中正嶺學報 第三十四卷 第一期 民國 94.11.
JOURNAL OF C.C.I.T., VOL.34, NO.1, NOV., 2005

Determining the optimum Manufacturing Target under the Uniform Quality Characteristic

Chung-Ho Chen

Department of Industrial Management, *Southern Taiwan University of Technology*

ABSTRACT

In 1998, Wu and Tang presented the uniform quality characteristic and adopted a asymmetric quadratic quality loss function for designing the optimum manufacturing target. However, they did not consider the condition of using a linear quality loss function in their model. In this paper, we further propose the modified Wu and Tang's model with the piecewise line quality loss and mixed quality loss for determining the optimum manufacturing target.

Key words: Uniform Distribution, Piecewise Line Quality Loss function, Mixed Quality Loss Function, Specification Limits, Manufacturing Target

考量均等品質特性之最佳製造目標值設定

陳忠和

南台科技大學工業管理系

摘 要

在1998年，Wu與Tang已提出具均等品質特性並採非對稱二次品質損失函數來考量的最佳製造目標值設定問題; 然而，他們卻未考慮產品具線性品質損失函數的情況於其模式中。本研究將進一步呈現具逐段線性品質損失與混合品質損失函數的修正的Wu與Tang模式以決定最佳的製造目標值。

關鍵詞: 均等分配，逐段線性品質損失函數，混合品質損失函數，規格界限，製造目標值

文稿收件日期 92.11.12 ; 文稿修正後接受日期 94.1.20.
Manuscript received Nov. 12, 2003; revised Jan. 20, 2005.

I. INTRODUCTION

The traditional product quality is defined as the quality characteristic of product within specifications. Hence, the step loss function is used in measuring the performance of product. In 1986, Taguchi [1] proposed the quadratic quality loss function and redefined the product quality as the total losses of the society.

There are considerable attentions paid to the study of economic selection of process mean. The selection of optimum process mean will directly affect the process defective rate, scrap or rework cost, and the loss to the customer. Recently, Wu and Tang [2], Li [3-5], Maghsoodloo and Li [6], Phillips and Cho [7], Li and Chou [8], Li and Wu [9], Duffuaa and Siddiqi [10], and Rahim and Tuffaha [11] have addressed the different problems of unbalanced tolerance design with the quadratic quality loss functions.

The regular quadratic quality loss function is patently inappropriate in some situations. Trietsch [12] remarks that 'One such case occurs when the expected cost of exceeding the tolerance limits is not equal to the right and to the left of the target. Missing by cutting too much, for instance, may imply scrap, while cutting too little only causes rework. When this is the case one possible response is fitting a loss function that is not symmetric, and not necessarily quadratic.'

There are the nominal-is-best, the smaller-the-better, and the larger-the-better types for describing the product characteristic. For the nominal-is-best characteristic, the product has the both-sided specification limits, i.e., the lower specification limit and the upper specification limit , and the constant target value. For the smaller-the-better characteristic, the product has a one-sided upper specification limit and the target value is equal to zero. For the larger-the-better characteristic, the product has a one-sided lower specification limit and the target value approaches infinity.

Many examples describe the nominal-is-best characteristic of product, e.g., television color intensity and the diameter of plastic pipes. The distribution of the product quality characteristic is usually assumed to be normal in the selection of optimum process mean. Chandra [13] pointed out that "The beta distribution could be more flexible when compared with the normal probability distribution because it can accommodate different finite ranges and different shapes from left skewed to symmetrical to right skewed. This makes the beta distribution a much better candidate for fitting real-life distributions of quality characteristics".

In the real world, however, many quality characteristics often show various types of skewed distribution. Hence, one needs a practical distribution fitting real situations very well. The Weibull distribution has been successfully applied in describing the lifetime of certain electronic systems. It is an important distribution for studying the reliability and failure rates of systems. This makes the Weibull distribution a much better candidate for fitting the larger-the-better quality characteristics. The log-normal distribution has been used for random quantities of pollutants in water or air and for other phenomena with skewed distributions [14]. Hence, the log-normal distribution can be applied in describing the smaller-the-better characteristic of product.

