Heuristics Approach for Portfolio Selection with Military Investment Assets

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ABSTRACT

The main purpose of this research is to solve a military investment problem by applying heuristic algorithms to a mean-variance risk model. This problem is essential to national defense not only in investing a yearly budget efficiently but also in maintaining a stable armed force. Decision makers need a strategic method in order to maximize the military capability and minimize the risk as well. This paper presents a new definition on military investment assets to construct the portfolio selection model. We compare the results of three heuristics based on genetic algorithm, tabu search and simulated annealing, and explain how the asset allocations are affected by the risk. The empirical results show that our heuristics are efficient methods in terms of solving the defined military investment problem.

Keywords: military investment assets, mean-variance, portfolio selection, heuristic algorithms

啟發式方法應用於軍事投資資產選擇

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摘要

本研究的主要目的是應用啟發式方法在均異風險模式，以求解軍事投資問題。在年度預算的有效投資與穩固武力的維持上這是國防的重要議題。決策者需要一個策略方法促使軍事能力的最大化與風險的最小化。本文給予軍事投資資產一個新定義以建構資產配置選擇模式。我們比較基因演算、限制搜尋與模擬退火三種演算法的結果，並說明風險對投資配置所造成的影響。對求解該軍事投資問題而言，結果顯示我們的啟發式演算法是一個有效的方法。

關鍵字：軍事投資資產，均異模式，資產選擇，啟發式演算法

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1. I. INTRODUCTION

National defense budget has been considered as necessary expense for the national security, but each country allocates amount of budget expenditure depending on own needs. In general, a government or firm always prefers to have the profit on its portfolio as large as possible. At the same time, the risk should be as small as possible. However, a high return always accompanied with a higher risk. How can we have either less risk than others for a given level of return, or have more return than others for the same risk? It is necessary to find a solution of weighted allocation method for military investment assets in a risk scenario and to provide an objective fact for decision makers in investment options. Under this trend, the government sectors and large enterprises gradually use more reasonable investment tools to manage investment assets. It is difficult to present a steady, rational and positive investment policy for military or large enterprises in diversified environment. Therefore these observations motivate authors' efforts to give a new definition of effectiveness measure for the military investment assets and discuss the portfolio selection problem.

The purpose of this paper is to consider military investment assets can be successfully solved by the heuristic algorithms if we use the mean-variance model. Portfolio selection is an optimization problem that plays a vital part in financial management and investment decision-making. It discusses the problem of how to allocate one's capital to a large number of securities so that the investment can bring a most profitable return. In past times, investors talked about risk without a measurable term to define it. Until 1952, Markowitz [1] stated that variance could be regarded as risk and numerous models have been developed based on this measurement afterward [2-4]. The standard mean-variance model assumed that the total return of a portfolio can be described using the mean return of the assets and the variance of return (risk) between these assets. The portfolios that offer the minimum risk for a given level of return form what is called an efficient frontier. For every level of desired mean return, this efficient frontier gives us the best way of investing our money.

In this paper we apply heuristics approach in order to trace out the efficient frontier associated to the portfolio selection problem, which assume that the return of a portfolio can be described using the annual amount of budget and the measure of effectiveness between military investment assets. In addition, the mean-variance model is extended to include cardinality constraints. These constraints ensure that a given number of different assets are invested in the portfolio as well as the proportion of the entire money to be invested in each asset.

The rest of the paper is organized as follows. First, we present the standard mean-variance model formulation with cardinality constraints for the portfolio selection problem in Section II. Then in Section III we employ heuristic algorithms to solve the mean-variance model with military investment assets. In Section IV we provide return data of a portfolio, which is effectiveness measure divide by budget. In Section V, we used data sets collected in the military investment returns to illustrate the optimization idea and experimental results. Finally, we draw some conclusions in Section VI.

2. II. MEAN-VARIANCE MODEL FOR PORTFOLIO SELECTION PROBLEM

The mean variance model is based upon assumptions that an investor is risk averse, the distribution of the rate of return is multivariate normal and the utility of the investor is quadratic function of the rate of return.

First of all, let us review the popular Markowitz mean-variance model [1] for portfolio selection problem. Let N be the number of different assets, μ_i be the mean return of asset *i*, σ_{ij} be the covariance between returns assets *i* and *j*, $\lambda \in \{0, 1\}$ be the risk aversion parameter. The decision variable w_i is the proportion ($0 \le w_i \le 1$) of the portfolio held in asset *i*. Using this notation, the mean-variance model for the portfolio selection problem is formulated as follows.

Minimize

$$\lambda\left[\sum_{i=1}^{N}\sum_{j=1}^{N} w_i w_j \sigma_{ij}\right]-(1-\lambda)\left[\sum_{i=1}^{N} w_i \mu_i\right] \quad (1)$$

Subject to

$$\sum_{i=1}^{N} w_i = 1 \quad (2)$$

$$0 \le w_i \le 1,\ i = 1, ..., N \quad (3)$$

In equation (1), the case $\lambda = 0$ represents maximum expected return (without considering the variance) and $\lambda = 1$ represents minimum risk (regardless of the mean returns). Values of λ satisfying $0 < \lambda < 1$ represent an explicit tradeoff between risk and return, generating solutions between the two extremes $\lambda = 0$ and $\lambda = 1$. By solving the above equations (1)-(3) for varying values of λ, we can trace out the efficient frontier. This efficient frontier is a curve that lies between the global minimum risk portfolio and the maximum return portfolio. In other words, the portfolio optimization problem is to find all the efficient portfolios along this frontier.

With the purpose of generalizing the standard mean-variance model, we use a similar formulation to include cardinality constraints [5, 6].

