Combined Arms System Dynamics Model for Modern Land Battle

Pei-Leen Liu[1], Huai-Ku Sun[2], and Yue-Tarng You[2*]

[1] *The Institute of Resources Management and Decision Science, Management College, National Defense University*
[2] *Department of Defense Science, Chung Cheng Institute of Technology, National Defense University*

ABSTRACT

This study first combines Lanchester mixed forces model with system dynamics to develop a model that is able to deal with modern combined arms land battle. By conceptualizing the understanding gained from experience, a system dynamics model for modern combined arms land battle is developed. To verify and demonstrate the model developed here, two strategies are employed to compare with the latest Lanchester mixed forces model and five common tactic scenarios are adopted to demonstrate how different tactics and weapons affect the force strength and how combined arms produces synergy. The results verify that the model developed here is more reasonable and effective than the latest Lanchester mixed forces model and better applies to evaluate modern combined arms land battle. In addition, the results show that it is possible to reserve inferior situation by managing adaptation and provide new thinking for a commander to manage adaptation by quantitative analysis rather than subjective decision.

Keywords: system dynamics, combined arms, land battle

現代聯合兵種地面作戰系統動態模型

劉培林 [1]　孫懷谷 [2]　游玉堂 [2*]

[1] 國防大學管理學院資源管理及決策研究所
[2] 國防大學理工學院國防科學研究所

摘　　要

本研究首創結合藍徹斯特混合兵力方程式與系統動態學，將來自經驗的理解概念化，再以系統動態學逐步發展的程序，發展現代聯合兵種地面作戰模型。為驗證與展示所發展的模型，本研究採用兩個策略比較最近發表的藍徹斯特混合兵力模型，並採用五個常用的戰術情境，展示不同戰術和武器對兵力強度的影響和聯合兵種所產生綜效。模擬結果證實本研究發展的聯合兵種地面作戰系統動態模型較最新發表的戰鬥模型更合理且有效率。此外，結果也顯示以權變的戰術作為可扭轉劣勢，提供作戰指揮官以量化分析輔助決策的新思維。

關鍵詞：系統動態學，聯合兵種，地面作戰

文稿收件日期 100.7.13;文稿修正後接受日期 100.12.16; *通訊作者
Manuscript received July 13, 2011; revised December 16, 2011; * Corresponding author

I. INTRODUCTION

In 1916, Lanchester first proposed a set of differential equations to describe the time dependence attrition [1]. After decades, it has been expended to a (n, m) mixed forces model [2]. In the meantime, system dynamics was employed to model the Lanchester model in 1981 [3]. Since then, it was applied to many fields, such as management of aircraft survivability [4]; creation of a feedback perspective for army situations and provision of insights into defense problems [5]; describtion of submarine operations, maintenance schedules, and aircraft carrier survivability [6][7]; development of a rapid assessment method for air-land conflicts in Europe [8], and development of a framework using system dynamics to model Lanchester equations [9].

However, as modern warfare continues to evolve, Lanchester equations are limited by its analytic nature and are insufficient to describe the interaction of different tactics in modern combined arms land battle. Besides, those previous studies employed system dynamics focused on force scale, overlooking the lethality of different weapons and the synergy produced by combined arms. Consequently, this study, different from those previous studies, first combines Lanchester mixed forces equations with system dynamics and focuses on the lethality of weapons to develop a combined arms system dynamics model for modern land battle. By simulating, the model proposed here is compared with the latest Lanchester mixed forces model and demonstrates how different weapons and tactics affect the force strength and how combined arms produce synergy. The results are shown in later sections.

II. THE BASIC THEORY OF THE SYSTEM DYNAMICS MODEL FOR MODERN LAND BATTLE

System dynamics model for a modern land battle is on the basis of famous Lanchester square law, which is shown as follows [1]:

$$\begin{cases} \dfrac{db}{dt} = -rc \\ \dfrac{dr}{dt} = -bk \end{cases} \qquad (1)$$

where b and r represent the blue and the red force respectively, db/dt and dr/dt are the attrition of the blue and the red force respectively, c and k are the kill rate of the red and the blue force, respectively.

Since the presentation of the Lanchester equations, they have been expanded to a large number of Lanchester-type equations in later research. In 2008, it is expanded to a mixed forces model, which is given bellow [2]:

$$\begin{cases} \dfrac{dR_\alpha(t)}{dt} = -\left(\sum_{i=1}^{n} b_i^\alpha B_i(t)\right)\dfrac{R_\alpha(t)}{R(t)} \\ \dfrac{dB_i(t)}{dt} = -\left(\sum_{\alpha=1}^{m} r_\alpha^i R_\alpha(t)\right)\dfrac{B_i(t)}{B(t)} \end{cases} \qquad (2)$$

where $B(t) = \sum B_i(t)$, $R(t) = \sum R_\alpha(t)$, $dR_\alpha(t)/dt$ is the attrition of α th unit of $R(t)$, $dB_i(t)/dt$, b_i^α is the attrition of i th unit of $B(t)$, r_α^i is the kill rate of each attacking unit (lower index) against each defender unit (upper index), $B_i(t)$ is the i th unit of $B(t)$ and $R_\alpha(t)$ is α th unit of $R(t)$.

