Embedding Pyramid Networks into Crossed Cubes

Chia-Ching Wu[1], Kuang-Shyr Wu[2], Ping-Yu Kuei[1], Yang-Hsiung Ko[3], and Chien-Ping Chang[2]*

[1]*Department of Electrical and Electronic Engineering, Chung Cheng Institute of Technology, National Defense University*
[2]*Department of Computer Science and Information Engineering, Ching Yun University*
[3]*Department of Power Vehicle and Systems Engineering, Chung Cheng Institute of Technology, National Defense University*

ABSTRACT

The pyramid network is one of the important architectures in parallel and distributed computer systems. Some topological properties of pyramid network have been developed to appear literatures. Then, an n-dimensional crossed cube, is a variation of the hypercube. Like the ordinary hypercube, the n-dimensional crossed cube is a regular graph with 2^n nodes and $n2^{n-1}$ edges. The diameter and wide diameter of the crossed cube is approximately half that of the ordinary hypercube. These advantages of the crossed cube motivated the study of how well it can simulate other networks such as the pyramid networks. In this paper, we show that the pyramid networks can be embedded in crossed cubes with dilation 2.

Keywords: crossed cubes, pyramid networks, hypercubes, parallel computing

金字塔網路嵌入交錯式立方體之研究

吳家慶[1]　吳匡時[2]　桂平宇[1]　葛揚雄[3]　張劍平[2]*

[1]國防大學理工學院電機電子學系
[2]清雲科技大學資訊工程系
[3]國防大學理工學院動力及系統工程系

摘　　要

金字塔網路(Pyramid Network)結構在平行處理網路系統上具有重要的地位，部分金字塔網路的拓樸特性已在文獻中探討。一個 n 維交錯式立方體(Crossed Cube)是超立方體(Hypercube)的一種變形，如同超立方體一樣，交錯式立方體是一個規則圖形具有 2^n 結點及 $n2^{n-1}$ 條連線。它具備一些很吸引人的拓樸特性，其中直徑(Diameter)、寬度直徑(Wide Diameter)長度大約是超立方體相對應拓樸特性的一半。這些特性，引發我們探討其他網路系統可嵌入交錯式立方體之動機。本論文中，我們證明金字塔網路(Pyramid networks)能嵌入交錯式立方體結構內且其擴張數值(Dilation)為 2。

關鍵詞：交錯式立方體，金字塔網路，超立方體，平行計算

文稿收件日期 100.5.6.；文稿修正後接受日期 100.9.6.；*通訊作者
Manuscript received May 6, 2011.; revised September 6, 2011.; * Corresponding author

Ⅰ. INTRODUCTION

In parallel and distributed computer systems, the simulation of one topology by another and the architecture of computations in a network of processors are key issue. The problem of simulation one network by another is modeled as a graph embedding problem where the set of nodes represents the processors and the set of edges represents communication links between processors. The pyramid networks is one of the important networks applied in parallel and distributed computer systems, these have been designed and constructed for specific tasks, and have traditionally been applied for image processing, parallel and networks computing [1, 2]. Certain special mapping functions enable networks to be embedded within other networks [3-8]. Several advanced algorithm that naturally lend themselves to the pyramid structure are examined by Stout [7]. Paths (linear arrays) and cycles (rings) are the simplest networks often adopted in parallel computing and local area networks. Paths and cycles are appropriate for developing simple algorithms.

The topological structure of an interconnection network can be modeled by a graph [9]. The vertices and edges of a graph respectively correspond to nodes and links of an interconnection network. It is practical importance to study the problem of how to embed a pyramid network into a host graph because it mean that a pyramid parallel algorithms can be executed on this network efficiently and in parallel and, hence, the system resources can be fully utilized. To our knowledge, the problem of how to embed a pyramid network into a crossed cube has yet to be investigated. The embedding of pyramid networks into crossed cubes is studied in this paper. We will prove that there exists a constant dilation of the pyramid networks into the crossed cubes, the pyramid nodes are mapped to every node of crossed cube.