Cho and Leonard [15] presented that the piecewise linear quality loss function for product is roughly proportional to the deviation of the quality characteristic from its specification limits. It is usually applied in the filling/canning problem for determining the optimum manufacturing target, for example, Calsson [16], Golhar and Pollock [17], Misiorek and Barnett [18], and Lee, et al. [19] etc..

Wu and Tang [2] assumed that the uniform quality characteristic of product and adopted the asymmetric quadratic quality loss function for determining the optimum manufacturing target. However, they did not consider the condition of using a linear quality loss function in their model. In this paper, we further propose the modified Wu and Tang's [2] model with the piecewise linear quality loss and mixed quality loss function for determining the optimum manufacturing target.

The piecewise linear quality loss of product only considers the costs of out-of specification and the mixed quality loss of product includes the quadratic quality loss within specifications and the piecewise line quality loss out-of specification.

II. WU AND TANG'S COST MODEL WITH THE ASYMMETRIC QUADRATIC QUALITY LOSS FUNCTION

According to Wu and Tang [2], the average quality loss of product per item is

$$C_{T_0} = \int_{m-T+x}^{m} f(y)k_1(y-m)^2\,dy + \int_{m}^{m+T+x} f(y)k_2(y-m)^2\,dy = \frac{1}{6T}[k_1(T-x)^3 + k_2(T+x)^3] \quad (1)$$

k_1 and k_2 denote the constant called quality loss coefficient at the specification limit. m denotes the design target. x denotes a positive distance of manufacturing target that is away from the design target. T denotes the tolerance zone.

$$f(y) = \frac{1}{2T}, m - T + x \le y \le m + T + x. \quad (2)$$

We can use the coefficients k_1 and k_2 for the two directions of deviation from the design target. If tolerances are both sides are Δ_1 and Δ_2, respectively, then the quality loss at the specification limit is defined as A_1 and A_2, respectively. We obtain $k_1 = \frac{A_1}{\Delta_1^2}$ and $k_2 = \frac{A_2}{\Delta_2^2}$.

In order to determine the optimal x value, Wu and Tang [2] took the derivative of Eq. (1) and set the first order derivative equal zero. Hence, the optimal x^* is

$$x^* = \frac{\sqrt{k_1/k_2}-1}{\sqrt{k_1/k_2}+1}T,\; k_1 > k_2 \quad (3)$$

and

$$C^*_{T_0} = \frac{4}{3}\frac{k_1k_2}{(k_1 + k_2 + 2\sqrt{k_1k_2})}T^2 \quad (4)$$

III. MODIFIED WU AND TANG'S COST MODEL WITH THE PIECEWISE LINEAR QUALITY LOSS FUNCTION

Cho and Leonard [15] presented the piecewise linear quality loss function for the nominal-the-best quality characteristic is as follows:

$$L(y) = \begin{cases} 0 & \text{if } T_L \le y \le T_U \\ D_L(T_L - y) & \text{if } y < T_L \\ D_U(y - T_U) & \text{if } y > T_U \end{cases} \quad (5)$$

T_U denotes the upper specification limit. T_L denotes the lower specification limit. D_U denotes the quality loss coefficient when the quality characteristic exceeds T_U. D_L denotes the quality loss coefficient when the quality characteristic is less than T_L. $L(y)$ denotes the quality loss function.

We consider the modified Wu and Tang's [2] model with the piecewise linear quality loss function. The modified model is to determine the optimum x^* that minimizes C_{T_1}.

$$C_{T_1} = \int_{m-T+x}^{T_L} D_L(T_L - y) f(y)dy + \int_{T_U}^{m+T+x} D_U(y - T_U) f(y)dy$$

$$= \frac{D_L T_L}{2T}[T_L - (m - T + x)] - \frac{D_L}{4T}[T_L^2 - (m - T + x)^2] + \frac{D_U}{4T}[(m + T + x)^2 - T_U^2] - \frac{D_U T_U}{2T}[(m + T + x) - T_U] \quad (6)$$

By differentiating C_{T_1} with respect to *x*, we have:

$$\frac{dC_{T_1}}{dx} = -\frac{D_L T_L}{2T} + \frac{D_L}{2T}(m - T + x) + \frac{D_U}{2T}(m + T + x) - \frac{D_U T_U}{2T} \quad (7)$$

