cumulative velocity increment is

$$\Delta V = \int_0^{t_f} |a_N| dt \qquad (40)$$

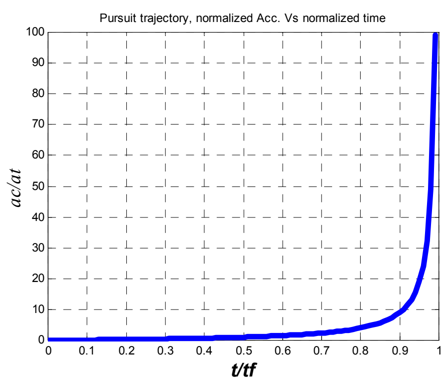

Fig. 5. Normalized missile acceleration due to target maneuver for pursuit guidance.

Which is related to the corresponding propellant mass required for effective intercept in exoatmospheric flight.

V. ILLUSTRATIVE EXAMPLE

In this section, we consider realistic examples that illustrate the performance of the proposed skill and compare it to Howe's closed form solution and the numerical solution. A two-dimensional missile-target engagement for numerical was set up using the second-order Runge-Kutta numerical integration technique [9,10]. The same data are used in each of the various solution, the missile can maneuver in the range of $|a_c| \leq$ 40g where g is the acceleration caused by gravity. The tests use two scenarios. Two types of target engagement: tail chase pursuit ($\beta_0 = \pi/2$) and tail chase pursuit ($\beta_0 = \pi/4$) are considered. In the examples, we are also concerned with missile acceleration and miss distance as well as with the effectiveness of the proposed schemes of exact solution.

5.1 Tail chase pursuit ($\beta_0 = \pi/2$)

Assume that the target moves with fixed velocity, $V_T = 300\, m/\sec$, and fixed heading, $\theta_T = 0$. Assume that the missile with fixed velocity, $V_M = 900\, m/\sec$, is launched at an initial range, $R_0 = 2000m$, and initial line-of-sight angle, $\beta_0 = \pi/2$. Assume that the missile has a maximum lateral acceleration capability of 40g. Notice that when Howe's skill is employed, substitute $\gamma = V_M / V_T = 3$ and $\sin\beta = 2\sin(\beta/2)\cos(\beta/2)$ into the Eq. (7), we obtain

$$\dot{\beta} = -\frac{4V_T \sin^2\frac{\beta}{2}\cos^2\frac{\beta}{2}}{K\left(\sin\frac{\beta}{2}\Big/\cos\frac{\beta}{2}\right)^3} = -\frac{4V_T\cos^5\frac{\beta}{2}}{K\sin\frac{\beta}{2}} \qquad (41)$$

Then as $\beta_0 = \pi/2,\ \sin\beta_0 = 1$ and $\tan(\beta_0/2) = 1$, the Eq. (6) becomes

$$R_0 = K \tag{42}$$

Substitute the Eq. (41) into the Eq. (42), the LOS rate, $\dot{\beta}$, is

$$\dot{\beta} = -\frac{4V_T\cos^5\frac{\beta}{2}}{R_0\sin\frac{\beta}{2}} = -\frac{4V_T\cos^6\frac{\beta}{2}}{R_0\sin\frac{\beta}{2}\cos\frac{\beta}{2}} \tag{43}$$

By noting that

$$\cos^2\frac{\beta}{2} = \frac{1+\cos\beta}{2} \quad \text{and} \quad \sin\frac{\beta}{2}\cos\frac{\beta}{2} = \frac{\sin\beta}{2}$$

Eq. (43) can be written as:

$$\dot{\beta} = -\frac{V_T(1+\cos\beta)^3}{R_0\sin\beta} \quad \text{or}$$

$$\frac{\sin\beta}{(1+\cos\beta)^3}d\beta = -\frac{V_T}{R_0}dt \tag{44}$$

Integrating both side of Eq. (44), then we can write

$$\frac{1}{2(1+\cos\beta)^2}\Big|_{\beta_0}^{\beta} = -\frac{V_T t}{R_0}\Big|_0^t$$

$$\Rightarrow \frac{1}{2(1+\cos\beta)^2} - \frac{1}{2} = -\frac{V_T t}{R_0}$$

$$\Rightarrow \cos\beta = \sqrt{\frac{1}{1-\frac{2V_T t}{R_0}}} - 1$$

$$\Rightarrow \beta = \cos^{-1}\left(\sqrt{\frac{1}{1-\frac{2V_T t}{R_0}}} - 1\right) \tag{45}$$

Substitute Eq. (42) into Eq. (6), we have

$$R = \frac{R_0(\tan\frac{\beta}{2})^{\gamma}}{\sin\beta} \tag{46}$$

In this case, by using the Eqs. (44) and (45) repeatedly, we can calculate the position of target and missile relative to target. The trajectory of the course is plotted in Fig. 6.