The rest of this paper is organized as follows. Section 2 summarizes some known results on the pyramid networks and introduces notation used in this paper. In Section 3, we prove our result. In the final section, we give our concluding remarks.

Ⅱ. PRELIMINARIES AND NOTATIONS

A pyramid network is a hierarchical structure based on mesh networks. A mesh network (also called a mesh) is defined by the Cartesian product $P_m \times P_n$ denoted by $M(m,n)$, where P_m and P_n are undirected paths with m and n nodes, respectively. The mesh node set is represented as $V(M(m,n)) = \{(x,y) \mid 1 \le x \le m, 1 \le y \le n\}$, and the mesh link set is represented as $E(M(m,n)) = \{(x_1,y_1),(x_2,y_2)$, (x_1,y_1) and (x_2,y_2) in $V(M(m,n)) \mid |x_1 - x_2| + |y_1 - y_2| = 1$ and $x_1 = x_2$ or $y_1 = y_2\}$. The maximum and minimum degrees of a mesh are 4 and 2, respectively.

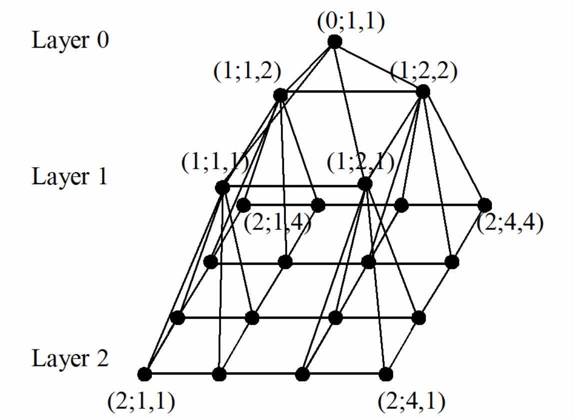

Fig. 1. The pyramid network PM[2].

Definition 1. The n-layer pyramid network is denoted by $PM[n]$. All nodes in i-th layer of $PM[n]$ form a $M(2^i,2^i)$, where $0 \le i \le n$. The node set of $PM[n]$ is $V(PM[n]) = \{(k;x,y) \mid 0 \le k \le n, 1 \le x \le 2^k, 1 \le y \le 2^k\}$. The link $(k_1;x_1,y_1)$ $(k_2;x_2,y_2)$ of $PM[n]$ satisfies one of the following statements [1]:

(1) $k_1 = k_2$, $|x_1 - x_2| + |y_1 - y_2| = 1$, where $x_1 = x_2$ or $y_1 = y_2$,

(2) $k_2 = k_1 + 1$, $x_1 = \lfloor x_2/2 \rfloor$, $y_1 = \lfloor y_2/2 \rfloor$.

The link satisfying (1) is called a mesh-link, and satisfying (2) is called a layer-link, let $(k_1;x_1,y_1)$ and $(k_2;x_2,y_2)$ denoted the two end nodes of a layer-link in $PM[n]$. Then,

$(k_1;x_1,y_1)$ is the parent of $(k_1+1;x_2,y_2)$ and $(k_1+1;x_2,y_2)$ is a child of $(k_1;x_1,y_1)$. An internal mesh-link (u,v) is such that nodes u and v have a common parent. Otherwise, the rest of the mesh-links are external mesh-links. Figure 1 shows the structure of $PM[2]$, some of the nodes labeled are omitted for clarity.

Furthermore, $PM[n]$ can be considered as a 4-ary rooted tree, in which every node in the same layer is connected as a mesh [1]. Thus, we have the following topological properties of $PM[n]$.

(1)The apex (0;1,1) is connected to its four children. The degree of the apex is 4.

(2)Every node at layer 1 to layer $n-1$ is connected with one layer-link to its parent, with at most four mesh-links to its neighbors at the same layer, and with four layer-links to its children. In this case, the maximum and minimum degrees are 9 and 7, respectively.