Minimize

$$\lambda\left[\sum_{i=1}^{N}\sum_{j=1}^{N} w_i w_j \sigma_{ij}\right]-(1-\lambda)\left[\sum_{i=1}^{N} w_i \mu_i\right] \quad (4)$$

Subject to

$$\sum_{i=1}^{N} w_i = 1 \quad (5)$$

$$\sum_{i=1}^{N} z_i = K \quad (6)$$

$$\varepsilon_i z_i \le w_i \le \delta_i z_i,\ i = 1, ..., N \quad (7)$$

$$z_i \in \{0,1\},\ i = 1, ..., N \quad (8)$$

In addition to those notations previously defined, let

K is the desired number of different assets in the portfolio with no null investment

ε_i is the minimum proportion that must be held of asset i ($i = 1, ..., N$) if any of assets i is held

δ_i is the maximum proportion that can be held of asset i ($i = 1, ..., N$) if any of assets i is held

z_i = 1 if any of asset i ($i = 1, ..., N$) is held
= 0 otherwise

Equation (5) ensures that the proportions add to one. Equation (6) ensures that exactly K assets are held. Equation (7) constraints define lower and upper limits on the proportion of each asset which can be held in the portfolio. It ensures that if any of assets i is held ($z_i = 1$) its proportion w_i must lie between ε_i and δ_i, while if none of asset i is held ($z_i = 0$) its proportion w_i is zero. Equation (8) is integrality constraint. By a risk aversion parameter λ, we could use this program (equations (4)-(8)) to trace out the cardinality constrained efficient frontier (CCEF) in an exactly analogous way.

The portfolio selection problem is an instance from the family of quadratic programming problems when the standard Markowitz mean-variance model is considered. But if this model is generalized to include cardinality constraints, then the portfolio selection problem becomes a mixed quadratic and integer programming problem. Although exact algorithms can be tackled using linear and/or integer programming, the form of the objective function and/or the constraints of the optimization problems are restricted (usually linear) due to the limiting of these exact algorithms. For other non-linear problems, the formulations are also required to be differentiable in order to apply calculus-based method. These limitations prevent us from building more realistic models to simulate the real world problems. Therefore, in recent years many researchers have emphasized heuristic methods to overcome the disadvantage of those classical approaches. The cardinality constrained portfolio selection problem, as far as we are aware, has no exact algorithm reported in the literature. On the other hand, some heuristic methods based mainly on genetic algorithm (GA), tabu search (TS) and simulated annealing (SA) have been well developed and widely applied in portfolio selection optimization problems [5, 7, 8].

III. HEURISTIC ALGORITHMS

A heuristic algorithm is a technique which seeks good solutions at a reasonable computational cost without being able to guarantee either feasibility or optimality, or even in many cases state how close to optimality a particular feasible solution is. Here we present three well-known heuristic algorithms which are GA, TS and SA. There can trace out the CCEF for the mean-variance model concerning military investment assets.

3.1 Genetic Algorithm

Based on the Darwin principle "the fittest survive" in nature, Genetic algorithm was first initiated by Holland's [9] and has rapidly become the best-known evolutionary techniques [10, 11]. Since the pioneering method by Holland, numerous related GA-based portfolio selection approaches have been published. Arnone, Loraschi and Tettamanzi [12] presented a GA for the unconstrained portfolio optimization problem with the risk associated with the portfolio being measured by downside risk. Kyong, Tae and Sungky [13] also used GA to support portfolio optimization for index fund management. Lin and Liu [14] proposed that GA for portfolio selection problems with minimum transaction lots. Recently, GA has attracted much attention in portfolio optimization problems.

In GA, the decision variables of a problem are usually represented by genes. The possible outcomes of a variable are named alleles. Individual solutions are then encoded in a string called a chromosome, which has a finite length over a finite alphabet. The alleles of these genes are often integer values with a range between 0 and 9. These chromosomes then can represent points in the search space of candidate solutions. In order to breed better solutions, each chromosome is evaluated by its fitness which shows how good it is in solving the optimization problem. This fitness may come from an appropriate evaluation or transformation of the objective function of the problem. The higher the fitness of a solution, the higher the chance of being selected for reproduction and hence to contribute to the subsequent generation. Therefore, GA can be interpreted as a method for searching for highly fit chromosomes on a fitness landscape.

A GA process starts with an initial population of a fixed number of solutions and maintains the same size population at each iteration. The new generation is obtained from the current one through a four step procedure. The better individuals from the population are first selected according to their fitness. Then a crossover operator is performed on pairs of them to produce new offspring. In the meantime, a mutation operator is randomly applied to a small proportion of these offspring to increase the variation of solutions. The last step is the replacement of poor individuals in the current population by the children. A GA algorithm repeats this procedure until a predetermined number of generations (or iterations) have been performed.

In a GA process, one should adopt a mechanism to allow the fitter solutions a better chance of being selected for reproduction. The crossover operator can be executed by combining the pieces of chromosomes from the parents, such as exchanging part of their genes or choosing each gene one by one randomly from one of them. Mutation can be achieved by simply altering some of the genes in a chromosome. Both the likelihood of crossover being applied to chromosomes and the chance of mutation at each position of every chromosome are defined by rates, called the crossover probability and the mutation probability. The useful method of how to apply these four components is discussed in next section. Here we show the basic steps of a simple GA:

Generate an initial solution
Evaluate fitness of individuals in the population
Repeat
  Select parents from the population
  Recombine parents to produce children
  Evaluate fitness of the children
  Replace some or all of the population by the children
Until a satisfactory solution has been found

3.2 Genetic Algorithm for Portfolio Optimization

The proposed genetic algorithm for

portfolio optimization problem based on the GA steps discussed in the previous section. This section we will describe in detail how to implement the proposed method.

3.2.1 Population initialization

This paper used a population size of 100. Parents were chosen by binary tournament selection which works by forming two pools of individuals, each consisting of two individuals drawn from the population randomly. The individuals with the best fitness, one taken from each of the two tournament pools, are chosen to be parents.