To implement the exact solution of pursuit course, the Eqs. (21), (22) is employed directly. The acceleration encountered by the missile is $a_{\max}$ = 40*g*. By using this formula, $|\dot{\theta}| = |a_c|_{\max}/V_M$, $\dot{\beta}_{sat_0} = \dot{\theta}_{\max}$ is obtained. From the Eq. (45), $V_T(1+\cos\beta)^3/(R_0\sin\beta) = \dot{\beta}_{sat_0}$ yields β_{sat_0}. From the Eq. (46), the range R_{sat_0} can be determined. From the Eq. (46), we have t_{sat_0}, thus, $\tau_{sat_0} = V_T t_{sat_0}/R_{sat_0}$. By using of $\Omega_{\max} = R_{sat_0}\dot{\theta}_{\max}/V_T$, $\Omega_{\max}$ is found. The corresponding result from the Eqs. (21), (22) are:

$$\begin{cases} X = R_{sat_0}x \\ Y = R_{sat_0}y \\ X_T = X_{Msat_0} + R_{sat_0}\cos\beta_{sat_0} + V_T(t - t_{sat_0}) \\ Const \end{cases} \tag{47}$$

Now, for every $\tau = \tau_{sat_0} + n\Delta\tau$, we can calculate *x, y* and *r*. t_{sat_0} From which, we can calculate the target and missile position as function of dimensionless time τ ,Now, for every $\tau = \tau_{sat_0} + n\Delta\tau$, we can calculate *x, y* and *r*. t_{sat_0} From which, we can calculate the target and missile position as function of dimensionless time τ, $t = R_{sat_0}\tau/V_T$. The results have been obtained and also plotted in

Fig. 6. Sample missile-target trajectories for this case with different solutions are depicted in Fig. 6. We can see that the exact solution and numerical solution have the similar trajectory, the Howe solution cause the missile course to lead the numerical solution course slightly in final stage. Otherwise the trajectories are virtually identical before the G-limit happens. The missile acceleration saturation problem is the most guidance system non-linearity in the performance comparison. In Fig. 7, where the lateral accelerations are plotted versus the time for the different solutions, it is evident that the proposed solution appears a saturation turning rate. This is the major advantage of the exact solution that can use a reasonable lateral acceleration in the final stage when a target must be destroyed. However, Fig. 7 shows that there are significant difference between the acceleration profiles for the exact solution and the Howe solution, then, we observed that the exact solution is more effective to describe the missile acceleration saturation phenomenon. In actual case certain acceleration limitation were required of missile, if the missile does not have the acceleration limitation by the guidance law, a miss will result. The acceleration level also affects the energy cost required for effective intercept of target, as shown in Fig. 8. It shows that a lower energy cost is required with a G-limit acceleration level. The miss

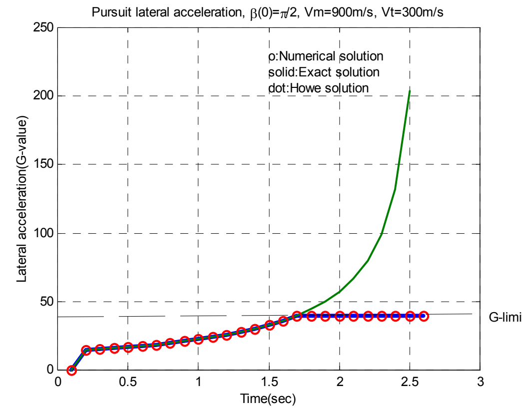

Fig.7. Pursuit a tail chase ($\beta_0 = \pi / 2$) target, lateral acceleration profile.