(3)Every node at layer n is connected with one layer-link to its parent, and with at most four mesh-links to its neighbor at the same layer. In this case, the maximum and minimum degrees are 5 and 3, respectively.

(4) $|V(PM[n])| = \dfrac{4^{n+1}-1}{3}$ and $|E(PM[n])| = 4^{n+1} - 2^{n+2}$.

The topology of a network is usually denoted by a graph $G(V,E)$, where nodes represent processors and edges represent links between processors. The embedding G into H means that the guest graph G maps each node into a unique node in the host graph H. Let $V(G)$ and $E(G)$ denote the node sets and the edge sets of a graph G. When H is connected, each edge of G is mapped to a path of H. The dilation of the mapping is the maximum distance between the images of any pair of nodes in the host graph that are adjacent in the guest graph. In this paper, we address the problem of embedding pyramid networks into crossed cubes. The research reported here is motivated by the importance of the crossed cube as a host graph and the pyramid network as a guest graph.

The crossed cube CQ_n has been studied extensively in recent years because the crossed cube is a variant of hypercube and it has many attractive properties [10-16] such as diameter, wide diameter and fault diameter, conditional fault diameter are approximately half of those of the hypercube [10, 11]. The power of the crossed cube simulates trees, paths and cycles are stronger than those of the hypercube [10, 14]. Furthermore, the connectivity, diagnosability, bisection width are equal to those of the hypercube. The embedding pyramid network into crossed cube is studied in this work. The n-dimensional crossed cube is a regular graph with 2^n nodes and $n2^{n-1}$ edges. Let x and y be two nodes, we use $d(x,y)$ to denote the *distance* between x and y in G. To define crossed cubes, we first introduce the notation of "pair related". The notation is $x_1x_0 \sim y_1y_0$. Let $T=\{(0,0),(0,1),(1,1),(1,0)\}$. Two binary strings $x=x_1x_0$ and $y=y_1y_0$ are *pair related* if and only if $(x,y)\in T$.

Definition 2. The n-dimensional crossed cube is a complete graph with two nodes labeled by 0 and 1, respectively. CQ_n consists of two identical subcubes CQ_{n-1}^0 and CQ_{n-1}^1. The node $u=0u_{n-2}...u_0 \in V(CQ_{n-1}^0)$ and $v=1v_{n-2}...v_0 \in V(CQ_{n-1}^1)$ is an edge in CQ_n if and only if

(1) $u_{n-2}=v_{n-2}$ if n is even, and

(2) $(u_{2i+1}u_{2i}, v_{2i+1}v_{2i}) \in T$ for $0 \le i < \lceil (n-1)/2 \rceil$.

Figure 2 shows CQ_3 and CQ_4. For $k<n$, the k-prefix of u, $P_k(u)$, is defined as $u_{n-1}u_{n-2}\cdots u_{n-k}$. Each node in CQ_n can thus write $u=u_{n-1}\cdots u_0 = p_k(u)u_{n-k-1}\cdots u_0$.

Let x be an l-bit string with $l \le n$. $CQ_n(x)$ denote the subgraph of CQ_n induced by the set of nodes with prefix x. It was shown in [12] that $CQ_n(x)$ is isomorphic to $CQ_{n-|x|}$. Let x and y be two distinct l-bit strings with $l<n$. If $CQ_n(x)$ and $CQ_n(y)$ can be joined by an edge in CQ_n, then $CQ_n(x)$ and $CQ_n(y)$ are called *adjacent* subgraphs of CQ_n. Let $CQ_n(x,y)$ denote the subgraph of CQ_n induced by $CQ_n(x)\cup CQ_n(y)$. It was proven in [5] that $CQ_n(x,y)$ is isomorphic to $CQ_{n-|x|+1}$ if $CQ_n(x)$ and $CQ_n(y)$ are adjacent subgraphs of CQ_n.