3.2.2 Fitness objective function evaluation

Using fitness objective function evaluation to try and ensure that the evaluated solution is feasible. Here we used mean-variance objective function $f = \lambda\left[\sum_{i=1}^{N}\sum_{j=1}^{N} w_i w_j \sigma_{ij}\right] - (1-\lambda)\left[\sum_{i=1}^{N} w_i \mu_i\right]$ as a fitness function to calculate the feasible solution in the portfolio optimization problem. The chromosome representation of a solution has two distinct parts, a set Q of K distinct assets and K real numbers s_i $(0 \le s_i \le 1), i \in Q$. Now given a set Q of K assets a fraction $\sum_{j \in Q} \varepsilon_j$ of the total portfolio is already accounted for and so we interpret s_i as relating to the share of the free portfolio proportion $1 - \sum_{j \in Q} \varepsilon_j$ associated with asset $i \in Q$. Hence the proportion associated with asset i in the portfolio is given by $w_i = \varepsilon_i + \left(s_i \Big/ \sum_{j \in Q} s_j \right)\left(1 - \sum_{j \in Q} \varepsilon_j\right)$, i.e. the minimum proportion plus the appropriate share of the free portfolio proportion.

Not all possible solutions correspond to feasible solutions (because of the constraint (equation (7)) relating to the limits on the proportion of an asset that can be held). In GA evaluation we can automatically ensure that the constraints relating to the lower limits ε_i are satisfied in a single algorithmic step. However we need an iterative procedure to ensure that the constraints relating to the upper limits δ_i are satisfied.

3.2.3 Reproduction, Crossover, and Mutation

In this section we describe how the genetic operators are modified and how they performed in our algorithm. Children in our GA are generated by uniform crossover. In uniform crossover two parents have a single child. If an asset i is present in both parents it is present in the child (with an associated value s_i randomly chosen from one or other parent). If an asset i is present in just one parent it has probability 0.5 of being present in the child. Children are also subject to mutation, multiplying by 0.9 or 1.1 (chosen with equal probability) the value $(\varepsilon_i + s_i)$ of a randomly selected asset i. This mutation corresponds to decreasing or increasing this value by 10%.

3.2.4 Replacement

We used a steady-state population replacement strategy. With this strategy each new child is placed in the population as soon as it is generated. We choose to replace the member of the population with the worst objective function value.

3.2.5 Termination criterion

With regard to the number of iterations we used 1000N for GA heuristic. These values mean that the heuristic evaluates exactly 1000N solutions for each value of λ.

3.3 Tabu Search

Compared with genetic algorithm, tabu search is a rather new method. Tabu search was first introduced by Glover [15-17]. The word tabu or taboo has the meaning of being prohibited. As in its title “tabu”, TS imposes restrictions via some flexible memory structures to help the search process avoid local optimum and explore the search space more efficiently. TS have been widely applied to different optimization problems such as nonlinear covering problems, shop scheduling, quadratic assignment and traveling salesman problems. For portfolio optimization problem, Glover, Mulvey and Hoyland [18] applied TS to the problem involving rebalancing a portfolio to maintain a fixed proportion in each asset category. Another financial application is Consiglio and Zenios [19] who implemented TS to optimize a callable bond design. Recently, Aldaihani and Aldeehani [20] proposed a tailored tabu search heuristic algorithm to solve two mathematical models for balancing the trade off between risk and return involved in the portfolio of emerging stock markets.

The main issue in TS is the use of flexible memory structures for utilizing historical information. These memories are implemented with regard to short term and long term components. In the short term scheme, it guides the search to escape from local minima. On the other hand, it diversifies or intensifies the search by means of the long term memory. To construct these memories, some attributes from the solution space are required to attain tabu status for the search process. These attributes can be variable values or certain functions. Solutions containing one or more of these attributes are considered tabu and usually restricted from selection.

Handling tabu status in the short term can be achieved by a recency-based memory structure called a tabu list. It assigns tabu status to the attributes of moves just performed and hold them tabu for a certain period. Thus the search process can avoid repetition of moves. The duration of an attribute remaining tabu (i.e. tabu tenure) is measured in terms of the number of iterations. Usually tabu tenure is a relatively small value (say 7) in terms of the number of search iterations. The implementation of tabu list may guide the search, but it could also be too restrictive to some potential moves which provide a fruitful direction towards the optimal solution. This situation can be avoided by certain aspiration criteria. A move with tabu status is allowed as long as it satisfies these aspiration conditions. The basic steps of a simple TS algorithm are can described as follows:

> Generate an initial solution
> Initiate the tabu status
> **Repeat**
>> Search a set of neighbor solutions of the current solution
>> Evaluate function values of these solutions
>> Apply aspiration criterion
>> Choose the best one among non-tabu solutions
>> Replace the current solution by the best one
>> Update tabu status
>
> **Until** a termination criterion has been met

3.4 Tabu Search for Portfolio Optimization

The tabu search for portfolio optimization problems based on the TS steps discussed in the previous section. This section we will describe in detail how to implement the proposed method.

3.4.1 Initialize feasible solutions

This procedure first randomly generates 1000 solutions. Each of these solutions consisted of a set *Q* of *K* randomly generated distinct assets. Associated with each asset $i \in Q$ was a value s_i randomly generated from [0, 1]. We adopted an algorithm mentioned below to evaluate each of TS solution into a feasible solution. The best solution found was as a starting point.

3.4.2 Fitness objective function evaluation

In our fitness objective function evaluation we used the same solution representation as in our proposed GA, as well as 3.2.2 in order to try and ensure that the evaluated solution was feasible.