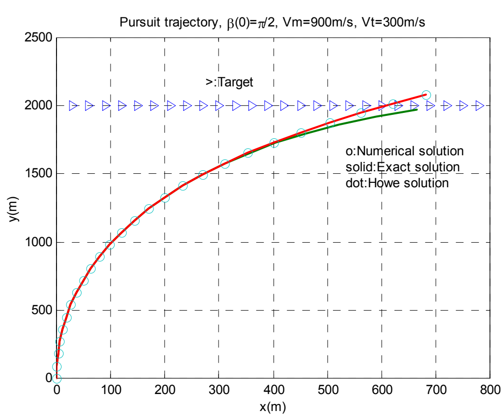

Fig. 6. Pursuit trajectories for a tail chase ($\beta_0 = \pi / 2$) target.

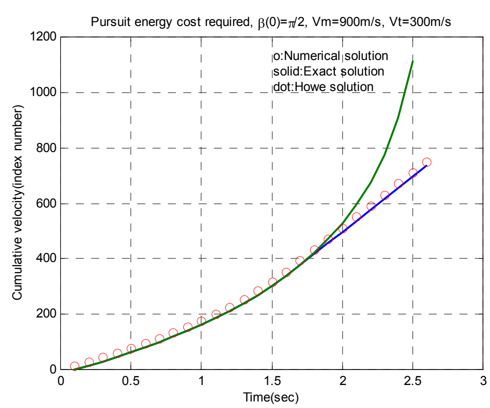

Fig.8. Pursuit a tail chase ($\beta_0 = \pi / 2$) target, energy cost required.

distance results for different solutions were recorded. Fig. 9 displays that the exact solution and numerical solution results are in close proximity, which demonstrates that the exact solution accurately captures the interaction between guidance system dynamics and the missile acceleration saturation for miss distance.

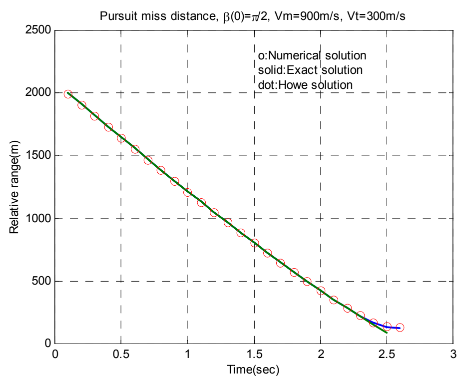

Fig.9 Pursuit a tail chase ($\beta_0 = \pi/2$) target, miss distance.

5.2 Tail chase pursuit ($\beta_0 = \pi/4$)

The missile velocity is chosen as case A. Assume that the missile has a maximum lateral acceleration capability of 20*g*. The initial conditions of the engagement are given by $\theta_0 = 45\deg$, $x_{T0} = 2000m$. The target is outgoing with a constant speed as case A. From the Eq. (6), given the missile initial range R_0 and the initial LOS angle θ_0 , we can compute the constant K , which are

$$K = \frac{\sqrt{2} \times 2000 \times \sin\frac{\pi}{4}}{(\tan\frac{\pi}{8})^3} = 28142 \tag{48}$$

From the Eq. (44), we have $\left.\frac{1}{2(1+\cos\beta)^2}\right|_{\frac{\pi}{4}}^{\beta} = -\left.\frac{V_T t}{K}\right|_0^t$, or the functional relationships are:

$$\beta = \cos^{-1}\left[\sqrt{\frac{1}{0.3432 - \frac{2V_T t}{28142}}} - 1\right] \tag{49}$$

Substituting Eq. (49) into Eq. (6) gives

$$R = \frac{28142\,(\tan\frac{\beta}{2})^3}{\sin\beta} \tag{50}$$

The recursive target location and missile position algorithm for the pursuit trajectory was employed as Eqs. (49) and (50). We can gain information about the shape of the trajectory graph if we know the Eqs. (49) and (50). Finally, we plot the points and use the information about how the LOS line shorten and rotate to complete the sketch shown in Fig.10. We can see from the closed proximity of the numerical and exact curves that both solutions are in close agreement. The lateral