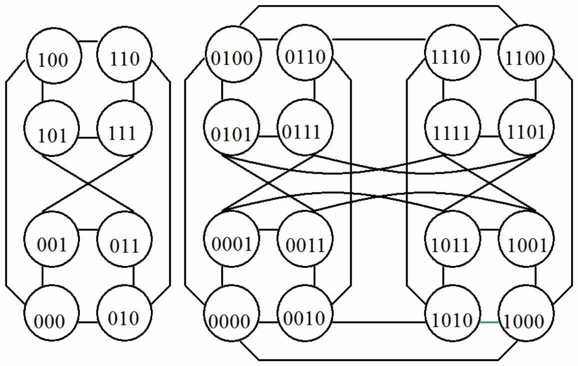

Fig. 2. CQ_3 and CQ_4.

Definition 3. Let (u,v) be an edge of CQ_n. When nodes u and v have a leftmost differing bit at position j, we say that v is the j-neighbor of u and the edge (u,v) is an edge of dimension j.

The $N_i(u)$ represents the i-neighbor of u. For example, let $u = 10101$. $N_4(10101)$, $N_3(10101)$, $N_2(10101)$, $N_1(10101)$, $N_0(10101)$ (i.e., the 4-, 3-, 2-, 1-, and 0-neighbors of u) are given by 01111, 11111, 10011, 10111, and 10100, respectively.

Ⅲ. EMBEDDING PYRAMID NETWORKS INTO CROSSED CUBES

In this section, we will show that crossed cubes can efficiently execute pyramid algorithm. At this point, we are able to simulate pyramid networks by specifying certain nodes of a crossed cube to map all the node of a pyramid network. We present that to embed an *k*-dimensional pyramid network into the crossed cube, the minimum dimension of the crossed cube are needed in following lemma.

Lemma 1. *Any embedding of k-dimensional pyramid network into a crossed cube requires at least an* $(2k+1)$ *-dimensional crossed cube for* $k \geq 1$.

Proof: An $PM[k]$ have $\frac{4^{k+1}-1}{3}$ nodes, then $|V(CQ_{2k})| = 2^{2k} < \frac{4^{k+1}-1}{3} < 2^{2k+1} = |V(CQ_{2k+1})|$. As the result, the nodes of CQ_{2k+1} are enough mapping all nodes of *k*-dimensional pyramid network.

For example, the $|V(PM[2])| = 21$ nodes at least require $|V(CQ_5)| = 32$ nodes to map it.

Lemma 2. *Any embedding of k-dimensional pyramid network into a crossed cube must have maximum dilation greater than one for any integer* $k \geq 1$.

Proof: The CQ_3 and CQ_4 are shown in figure 2, respectively. Because the crossed cube contains no triangle (i.e., cycle with length 3) in graph, there must be path with length 2 for a pair of nodes in this triangle. Thus, we have the dilation of crossed cubes greater one. □

For the next level up of pyramid networks, notice that these nodes are parents of the apexes of subpyramids in 4 nodes, where the 4 nodes form a square. One of these nodes will also do the work of simulating the node at the next level. We use a top-down approach to recursively construct this embedding, we present a method to connect the four distinct nodes at layer $m+1$ from each node at layer m as follows.

Definition 4. The embedding pyramid network into crossed cube is defined recursively as follows: Let u be a node of crossed cubes which mapping at layer m in $PM[n]$, D be a set of nodes of crossed cubes which mapping at layer $m+1$ in $PM[n]$, where $0 \leq m < n$. There exists four layer-links from u to a node set $v \in D$ and four internal mesh-links between nodes in D at layer $m+1$.