3.4.3 Neighborhood structure

The algorithm starts from a feasible initial solution, and moves the current solution to the best neighborhood which is not forbidden. The move operator corresponds to taking all assets present in portfolio of *K* assets and multiplying their values by 0.9 and 1.1. This means that the number of neighbors which we need to evaluate is 2*K*. The tabu list is a matrix of 2*N* integer values which indicates for each of the *N* assets whether a particular move (multiplying by 0.9 or 1.1) is currently tabu or not. From the neighborhood the best solution is chosen to become the new starting solution for the next iteration and the process repeats. The best solution is termed as the local best solution.

3.4.4 Tabu tenure and aspiration criteria

In order to prevent cycling a list of “tabu moves” is employed. It defines a control mechanism to determine restriction condition in search processing. Typically this list prohibits certain moves which would lead to the revisiting of a previously encountered starting solution. This list of tabu moves is updated as the algorithm proceeds so that a move just added to the tabu list is removed from the tabu list after a certain number of iterations (the “tabu tenure”) have passed. It is common to allow tabu moves to be made if they lead to an improved feasible solution which is better the current best solution (an aspiration criterion).

3.4.5 Termination condition

Update tabu status until a termination condition has been met. With regard to the number of iterations we used 500(*N* /*K*) for heuristic TS. These values mean that the heuristic evaluates exactly 1000N solutions for each value of λ .

3.5 Simulated Annealing

Simulated annealing originated in an algorithm to simulate the cooling of material in a heat bath [21] but its use for optimization problems originated with Kirkpatrick, Gelatt and Vecchi [22] and Cerny [23].

The major advantage of SA over classical local search methods is its ability to avoid getting trapped in local minima while searching for a global minimum. SA surveys and descriptions of application can be found in Osman and Laporte [24], Aarts and Lenstra [25] or Crama and Schyns [2].

SA searches a new solution by examining a solution S chosen at random from the neighborhood of the current solution S^* . The movement from solution S^* to S occurring or not depends on both function values $f(S^*)$ and $f(S)$. The selection rule is: (a) if solution S does improve the result (i.e. $f(S) \le f(S^*)$ for a minimization problem), then S replaces S^* becoming the next current solution (i.e. $S^* = S$); (b) otherwise, the selection between S^* and S depends on the acceptance probability P calculated by some probabilistic law. More specifically, the probability that the neighbor solution S replaces the current one is P . On the other hand, solution S^* remains as the current one with the complementary probability $1-P$.

The probability P to accept a deteriorated solution is a Boltzmann-like distribution which is usually used as an analogy of the annealing process. In general, it is opposite to the size of deterioration and decreased with time (i.e. iterations). The mathematical term of this probability can be defined as $e^{-\Delta/T}$, where Δ is the value of objective function deteriorated from the current solution (i.e. $f(S)-f(S^*)$) and T is the current temperature. The use of the probability distribution implies that the solution goes uphill as well as downhill. Thus, SA has the ability to avoid being trapped in a local optimum. Movement towards a deteriorated solution occurs frequently when the temperature is high but becomes less likely if the temperature is low. Hence the temperature should be a non-increasing function in order to explore the solution space in the beginning and to pursue the optimal solution at the end of optimization process.

The acceptance probability should be relatively high, for example 0.9, at the start to allow all possible movements in a neighborhood. Therefore an initial temperature T_0 should also be properly chosen to ensure the probability is high enough to perform a random search. The acceptance of a deteriorated solution becomes more and more selective when temperature is gradually decreased through the optimization process. Eventually the temperature becomes very close to zero and only the improving move is allowed in the process (i.e. similar to decent search). The basic steps of a simple SA algorithm are shown below.

Generate an initial solution
Select an initial temperature and a cooling factor

Repeat

Examine a random neighbor solution of the current solution
Compare function values of both solutions
If improved, then replace the current solution by the neighbor solution
Else, draw a random probability and calculated the acceptance probability

If accepted, then replace the current solution by the neighbor
Else, retain the current solution
Cool the temperature by a specific rule
Until a termination criterion has been met

3.6 Simulated Annealing for Portfolio Optimization

The approach of our SA for portfolio optimization problem is similar to those of TS. We used the same solution representation as mention in section 3.2.2 to ensure that the evaluated solution was feasible. This SA procedure first randomly generates 1000 solutions. Each of these solutions consisted of a set Q of K randomly generated distinct assets. Associated with each asset $i \in Q$ was a value s_i randomly generated from [0, 1] in order to meet the cardinality constraint. The best solution found was used as a starting point.

3.6.1 Cooling schedule

In SA the temperature is reduced over the course of the algorithm according to a "cooling schedule" which specifies the initial temperature and the rate at which temperature decreases. A common cooling schedule is to reduce the temperature T by a constant factor α ($0<\alpha<1$) using $T = \alpha T$ at regular intervals. Therefore, the initial temperature of our SA is derived from the objective value of initial starting solution and α is set equal to 0.95.

3.6.2 Neighborhood structure

The move operator corresponds to taking all assets present in portfolio of K assets and multiplying their values by 0.9 and 1.1. This means that the number of neighbors which can be randomly chosen is $2K$. In our SA heuristic, we did $2N$ iterations at the same temperature.

3.6.3 Acceptance probability

This probability is related to what is known as the "temperature". More precisely, a move that worsens the objective value by Δ is accepted with a probability proportional to $e^{-\Delta/T}$, where T is the current temperature. The higher the temperature T, the higher the probability of accepting the move. Hence this probability decreases as the temperature decreases.

The algorithm terminates after evaluating exactly $1000N$ solutions for each value of λ. This implies that the same number of solutions will be examined in each of the three heuristics.

