Determining the optimum Process Mean for the Product with Screening Limits

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ABSTRACT

In 2000, Lee et al. presented a filling problem of determining the optimum process mean and screening limits. They considered three grades of product, assumed the normal quality characteristic, and adopted the piecewise linear profit function for measuring the profit per item. However, in the real world, many quality characteristics often show various types of skewed distributions. In this paper, we propose a modified Lee et al.'s model with beta distribution for determining the optimum process mean.

Keywords: piecewise linear profit function, beta distribution, process mean, process control

產品具篩選界限之最佳製程平均數設定

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摘 要

在2000年, Lee等人提出有關裝填業產品具篩選界限設定最佳製程平均數的問題, 其模式為考量將產品分成三種等級、假設品質特性為常態分配、並採逐段線性利潤函數來量測每單位產品的利潤。然而在實際情況下, 很多品質特性通常呈現不同型態的偏態分配, 本研究將進一步提出具有貝他品質特性的Lee等人的修正模式以決定最佳的製程平均數。

關鍵詞: 逐段線性品質損失函數，貝他分配，製程平均數，製程監控

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I. INTRODUCTION

Statistical process control (SPC) is a preventive method for quality monitor and improvement. The control charts, process capacity analysis, and optimum process level setting are three important issues in SPC. The technique of control charts is used for process troubleshooting, process capability analysis is used in evaluating the status of product, and optimum process level setting is used for selecting the desirable manufacturing target.

The selection of optimum process level/mean directly affect the process defective rate, scrap or rework cost, and the loss to the customer. There are considerable attentions paid to the study of economic selection of process mean. The piecewise linear profit function of quality characteristic is usually applied in the filling/canning problem for determining the optimum manufacturing target and other important parameters, e.g., Hunter and Kartha [1], Carlsson [2], Bisgaard et al. [3], Golhar [4, 5], Golhar and Pollock [6, 7], Misiorek and Barnett [8], and Lee, et al. [9, 10], and so forth.

Lee et al. [9, 10] presented the problem of jointing determination of optimum process mean and screening limits. The quality characteristic of performance variable or surrogate variable is considered as the screening variable. Their models involved selling and discounted prices as well as production, inspection, rework and penalty costs. The normal and bivariate normal distributions are assumed and used in Lee et al.'s [9, 10] models. The screenings for a product with three grades, single stage screening, and two stage screening are considered, respectively. The objective of their models is to maximize the expected profit per item.

The distribution of the product quality characteristic is usually assumed to be normal in the selection of optimum process mean. In the real world, however, many quality characteristics often show various types of skewed distribution. Hence, one needs a practical distribution fitting real situations very well. Chandra [11] pointed out that "The beta distribution could be more flexible when compared with the normal probability distribution because it can accommodate different finite ranges and different shapes from left skewed to symmetrical to right skewed. This makes the beta distribution a much better candidate for fitting real-life distributions of quality characteristics".

In this paper, we propose a modified Lee et al.'s [9] model with beta distribution for determining the optimum process mean and maximizing the expected profit per item. A numerical example is provided for illustrating Lee et al.'s [9] solution and our result.

II. LEE ET AL.'S MODEL WITH NORMAL DISTRBUTION

In Lee et al.'s [9] model considering performance variable, X, the objective is to maximize the expected profit per item and obtain the optimum process mean. The profit for an item depends upon the value of normal quality characteristic, X, whether or not it satisfies a critical specification limit. Each item is classified into three grades, A, B, and C. Let U and L be the pre-specified specification limits for grades, A, B, and C, respectively. If an item with $X \geq U$, it is sold at a fixed price a_1 to the primary market. If an item with $L \leq X < U$, it is sold at a fixed price $a_2 (< a_1)$ to the secondary market. If an item with $X < L$, it is reworked by the same manufacturing process at a rework cost $r (< a_2 < a_1)$.

It is assumed that the quality characteristic X has unknown mean μ and known standard deviation σ . Let the production cost per item be $m + nX$, where m is the fixed cost and n is the variable cost per item. Let k be the inspection cost per item and X_r be the quality characteristic of a reworked item. It is assumed that X and X_r are identically and independently distributed.