The nodes $v \in D$ in pyramid network are constructed as follows:

1. layer-link：

$$\begin{cases} (u, N_{2(n-m)-2}(u)), \\ (u, N_{2(n-m)-1}(N_{2(n-m)-2}(u))), \\ (u, N_{2(n-m)-2}(N_{2(n-m)}(u))), \\ (u, N_{2(n-m)}(N_{2(n-m)-2}(u))). \end{cases} \quad (1)$$

2. mesh-link：

$$\begin{cases} (N_{2(n-m)-2}(u), N_{2(n-m)-1}(N_{2(n-m)-2}(u))), \\ (N_{2(n-m)-1}(N_{2(n-m)-2}(u)), N_{2(n-m)-2}(N_{2(n-m)}(u))), \\ (N_{2(n-m)-2}(N_{2(n-m)}(u)), N_{2(n-m)}(N_{2(n-m)-2}(u))), \\ (N_{2(n-m)}(N_{2(n-m)-2}(u)), N_{2(n-m)-2}(u)). \end{cases} \quad (2)$$

By definition 4, we have established the connection method to embed the pyramid networks into crossed cubes. For example, let u=000 be a node of CQ_3 mapping at layer 0 in $PM[1]$. We use four layer-links to construct a set of nodes $v \in D$ in CQ_3 mapping at layer 1 in $PM[1]$. The four nodes are $N_{2(1-0)-2}(000) = 001$, $N_{2(1-0)-1}(N_{2(1-0)-2}(000)) = N_1(001) = 011$, $N_{2(1-0)-2}(N_{2(1-0)}(000)) = N_0(100) = 101$, and $N_{2(1-0)}(N_{2(1-0)-2}(000)) = N_2(001) = 111$.

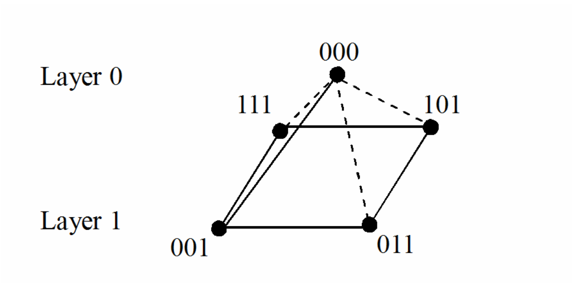

Fig. 3. Embedding PM[1] into CQ_3.

Thus, the layer-link from u to $v \in D$ are (000,001), (000,011), (000,101), and (000,111), and the distance between these are 1, 2, 2, and 2, respectively. The mesh-link in the node set $v \in D$ are (001,011), (011,101), (101,111), and (111,001), and the distance between each internal mesh-link are one. The topology of a $PM[1]$ is shown in figure 3, in which the dashed lines represent links of dilation 2 and the solid lines represent links of dilation 1, respectively.

It is clear that four nodes of $PM[m+1]$ are adjacent along the dimension of $PM[m]$ from the recursive definition of our proposed method, where $0 \le m < n$. To make some insightful observations that can help us devise a layer-link method that works out pyramid networks in a crossed cube, we will show that it can be directly used to generate a pyramid network for the given permutation.

Theorem 1. *The* $PM[n]$ *can be embedded in a crossed cube* CQ_{2n+1} *with dilation 2 for* $0 \le n < 2$.

Proof: For $n = 0$, the proof is trivial. The base case corresponds to the embedding of $PM[0]$ into CQ_1.

Now, let us check that the theorem holds for $n = 1$. By Lemma 1, the $PM[1]$ can be embedded in CQ_3. Every node of the pyramid networks at layer m to layer $m+1$ is connected with four layer-links to its children. Therefore, let $x = x_2x_1x_0$ be a node at layer $m = 0$, the relative positions of x at layer 0 and its neighbor set $y_i \in D = \{ x_t x_{t-1}\overline{x_{t-2}}, x_t\overline{x_{t-1}x_{t-2}}, \overline{x_t}x_{t-1}\overline{x_{t-2}}, \overline{x_t x_{t-1} x_{t-2}} \}$ at layer 1 are $x_2x_1\overline{x_0}$, $x_2\overline{x_1x_0}$, $\overline{x_2}x_1\overline{x_0}$, and $\overline{x_2x_1x_0}$ for $1 \le i \le 4$, where $t = 2(n-m) = 2$.