IV. TEST MILITARY INVESTMENT ASSETS DATA SETS

To test our heuristic algorithms we constructed ten military investment assets by considering options involved in ten different national defense items drawn from national defense report, maneuver exercises and military conference records. Specifically these items are defensive counter-measurement weapon system, armament replenishment, air strike weapon system, sea strike weapon system, ground force weapon system, defensive construction, military training, defensive vehicle, living facility and C^4ISR (Command, Control, Communication, Computer, Intelligence, Surveillance, and Reconnaissance system). Note that, as far as we are aware, there are no standard portfolio selection methods or quantitative analysis guidelines existed in the present decision-making of national defense investment. Through constructing the above ten military investment assets, we may develop a quantitative approach and plan annual national defense investment in a strategic level.

4.1 Annual Investment Budget

In order to find weighted allocation for portfolio selection regarding military investment assets, we surveyed and clustered the historical annual budget data of Ministry of National Defense R.O.C. from fiscal year Y_1 to Y_{16}. These data are associated with ten military investment assets in the following sections.

4.2 Measure of Effectiveness (MOE) on Military Investment Assets

In general, the MOE of a weapon system may contain six factors: strike ability, surveillance ability, mobility, replenishment ability, command and communication ability, and stability as criteria of measure. These factors can affect the decision-making of portfolio optimization for military investment. However, instead of considering all six factors, we only use strike ability to estimate the simplified MOE in this paper due to the national defense confidentiality. A more comprehensive approach on solving the military investment problem is to apply probability theory among these six abilities of MOE according to their definition and principle.

Here, prior to formulating all the MOE in the detail for different military assets, we summarize the notations that are used in these MOE as below.

A_n: is the number of different military investment assets, $n = 1,...,10$.

LIFPM: lock & intercept fires per minute
FFPM: firing fires per minute
FFPMC: firing fires per minute contained
FPMI: fires per minute interference
MOE: weapon system measure of effectiveness

Therefore, the MOE of ten military investment assets can be presented as follows:

A_1: Defensive counter-measurement weapon system (MOE)
= Sum (LIFPM per sq. ft. per vehicle * avg. effective range * annual procurement quantity)

A_2: Armament replenishment (MOE)
= Sum (FFPM per sq. ft. per vehicle * avg. effective range * annual procurement quantity)

A_3: Air strike weapon system (MOE)
= Sum (FFPM per sq. ft. per vehicle * avg. effective range * annual procurement quantity)

A_4: Sea strike weapon system (MOE)
= Sum (LIFPM per sq. ft. per vehicle * avg. effective range * annual procurement quantity)

A_5: Ground force weapon system (MOE)
= Sum (FFPM per sq. ft. per vehicle * avg. effective range * annual procurement quantity)

A_6: Defensive construction (MOE)
=Sum (FFPMC per sq. ft. per facility * avg. effective period * annual construction quantity)

A_7: Military training (MOE)
=Sum (training capacity per class (Combat, Logistic, Strategic) * no. of classes per year)

A_8: Defensive vehicle (MOE)
= Sum (FFPMC per sq. ft. per vehicle * avg. effective range * annual procurement quantity)

A_9: Living facility (MOE)
= Sum (FFPMC per sq. ft. per facility * avg. effective period * annual construction quantity)

A_{10}: C^4ISR (MOE)
= Sum (FPMI per sq. ft. per vehicle * avg. effective range * annual procurement quantity)

Parameters of all these MOE items were selected by referencing national defense consulting firms such as Rand Corporation, public annual military exercise newscasts, Jane's Information Group and experienced combat staffs that are familiar with war game scenario simulation. Since there is no uniform or standard model for this decision-making process, we only choose some major platforms to build the demonstrating data set for our heuristic approaches. For example, we pick three major types of warship on sea strike weapon system, but anyone may also select 2 or 4 platforms due to different scenarios.

4.3 Return of investment (ROI)

The parameter settings for ROI are defined as follow:

ROI = MOE / Budget

The amount of data is a 10*16 square matrix on ten military investment assets. The normalized ROI data set is shown in Table 1. We can acquire an efficient frontier (on risk and return coordinates) of these ten assets if the ROI data are applied to the standard mean-variance model through heuristic algorithms we developed above.

V. RESEARCH RESULTS

To illustrate the portfolio selection problem of the military investment assets and to test the effectiveness of the proposed heuristic algorithms, we construct the ROI data set for sixteen fiscal years and program algorithms in C++ language. All the results are run on a personal computer. The cardinality constraints are set to maintain all the assets in our portfolio with a minimum holding proportion of total budget being 1% for each asset. We describe all the figures and tables before a further discussion of them. The CCEF of mean-variance risk model for military investment assets is presented in Figure 1. It contains graphs of efficient points generated from three heuristic algorithms. Each point also reveals the best allocation of budget under the specific risk and return. Figure 2 is the compared CCEF of mean-variance model for 99 stocks in S&P100 index. In this case we choose daily price data from Jan. 2004 to Dec. 2006 and set the cardinality constraint to be 10. Table 2 shows the computational time (second) in each heuristic method. In order to compare those results in Figure 1 further, we adopt the definition of efficient point (has the higher return for a given risk or the lower risk for a desired return) and pool all heuristic results to form a better distribution of points over the frontier. The numerical analysis of pooled results is then illustrated in Table 3. To explain the influence of risk on decision-making, Table 4 shows the percentages of total budget which should be allocated to 10 military investment assets at a series of risk values. These numerical results are also illustrated in Figure 3 for an integrated display of the shift in asset allocations.

The results of three CCEF are nearly the same if we make a macroscopic comparison in Figure 1. Similar outcomes are presented in Figure 2 which draws CCEF with a different financial data set. These evidences show that our heuristics based GA, TS and SA can solve the defined military investment problem adequately. Note that, although an increase of return will accompany a rise in risk, the shape of efficient frontier in Figure 1 has a significant turn along the curve. A sharp rise in risk follows a steep increase in return after the turn. Therefore, efficient points near the turn may be favorable to decision makers who only consider trade-off between risk and return of a portfolio. The shape of CCEF of military investment assets also varies from that of S&P100 stocks. We believe this variation is due to the characteristic of ROI data. They are annual data with restricted information and more complicated than stock prices. With a larger sample size and further substantial war game data, this curve may become more smoothly.