From Lee et al. [9], we have the profit per item as follows:

$$P(X)=\begin{cases} a_1-m-nX-k, & X\geq U \\ a_2-m-nX-k, & L\leq X<U \\ P(X_r)-r-k, & X<L \end{cases} \quad (1)$$

$P(X)$ denotes the profit per item. $P(X_r)$ denotes the profit for a rework item. Assume that the reworked item has the same profit as the regular item, i.e., $E[P(X)] = E[P(X_r)]$. Hence, the expected profit per item is

$$E[P(X)]=\int_U^{\infty}(a_1-m-nx-k)f(x)dx+\int_L^U(a_2-m-nx-k)f(x)dx+\int_{-\infty}^L(E[P(X)]-r-k)f(x)dx \quad (2)$$

$f(x)$ denotes the normal probability density function with mean μ and standard deviation σ.

From Lee et al. [9], Eq. (2) can be rewritten as

$$E[P(X)]=\{a_1\Phi(\frac{\mu-U}{\sigma})+a_2[\Phi(\frac{U-\mu}{\sigma})-\Phi(\frac{L-\mu}{\sigma})]-(m+n\mu)\Phi(\frac{\mu-L}{\sigma})-r\Phi(\frac{L-\mu}{\sigma})-n\sigma\phi(\frac{\mu-L}{\sigma})-k\}/\Phi(\frac{\mu-L}{\sigma}) \quad (3)$$

$\Phi(z)$ denotes the cumulative probability of the standard normal random variable with probability density function $\phi(z)$.

Lee et al. [9] took the first derivative of Eq. (3) with respect to μ, set it equal to zero, and adopted the bisection method for finding the optimal μ that maximizes the expected profit per item.

III. MODIFIED LEE ET AT.'S MODEL WITH BETA DISTRIBUTION

Assume that the quality characteristic, X, follows a beta distribution. The objective is to maximize the expected profit per item, i.e.,

Maximize

$$\begin{aligned} E[P(X)]=&\int_U^{b+\delta}(a_1-m-nx-k)g(x)dx+\int_L^U(a_2-m-nx-k)g(x)dx+\int_{a+\delta}^L(E[P(X)]-r-k)g(x)dx \\ =&\int_U^{b+\delta}a_1g(x)dx+\int_L^U a_2g(x)dx-(m+k)\int_L^{b+\delta}g(x)dx-\int_L^{b+\delta}nxg(x)dx+(E[P(X)]-r-k)\int_{a+\delta}^L g(x)dx \\ =&a_1[1-B(\frac{U-\delta-a}{b-a};\alpha,\beta)\cdot\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}]+a_2\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}\cdot[B(\frac{U-\delta-a}{b-a};\alpha,\beta)-B(\frac{L-\delta-a}{b-a};\alpha,\beta)]-(m+k)\cdot \\ &[1-B(\frac{L-\delta-a}{b-a};\alpha,\beta)\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}]+(E[P(x)]-r-k)\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}\cdot \end{aligned}$$

$$B(\frac{L-\delta-a}{b-a};\alpha,\beta)-n\{(\delta+a)\cdot [1-B(\frac{L-\delta-a}{b-a};\alpha,\beta)\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}] + (b-a)[1-B(\frac{L-\delta-a}{b-a};\alpha+1,\beta)\cdot \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}]\} \quad (4)$$

$$g(x)=\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}\frac{1}{b-a}[\frac{(x-\delta)-a}{b-a}]^{\alpha-1}\cdot [\frac{b-(x-\delta)}{b-a}]^{\beta-1},$$
$$a+\delta\le L\le x\le U\le b+\delta \quad (5)$$

a denotes the minimum value of quality characteristic *X*. b denotes the maximum value of quality characteristic *X*. α denotes the shape parameter, $\alpha\ge 0$. β denotes the shape parameter, $\beta\ge 0$. δ denotes the location parameter.

$$B(z;\alpha,\beta)=\int_0^z s^{\alpha-1}(1-s)^{\beta-1}ds \quad (6)$$

$$\Gamma(z)=\int_0^\infty s^{z-1}e^{-s}ds \quad (7)$$

Let $t_1=\frac{U-\delta-a}{b-a}$, $t_2=\frac{L-\delta-a}{b-a}$, and $I_t(\alpha,\beta)=\int_0^t \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}y^{\alpha-1}(1-y)^{\beta-1}dy$.