Since the most significant differing double bit in D are x_2x_1, $x_2\overline{x_1}$, $\overline{x_2}x_1$, and $\overline{x_2x_1}$, there are four distinct nodes. The distance of layer-links are 1, 2, 2, and 2, respectively. Furthermore, by Definition 4, the distance of mesh-links are 1, 1, 1 and 1, respectively. Consequently, the dilation of the maximum distance between the images of any pair of nodes in CQ_3 that are adjacent in $PM[1]$ is two. Thus, the $PM[1]$ can be embedded in CQ_3 with dilation 2. □

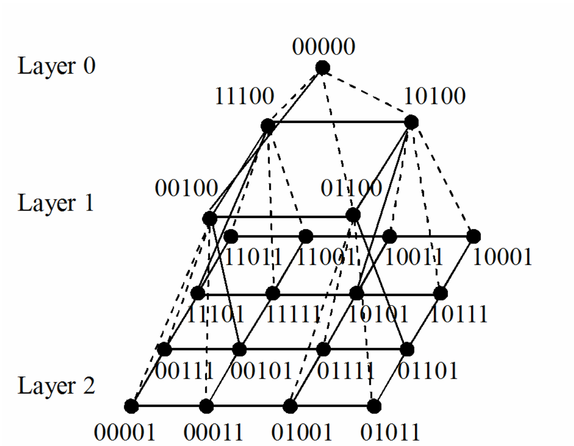

Fig. 4. Embedding PM[2] into CQ_5.

To illustrate theorem 2 for $n = 2$, for example, the relative positions of u at layer 0 and its connected a node set $v \in D$ at layer 1 are formally stated in definition 4. Let u=00000 be a node at layer 0 and its connected $v \in D$ at layer 1 are 00100, 01100, 10100 and 11100, the maximum distance between u and $v \in D$ is two. One of the nodes 00100 at layer 1 connects four nodes 00101, 00111, 00001 and 00011 at layer 2, and the others node sets at layer 2 are recursively constructed as definition 4.

The topology of a $PM[2]$ is shown in figure 4, in which the dashed lines represent links of dilation 2 and the solid lines represent links of dilation 1, respectively. Note that all of the nodes in figure 4 are distinct nodes. As a result, the $PM[2]$ can be embedded in CQ_5. It should be understood that the arrangement of nodes labeled and layer-links in figure 4, in which the binary strings represent the nodes of crossed cube and the lines represent the links of pyramid network, respectively. Figure 4 shows the topology of an embedding, after having embedded the top $PM[m]$ -subpyramid of $PM[m+1]$. We use a top-down approach to recursively construct the embedding as follows.

Theorem 2. *The $PM[n]$ can be embedded in a crossed cube CQ_{2n+1} with dilation 2 for any integer $n \geq 2$.*

Proof: For $1 \leq k \leq n-1$, let $x = x_{2n} \ldots x_1 x_0$ be a node at the (*k*-1)-th layer. It connects to a set of four nodes $y_i \in D = \{p_{2(k-1)}(x)x_t x_{t-1}\overline{x_{t-2}} \ldots x_0$, $p_{2(k-1)}(x)x_t \overline{x_{t-1}}\,\overline{x_{t-2}} \ldots x_0$, $p_{2(k-1)}(x)\overline{x_t} x_{t-1}\overline{x_{t-2}} \ldots x_0$, $p_{2(k-1)}(x)\overline{x_t}\,\overline{x_{t-1}}\,\overline{x_{t-2}} \ldots x_0\}$ at layer k for $1 \leq i \leq 4$, where $t = 2(n-k+1)$. Note that the formulation of binary string for each node should follow the notation of pair related. On the other hand, four nodes in D at layer k are distinct nodes whose most significant differing double bit are distinguished $x_t x_{t-1}$, $x_t \overline{x_{t-1}}$, $\overline{x_t} x_{t-1}$, and $\overline{x_t}\,\overline{x_{t-1}}$. It is easy to see that one node at layer $k-1$ and four nodes in D at layer k are distinct nodes. The maximum distance between x and nodes $y_i \in D$ is two as shown in the proof of Theorem 1.