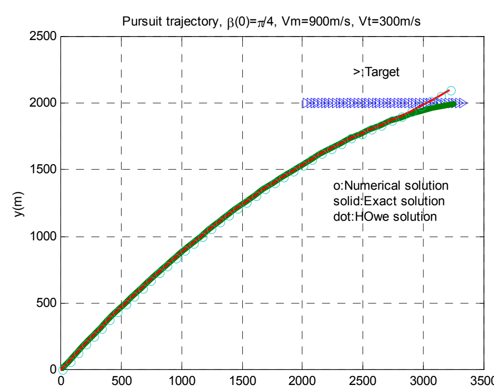

Fig. 10. Pursuit trajectories for a tail chase ($\beta_0 = \pi/4$) target.

acceleration behavior in case B is shown in Fig. 11. It is observed from the figure that the acceleration is followed G-limit value by using proposed solution. When the intercept point approach, the lateral acceleration in Howe's method show a sudden increase unlike proposed method. The results establish that acceleration profiles are qualitatively different under the two-dimensional engagement scenario compared to the Howe's solution. While pursuit guidance scheme involve in estimating the lateral acceleration constraint reasonably and accurately, the limit turning rate can be exploited for miss distance advantage. Figure 12 present the energy cost index in two-dimensional trajectory simulation of pursuit course. Herein we can approve with the exact and numerical solutions are in close agreement. Figure 13 shows the exact and numerical solutions of miss distance. The same manner appears in Fig. 13, which display the close proximity of

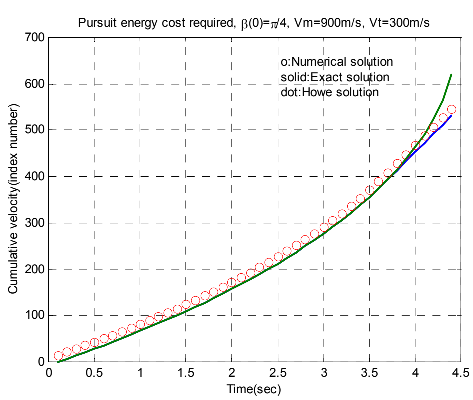

Fig. 11. Pursuit a tail chase ($\beta_0 = \pi / 4$) target,

lateral acceleration profile.

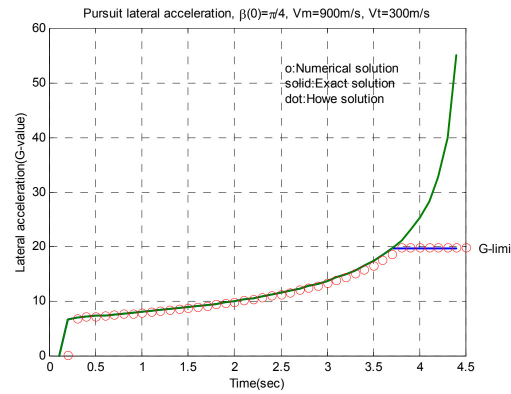

Fig. 12. Pursuit a tail chase ($\beta_0 = \pi / 4$)

target, energy cost required.

exact and numerical solution.

VI. CONCLUSIONS

The algorithms presented as a refinement to a classical solution derived by Howe helped to obtain good solution to the pursuit guidance problem. This paper has successfully demonstrated that the essential technique has been applied for the design of

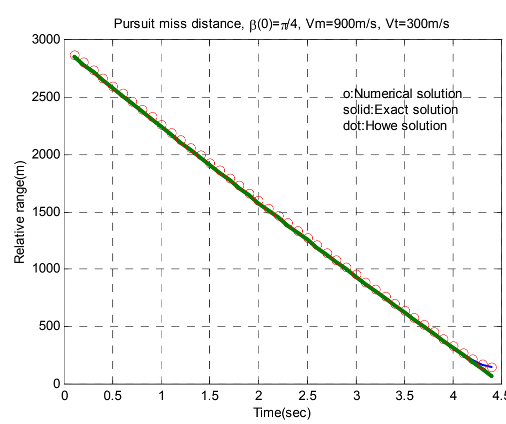

Fig. 13. Pursuit a tail chase ($\beta_0 = \pi / 4$).

target, miss distance.

a pursuit problem. Also, some significant characteristics, such as lateral acceleration profile, energy cost required and miss distance are investigated and discussed in detail under the effect of lateral acceleration constraint. Then in advance, numerical simulations demonstrate that the proposed exact solution has much lower estimation errors than Howe solution when near intercept. Most importantly, the proposed method allows missile to support defense and offense operations.

VII. ACKNOWLEDGEMENT

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