Obviously every risk value on the CCEF has an associated return value, but risk values always have positive correlation with return values. Since there is no standard criterion for the selection of risk values, it depends on the decision makers' subjective preference. Therefore we select seven different risk values form CCEF to demonstrate the variation of weighted allocations. The investors or decision makers may allocate military investment assets according to the subfigure of a desired risk value in Figure 3. These subfigures are arranged in three rows and the integrated figure of all risk values is arranged on the right of third row. Judging from these figures, one may invest more money on assets A_4, A_6, A_7, A_8 and A_{10} when the risk value is set to be 0.1 (low risk). Among them, assets A_4 and A_{10} excess the others in this portfolio. Both weighted allocations of A_4 and A_8 will increase with the risk value if it rises up to 0.19 (turning point). In the meantime, the allocation in A_{10} will decline rapidly by shifting its share to A_8. Afterward most of asset distributions will concentrate on A_8 till the risk reaches its maximum (0.29). The analysis of weighted allocations also can be made from Table 4 numerically. It is possible to distribute more weight on different assets if we adjust their minimum holding proportions to meet the investor's requirement. This will prevent the investment from concentrating in few military assets.

In Table 2, all the computational times are no more than 47 seconds for our heuristic algorithms. It means that these methods can solve the military investment problem easily with more items. Among them, the SA is the most efficient one followed by the TS and GA.

In order to compare the heuristic results of Figure 1 in detail, we can pool all the efficient points of these methods together to build a new efficient frontier. Not only it will improve the results form individual heuristic, but provide a benchmark of comparison. Table 3 presents, for each of the three heuristics, the number of efficient points that they individually contribute to the pooled set of efficient points. We also record the initial number of efficient points in each heuristic and compare them with the final numbers of points in the pooled efficient frontier. The percentage of points that survive the merge process is shown in the last column for each heuristic. It can be seen that of the 5234 pooled efficient points 3342 (63.85%) are contributed by the SA heuristic, 1881 (35.94%) are contributed by the TS heuristic and only 11 (0.21%) are contributed by the GA heuristic. Therefore, we may suggest that the SA heuristic is the best choice if only one method is allowed to solve this problem. Both its computational capability and performance are the best among three. Unfortunately the efficient points of GA are dominated by those of the two heuristics and almost play no role in the pooled data. Note here that the performance of heuristics may be related to the input data. However, in this case, a pooled efficient frontier of the TS and SA will provide better information to decision makers.

VI. CONCLUSIONS

In this paper, we stress the importance of military investment and preset a new approach to this problem. Ten military investment assets are defined in order to form a mean-variance model that takes the risk of investment into account. Based on the GA, TS and SA, three heuristic algorithms are developed to solve the portfolio optimization problem efficiently. Each method can produce an efficient frontier which contains different solutions with better return and risk values. They provide feasible choices for a decision maker according to his/her preference.

Although GA performs well in many fields, the empirical results show that it may not be the best method in allocating these military investment assets. In general, an examination of heuristic solutions is suggested by pooling all efficient points to form an improved frontier. Solutions of this frontier will be more attractive to decision makers. Furthermore, our research can provide a quantitative analysis which reveals more information for the proposed problem. Its potential of dealing with more complex investment should not be neglected.

The future research will focus on more realistic problems or more efficient algorithms such as:

(a) Include more items in military investment assets to disclose the true value of them. With these data, the asset allocations will be more accurate to decision makers.
(b) Take surveillance ability, mobility, replenishment ability and stability into consideration gradually to measure more practical MOE of weapon systems.
(c) Apply other algorithms to the military investment problems and compare their results with ours. We are interested in one particular method that can outperform the others.

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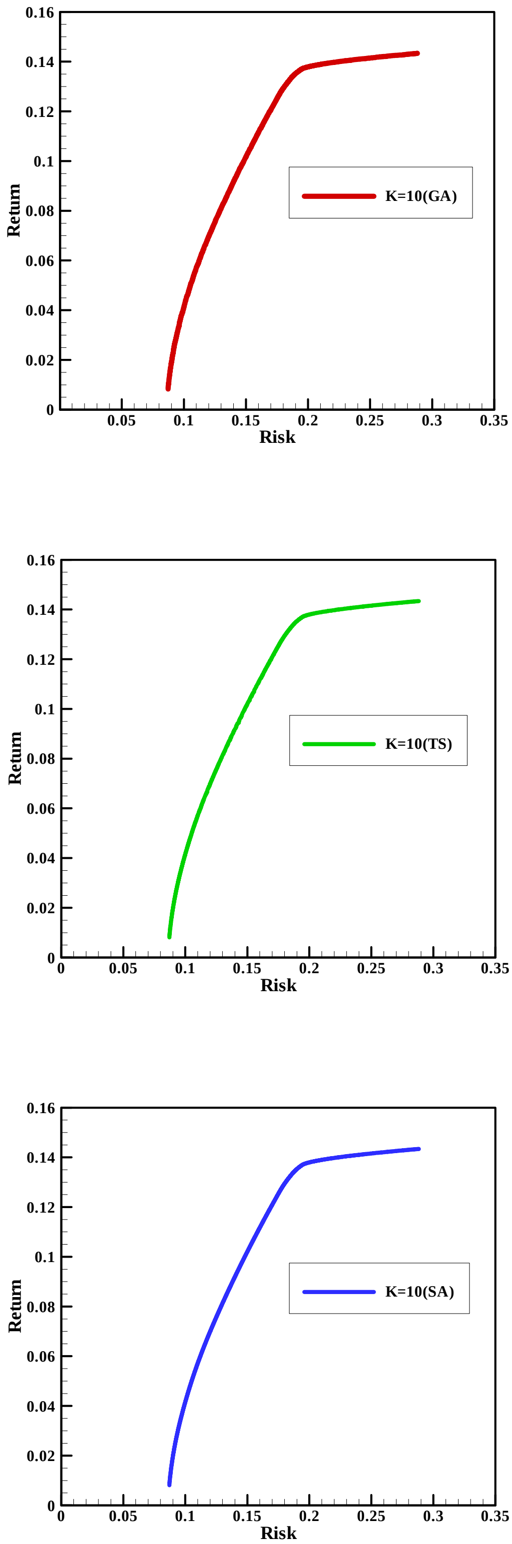