Each node in set $y_i \in D$ at layer k is connected by a node set D_i at layer $k+1$ for $1 \leq i \leq 4$. The node $y_1 = p_{2(k-1)}(x)x_t x_{t-1}\overline{x_{t-2}} x_{t-3} x_{t-4} \ldots x_0$ in D at layer k links a node set

$D_1 = \{p_{2(k-1)}(x)x_t x_{t-1}\overline{x_{t-2}} x_{t-3}\overline{x_{t-4}} \ldots x_0$,

$p_{2(k-1)}(x)x_t x_{t-1}\overline{x_{t-2}}\,\overline{x_{t-3}}\,\overline{x_{t-4}} \ldots x_0$,

$p_{2(k-1)}(x)x_t x_{t-1} x_{t-2} x_{t-3}\overline{x_{t-4}} \ldots x_0$,

$p_{2(k-1)}(x)x_t x_{t-1} x_{t-2}\overline{x_{t-3}}\,\overline{x_{t-4}} \ldots x_0\}$ at layer $k+1$.

Obviously, y_1 at layer k and D_1 at layer $k+1$ are distinct nodes whose the most significant differing double bit are different form $x_{t-2}x_{t-3}$, $x_{t-2}\overline{x_{t-3}}$, $\overline{x_{t-2}} x_{t-3}$, and $\overline{x_{t-2}}\,\overline{x_{t-3}}$, note that $x = p_{2(k-1)}(x)x_t x_{t-1} x_{t-2} x_{t-3} x_{t-4} \ldots x_0$ at layer $k-1$ and the node $p_{2(k-1)}(x)x_t x_{t-1} x_{t-2} x_{t-3}\overline{x_{t-4}} \ldots x_0$ at $k+1$ has most significant differing bit at bit $(t-4)$. Therefore, all the nodes, x at layer $k-1$, y_1 in D at layer k, and all the nodes $z_g \in D_1$ at layer $k+1$ are distinct nodes for $1 \leq g \leq 4$. Similarly, $y_2 = p_{2(k-1)}(x)x_t \overline{x_{t-1}}\,\overline{x_{t-2}} x_{t-3} x_{t-4} \ldots x_0$ in D at layer k links a node set $D_2 = \{$

$p_{2(k-1)}(x)x_t \overline{x_{t-1}}\,\overline{x_{t-2}} x_{t-3}\overline{x_{t-4}} \ldots x_0$,

$p_{2(k-1)}(x)x_t \overline{x_{t-1}}\,\overline{x_{t-2}}\,\overline{x_{t-3}}\,\overline{x_{t-4}} \ldots x_0$,

$p_{2(k-1)}(x)x_t \overline{x_{t-1}} x_{t-2} x_{t-3}\overline{x_{t-4}} \ldots x_0$,

$p_{2(k-1)}(x)x_t \overline{x_{t-1}} x_{t-2}\overline{x_{t-3}}\,\overline{x_{t-4}} \ldots x_0\}$ at layer $k+1$. Obviously, y_2 in D at layer k and the nodes $z_h \in D_2$ at layer $k+1$ are distinct nodes whose the most significant differing double bit are different form $x_{t-2}x_{t-3}$, $x_{t-2}\overline{x_{t-3}}$, $\overline{x_{t-2}} x_{t-3}$, and $\overline{x_{t-2}}\,\overline{x_{t-3}}$ for $1 \leq h \leq 4$. Also, node x at layer $k-1$ is not a node $z_h \in D_2$ at layer $k+1$ with different bit at bit $(t-4)$. Therefore, node x at layer $k-1$, y_2 in D at layer k, and all the nodes $z_h \in D_2$ at layer $k+1$ are different from each other.