The second derivative of C_{T_1} with respect to *x* is

$$\frac{d^2 C_{T_1}}{dx^2} = \frac{D_L + D_U}{2T} \quad (8)$$

Eq. (8) is positive. Hence, one sets the first derivative of C_{T_1} equal to zero, and solves for *x*. We obtain

$$x^* = \frac{D_L(T_L + T) + D_U(T_U - T)}{D_L + D_U} - m \quad (9)$$

IV. MODIFIED WU AND TANG'S COST MODEL WITH THE MIXED QUALITY LOSS FUNCTION

There is a simplified relationship between quality loss and the amount of deviation from the target value. When the quality characteristic, *y*, is within specification limits, the quadratic quality loss function is adopted to measure the product loss. When *y* exceeds either one of the specification limits, the quality loss is equal to the cost of the product's disposal. Assume that the product is reworked when $y < T_L$ and the product is scrapped when $y > T_U$. The rework cost may not be a constant because one needs to input excess material in the reworked process, which is linearly proportional to the deviation from T_L. The scrap cost may not be a constant because one needs to input excess space, people, and time in the scrapped process, which is linearly proportional to the deviation from T_U. Define a mixed quality loss function *L*(*y*) for the nominal-is-best quality characteristic to be

$$L(y) = \begin{cases} A_1 + D_L(T_L - y), & y < T_L \\ k_1(T - y)^2, & T_L \le y < T \\ k_2(y - T)^2, & T \le y \le T_U \\ A_2 + D_U(y - T_U), & y > T_U \end{cases} \quad (10)$$

A_1 and A_2 denote the quality loss at the lower and upper specification limit, respectively. *T* denotes the target value. k_1 and k_2 denote the constant called the quality loss coefficient on the left-hand and right-hand side of the target value, but still within specification limit. D_L denotes the quality loss coefficient when the quality characteristic is less than the lower specification limit, T_L. D_U denotes the quality loss coefficient when the quality characteristic exceeds the upper specification limit, T_U.

We consider the modified Wu and Tang's [2] model with the mixed quality loss function. The modified model is to determine the optimum x^* that minimizes C_{T_2}.

$$
\begin{aligned}
C_{T_2} = & \int_{m-T+x}^{T_L} [A_1 + D_L(T_L - y)] f(y) dy + \\
& \int_{T_U}^{m+T+x} [A_2 + D_U(y - T_U)] f(y) dy + \\
& \int_{T_L}^{m} k_1 (y-m)^2 f(y) dy + \\
& \int_{m}^{T_U} k_2 (y-m)^2 f(y) dy \\
= & A_1[T_L - (m - T + x)] + \\
& A_2[(m + T + x) - T_U] + \\
& \frac{D_L T_L}{2T}[T_L - (m - T + x)] - \\
& \frac{D_L}{4T}[T_L^2 - (m - T + x)^2] + \\
& \frac{D_U}{4T}[(m + T + x)^2 - T_U^2] - \\
& \frac{D_U T_U}{2T}[(m + T + x) - T_U] + \\
& \frac{1}{6T}[k_2(T_U - m)^3 - k_1(T_L - m)^3] \qquad (11)
\end{aligned}
$$

Differentiating C_{T_2} with respect to x, we have:

$$
\begin{aligned}
\frac{dC_{T_2}}{dx} = & [-A_1 + A_2] - \frac{D_L T_L}{2T} + \\
& \frac{D_L}{2T}(m - T + x) + \frac{D_U}{2T}(m + T + x) - \\
& \frac{D_U T_U}{2T} \qquad (12)
\end{aligned}
$$

The second derivative of C_{T_2} with respect to x is

$$
\frac{d^2 C_{T_2}}{dx^2} = \frac{D_L + D_U}{2T} \qquad (13)
$$

Eq. (13) is positive. Hence, one sets the first derivative of C_{T_2} equal to zero, and solves for x. We obtain

$$
\begin{aligned}
x^* = & [D_L(T_L + T) + D_U(T_U - T) + \\
& 2T(A_1 - A_2)]/(D_L + D_U) - \\
& m, \ A_1 > A_2 \qquad (14)
\end{aligned}
$$