Fig. 1 CCEF of mean-variance model for military investment assets.

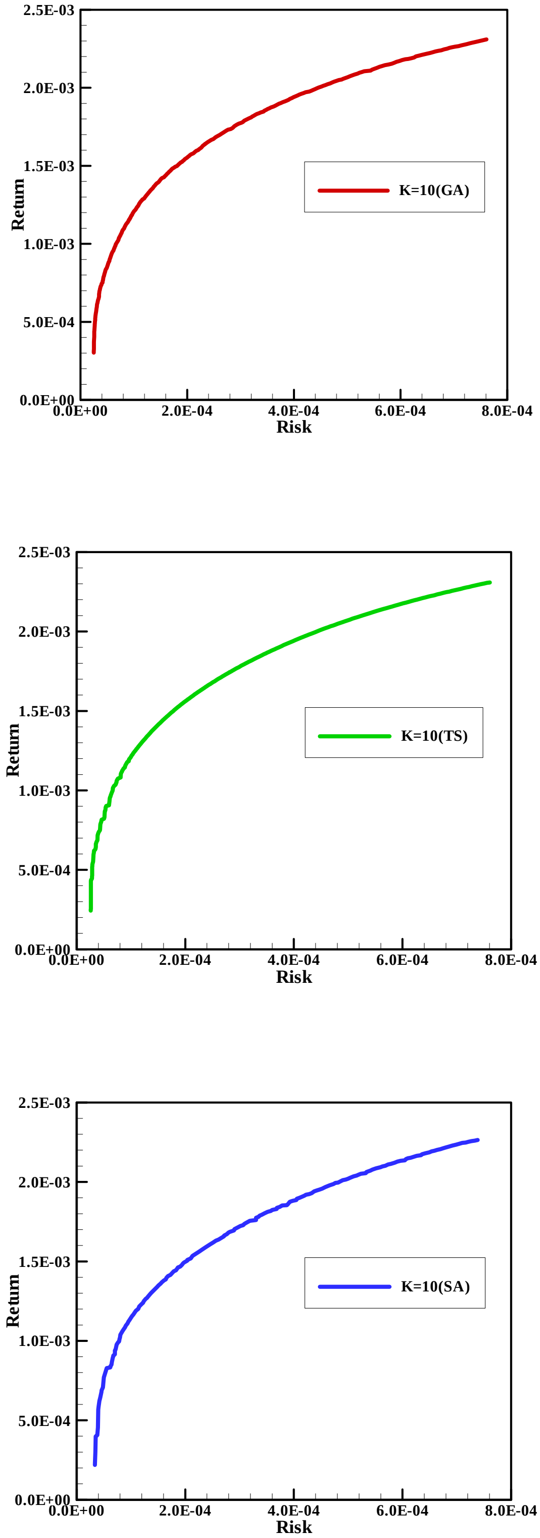

Fig. 2 CCEF of mean-variance model for S&P 100 data.