Similarly, for $1 \leq p \leq 4$ and $1 \leq q \leq 4$, using the layer-link method of our proposed approach can obtain x, $y_3 = p_{2(k-1)}(x)\overline{x_t} x_{t-1}\overline{x_{t-2}} x_{t-3} x_{t-4} \ldots x_0$, and $z_p \in D_3$, and x, $y_4 = p_{2(k-1)}(x)\overline{x_t}\,\overline{x_{t-1}}\,\overline{x_{t-2}} x_{t-3} x_{t-4} \ldots x_0$, and $z_q \in D_4$ are distinct nodes, respectively. Obviously, the node sets D_1, D_2, D_3, and D_4 are distinguished different from each other at the most significant differing double bit $x_t x_{t-1}$, $x_t \overline{x_{t-1}}$, $\overline{x_t} x_{t-1}$, and $\overline{x_t}\,\overline{x_{t-1}}$ in D at layer k, respectively. Each node in set D_i at layer $k+1$ are distinct nodes whose most significant differing double bit are distinguished $x_{t-2}x_{t-3}$, $x_{t-2}\overline{x_{t-3}}$, $\overline{x_{t-2}} x_{t-3}$, and $\overline{x_{t-2}}\,\overline{x_{t-3}}$.

Table 1. Embedding pyramid network PM[3] into crossed cube CQ_7

Layer 0	Layer 1	Layer 2	Layer 3			
0000000	0010000	0010100	0010101	0010111	0010001	0010011
		0001100	0001101	0001111	0001001	0001011
		0011100	0011101	0011111	0011001	0011011
		0000100	0000101	0000111	0000001	0000011
	0110000	0110100	0110101	0110111	0110001	0110011
		0111100	0111101	0111111	0111001	0111011
		0100100	0100101	0100111	0100001	0100011
		0101100	0101101	0101111	0101001	0101011
	1010000	1010100	1010101	1010111	1010001	1010011
		1011100	1011101	1011111	1011001	1011011
		1000100	1000101	1000111	1000001	1000011
		1001100	1001101	1001111	1001001	1001011
	1110000	1110100	1110101	1110111	1110001	1110011
		1111100	1111101	1111111	1111001	1111011
		1100100	1100101	1100111	1100001	1100011
		1101100	1101101	1101111	1101001	1101011

Consequently, all the nodes in $z_j \in D_i$ at layer $k+1$ are distinct nodes for $1 \le i \le 4$ and $1 \le j \le 16$. Since the maximum distance between $y_i \in D$ and its connected nodes $z_j \in D_i$ is two, the dilation is two. Thus, the pyramid networks $PM[n]$ can be embedded in CQ_{2n+1} with dilation 2. Hence, the proof is completed. □

We have proved that the pyramid networks can be embedded in its optimum crossed cubes with dilation 2. The proposed embedding algorithm can be used in pyramid networks applications.

An interconnection network plays a critical role of a multi-computer because the system performance is deeply dependent on network latency and throughput. When evaluating an interconnection network, one major concern is its graph embedding ability, which reflects how efficiently a parallel algorithm with structured task graph (guest graph) can be executed on this network (host graph). Table 1 is an example of embedding the pyramid network $PM[3]$ into the crossed cube CQ_7, the under line mark the most significant differing double bit at its adjacency from parent layer.

Ⅳ. CONCLUSIONS

Embedding of pyramid networks into crossed cubes is studied in this paper. The performance of an embedding can be measured by dilation. The smaller the dilation of an embedding is, the shorter the communication delay that the host graph simulates the guest graph. The embedding method can be used in many applications, such as supporting communication requirements in enhancing processor utilization and reducing communication time.

In this paper, we propose a method to embed pyramid networks into crossed cubes. We prove that a pyramid network $PM[n]$ can be embedded in a crossed cube CQ_{2n+1} with dilation 2 and a one-to-one mapping maximum dilation of two is possible. Exploiting this network for different parallel computing algorithms and their structure is another interesting area of future research.

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