V. NUMERICAL EXAMPLE AND DISCUSSION

Suppose that a rod is put together in sections. One considers the length, y, of the assembled rod is uniform. The probability density function (p.d.f.) shows a constant probability with respect to the quality characteristic y. The tolerance zone of the characteristic covers a range of $2T$. The different costs may occur for the assembled rod within the specification limits and out-of-specification. The design target m is equal to 1.5, the lower specification limit T_L is equal to 0.9, and the upper specification limit T_U is equal to 2.1. Assume that $T = 1$ and the positive distance of manufacturing target that is away from the design target is x. The p.d.f. is $f(y) = \frac{1}{2}, 0.5 + x \le y \le 2.5 + x$. We would like to find the optimum manufacturing target based on minimizing the average loss of a product per item.

5.1 Wu and Tang's Cost Model with the Asymmetric Quadratic Quality Loss Function

The asymmetric quadratic quality loss function is used in evaluating the product quality. Assume that the monetary loss per item at the lower specification limit is $A_1 = 1.5$ and the monetary loss per item at the upper specification limit is $A_2 = 1$. Hence, we have the coefficient of quality loss to the left of the design target is $k_1 = \frac{A_1}{\Delta^2} = \frac{25}{6}$ and the coefficient of quality loss to the right of the design target is $k_2 = \frac{A_2}{\Delta^2} = \frac{25}{9}$. By solving Eq. (3), we obtain that the optimum x value for Wu and Tang's [2] model with the asymmetric quadratic quality loss function is x^* = 0.1010 with $C^*_{T_0} = 1.12245$.

5.2 Modified Wu and Tang's Cost Model with the Piecewise Linear Quality Loss Function

The piecewise linear quality loss function is used in evaluating the product quality. Assume that the quality loss coefficient, D_U , when the quality characteristic exceeds T_U is 1 and the quality loss coefficient, D_L , when the quality characteristic is less than T_L is 1.5. By solving From Eq. (9), we obtain that the optimum x value for modified Wu and Tang's [2] model with the piecewise linear quality loss function is x^* = 0.08 with $C^*_{T_1} = 0.9485$.

5.3 Modified Wu and Tang's Cost Model with the Mixed Quality Loss Function

The mixed quality loss function is used in evaluating the product quality. Assume that the monetary loss per item at the lower specification limit is $A_1 = 1.5$ and the monetary loss per item at the upper specification limit is $A_2 = 1$. Hence, we have the coefficient of quality loss to the left of the design target is $k_1 = \frac{A_1}{\Delta^2} = \frac{25}{6}$ and the coefficient of quality loss to the right of the design target is $k_2 = \frac{A_2}{\Delta^2} = \frac{25}{9}$. The quality loss coefficient, D_U , when the quality characteristic exceeds T_U is 1 and the quality loss coefficient, D_L , when the quality characteristic is less than T_L is 1.5. By solving Eq. (14), we obtain that the optimum x value for modified Wu and Tang's [2] model with the mixed quality loss function is x^* = 0.48 with $C^*_{T_2} = 1.0932$.

5.4 Discussion

From the above numerical example, it shows that the average quality loss of product would be lower if a proper amount of movement for the manufacturing target was assigned. The average losses have a minimum value associated with a particular shift x^* . Process planning, including this adjustment, will produce the best economic benefit. Table 1 lists the results of the above numerical example. From Table 1, we obtain that (1) both the modified models have the smaller average loss of product per item than that of Wu and Tang's [2] model; (2) the modified model with the piecewise linear quality loss function has the smallest the positive distance and the average quality loss of product per item; (3) the modified model with the mixed quality loss function has the largest positive distance.

Table 1. Comparison of our results and Wu and Tang's solution.

	positive distance	average quality loss
A^*	0.101	1.12245
B^{**}	0.08	0.9485
C^{***}	0.48	1.0932

Notes:
*: Wu and Tang's solution
**: Result with piecewise loss function
***: Result with mixed loss function

V. CONCLUSIONS

In this article, we have presented the modified Wu and Tang's [2] model with the piecewise linear quality loss function and the mixed quality loss function. Further study will extend this method to another quality characteristic for determining the optimum manufacturing target.

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