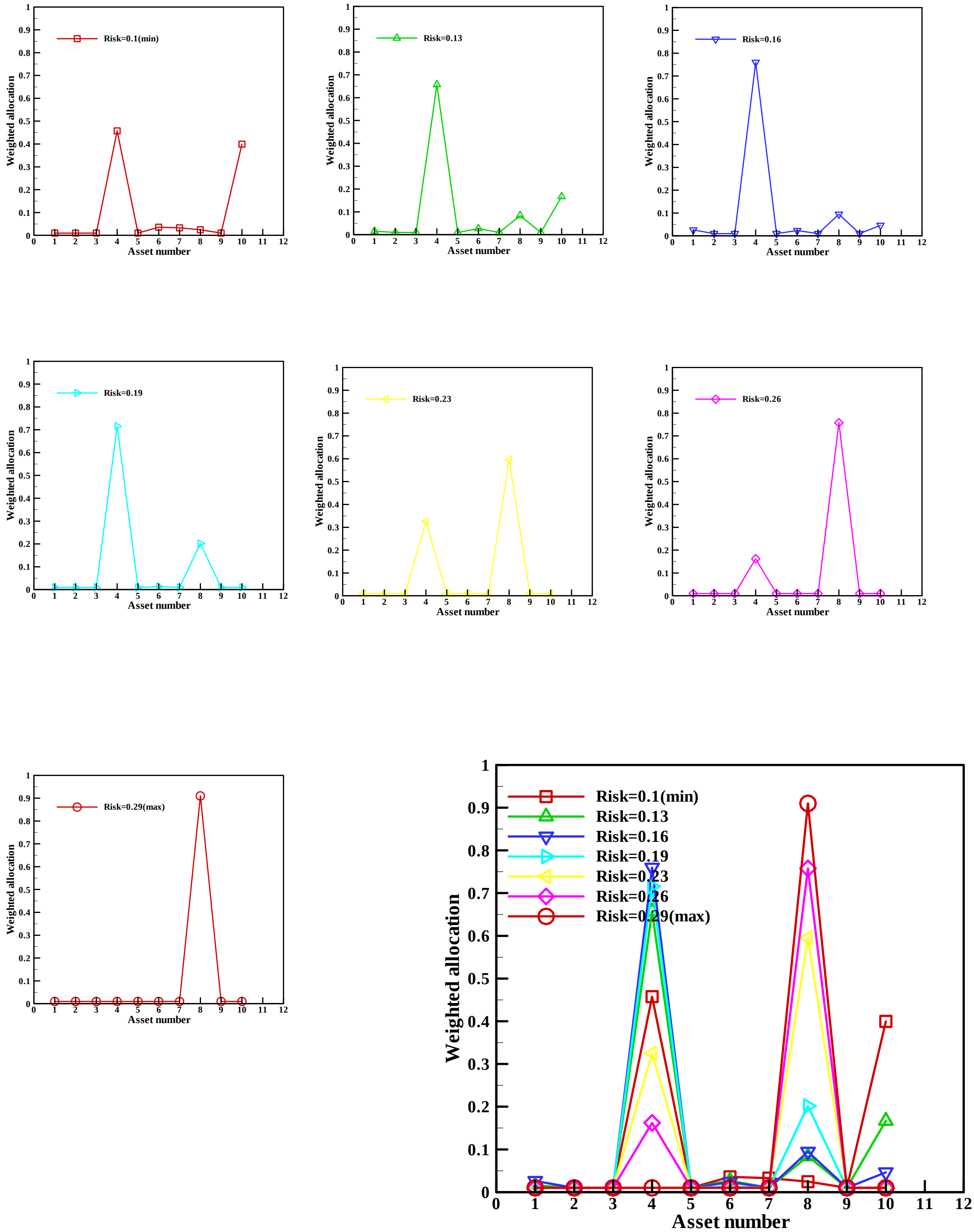

Fig. 3 Weighted allocations at different risk values for military investment assets.

Table 1. Sixteen annual return of investment

Year \ Item	A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8	A_9	A_{10}
Y_1	0.0782	0.0877	0.0718	0.0777	0.0257	0.0615	0.1475	0.0578	0.0090	0.5842
Y_2	0.1147	0.1482	0.0627	0.0875	0.0197	0.2506	0.3415	0.0828	0.0187	0.8873
Y_3	0.2479	0.2310	0.1276	0.1218	0.0743	0.3308	0.1812	0.1671	0.0488	0.8622
Y_4	0.5491	0.0593	0.1707	0.1480	0.1909	0.2108	0.1056	0.2495	0.1496	1.0000
Y_5	1.0000	0.0869	0.2468	0.1857	0.9521	1.0000	0.2577	0.4803	0.2642	0.7846
Y_6	0.3330	0.0931	0.6390	0.2945	1.0000	0.2641	0.3989	0.6554	0.9263	0.3319
Y_7	0.0951	0.1026	0.3896	0.4425	0.0236	0.4987	0.3679	0.4343	0.6311	0.3019
Y_8	0.1045	0.0552	0.4273	0.3726	0.0225	0.0736	0.1740	0.6210	0.8896	0.4804
Y_9	0.3871	0.2582	0.3726	0.3145	0.0899	0.1436	0.3834	0.8423	0.0119	0.4371
Y_{10}	0.0408	0.2218	0.8319	0.5065	0.0980	0.0587	0.6029	1.0000	0.0949	0.4549
Y_{11}	0.0630	0.0849	0.4872	0.5493	0.0152	0.0981	0.4235	0.4792	0.0516	0.5660
Y_{12}	0.1223	0.3468	0.4304	0.6141	0.0096	0.0979	0.5710	0.6448	0.0126	0.4979
Y_{13}	0.2370	0.1522	0.4466	0.3943	0.0478	0.2520	0.5893	0.7935	0.0184	0.4524
Y_{14}	0.0967	0.2047	1.0000	0.7415	0.0659	0.0374	1.0000	0.4302	0.0725	0.4997
Y_{15}	0.0882	0.6468	0.6126	0.8162	0.0096	0.0907	0.5592	0.6871	1.0000	0.6599
Y_{16}	0.1211	1.0000	0.5552	1.0000	0.0093	0.0734	0.5747	0.9519	0.0509	0.5676

Table 2. Computational time (s) of the heuristic algorithms for 10 military investment assets

Time \ Algorithm	Genetic algorithm	Tabu search	Simulated annealing
second	47	42	40

Table 3. Contribution to the pooled efficient frontier in numerical results

Algorithms	Number of points in the initial efficient frontier	Number of points in the pooled efficient frontier	Contribution percentage
Genetic Algorithm	2925	11	0.21%
Tabu Search	3418	1881	35.94%
Simulated Annealing	3420	3342	63.85%
Total		5234	

Table 4. Allocation of budget to 10 military investment assets (%) at different risk values

Risk Values \ Allocation of Budget \ Asset *i*	1	2	3	4	5	6	7	8	9	10
0.10	1.00	1.00	1.00	45.76	1.00	3.57	3.27	2.44	1.00	39.97
0.13	1.57	1.00	1.00	65.70	1.00	2.66	1.00	8.38	1.00	16.69
0.16	2.62	1.00	1.00	75.96	1.00	2.33	1.00	9.46	1.00	4.63
0.19	1.00	1.00	1.00	71.54	1.00	1.31	1.00	20.16	1.00	1.00
0.23	1.00	1.00	1.00	32.50	1.00	1.00	1.00	59.50	1.00	1.00
0.26	1.00	1.00	1.00	16.21	1.00	1.00	1.00	75.79	1.00	1.00
0.29	1.00	1.00	1.00	1.00	1.00	1.00	1.00	91.00	1.00	1.00