Hence, the problem of Eq. (4) can be rewritten as

Maximize

$$E[P(X)]=a_1\frac{[1-I_{t_1}(\alpha,\beta)]}{[1-I_{t_2}(\alpha,\beta)]}+a_2\cdot \frac{[I_{t_1}(\alpha,\beta)-I_{t_2}(\alpha,\beta)]}{[1-I_{t_2}(\alpha,\beta)]}-(m+k) -(r+k)\frac{I_{t_2}(\alpha,\beta)}{[1-I_{t_2}(\alpha,\beta)]}- n\{(\delta+a)+(b-a)\cdot \frac{[1-\frac{\alpha}{\alpha+\beta}I_{t_2}(\alpha+1,\beta)]}{[1-I_{t_2}(\alpha,\beta)]}\} \quad (8)$$

If α and β are both positive integers, then for $0\le w\le 1$,

$$I_w(\alpha,\beta)=\sum_{i=\alpha}^{\alpha+\beta-1}C_i^{\alpha+\beta-1}w^i(1-w)^{\alpha+\beta-1-i} \quad (9)$$

That is, one has $P(\alpha\le W\le\alpha+\beta-1)=1-F_W(\alpha-1)$. W has a binomial distribution with parameters $n=\alpha+\beta-1$ and $p=w$. Hence, if α and β are both positive integers, then the problem of Eq. (8) can be rewritten as

Maximize

$$E[P(X)]=a_1\frac{F_{T_1}(\alpha-1)}{F_{T_2}(\alpha-1)}+a_2\cdot \frac{F_{T_2}(\alpha-1)-F_{T_1}(\alpha-1)}{F_{T_2}(\alpha-1)}- (m+k)-(r+k)\frac{1-F_{T_2}(\alpha-1)}{F_{T_2}(\alpha-1)} -n\{(\delta+a)+(b-a)\cdot \frac{1-\frac{\alpha}{\alpha+\beta}[1-F_2(\alpha)]}{F_{T_2}(\alpha-1)}\} \quad (10)$$

$F_{T_i}(\alpha-1)$ denotes the cumulative binomial probability with parameters $n=\alpha+\beta-1$ and $p=t_i, i=1,2$. $F_{T_2}(\alpha)$ denotes the cumulative binomial probability with parameters $n=\alpha+\beta$ and $p=t_2$.

The above problem of Eq. (10) is to determine the optimal value of δ, δ^*, such that the expected profit per container is maximized. Hence, the optimum process mean is $\mu^*=\delta^*+a+(b-a)\dfrac{\alpha}{\alpha+\beta}$. Since the problem of Eq. (10) is one dimensional, one can use a simple interval search method for obtaining the optimum location parameter δ^* . δ^* could be different for the side of *a* and the side of *b*.

IV. NUMERICAL EXAMPLE

Consider a numerical example given in Lee et al. [9]. Let $a_1 = 3.25$, $k = 0.04$, $a_2 = 3.10$, $r = 0.1$, $m = 0.1$, $n = 0.06$, $U = 41.5$, and $L = 40$. The quality characteristic is normally distributed with the unknown mean μ and the known standard deviation $\sigma = 1.25$. By solving Eq. (3), the optimal process mean is $\mu^* = 43.443$ with $E[P(X)] = 0.4937$.

Assume that the quality characteristic follows a beta distribution with the unknown δ and the known parameters $a = 38$, $b = 45$, $\alpha = 1$ and $\beta = 1$. By solving the problem of Eq. (10), the optimal value for δ is $\delta^* = 2$ with $E[P(X)] = 0.2579$. Hence, the optimal process mean is $\mu^* = 43.5$.

V. CONCLUSIONS

Lee et al. [9] considered a filling problem where items are produced continuously, items have three grades, and every item is inspected. The optimum process mean needs to be determined in their models. In this paper, we have presented a modified Lee et al.'s [9] model with beta distribution for maximizing the expected profit per item. Further study will extend this method to modified Lee et al.'s [9] model with the customer's quality loss.

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