III. CONSTRUCTING A COMBINED ARMS SYSTEM DYNAMICS MODEL

Combined arms is an approach to battle that seeks to integrate different arms to achieve mutually complementary effects. This study adopts three main arms and combines system dynamics with Lanchester mixed force equations to develop a model that is able to deal with modern combined arms land battle.

3.1 The procedure of model construction

The application of system dynamics involves developing a cause and effect diagram of the main flows of resources and information in the system, which relate to an observed cause for concern. Generally, the procedure of constructing a system dynamics model can be divided into four steps [10, 11]:

(1)Identify the problems studied and confirm the key variables and their relationships.

(2)Formulate acceptable formal decision policies that describe how a decision is made from the available information streams. Then, develop causal loop diagrams based on the initial hypothesis, key variables, and their relationships. The major strength of these qualitative diagrams lies in the fact that the qualitative model can be validated and developed into a quantitative model to capture the underlying major feedback structure of the model. This is a critical process in maintaining understanding and simplicity.

(3)Transform the conceptual qualitative causal loop diagram into a quantitative flow diagram by identifying, formulating, and integrating the stocks, flows, and other variables in the system. Then, compare the results with all the experimental knowledge about the real system, and revise the model until it is acceptable as a representation of the actual system.

(4)Design the scenarios, including environmental conditions, decision rules, strategies, and policy interactions. Evaluate the optimum strategy on the behavior of the system and derive understandings and applicable policy insights by simulation.

3.2 The conceptual qualitative model

Assume that the landed red force consists of tanks, infantry fighting vehicles (IFVs), and antitank guided missiles (AGMs), and that the blue force assigns tanks, IFVs and AGMs force to counterthrust the red force. In order to make the model close to real war, this study discusses lots of factors, including endogenous and exogenous variables, in modern land battle. The key variables are identified by expert discussion and a systems thinking process, as shown in Table 1.

Table 1. The important variables in land battle

classification	variables
Endogenous variables	Surviving number of weapons, casualty, lethality, Fire distribution
Exogenous variables	Terrain, Weather, Air superiority, Morale, Surprise

In Table 1, endogenous variables are internal variables resulted from engaging opposing force. They are critical factors of operational potential. Exogenous variables are external factors that bring operational potential into full play.

To show how different weapons affect the force strength and how combined arms produce synergy, this study focuses on endogenous variables. Also, this study adopts terrain and weather factor proposed by Dupuy [12, 13] but ignore the others exogenous factors because they are generally regarded as the same in meeting engagement in real war, as shown in Appendix.

After confirming the factors and their relationships, the casual loop diagram can be developed by a stepwise conceptualization approach, as shown in Figure 1.

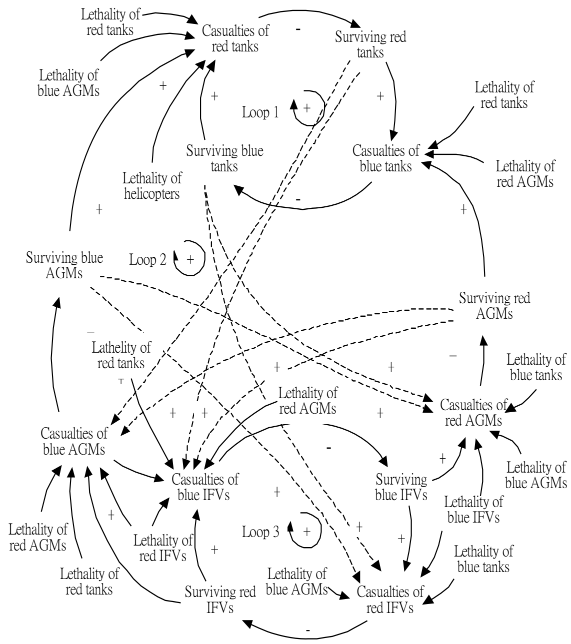

Fig. 1. The casual loop diagram of combine arms land battle.

Figure 1 shows a qualitative diagram that is employed to maintain the relationship between the system dynamics model and reality. Causal loop 1 describes the cause and effect of engaging tanks. On the left side, the surviving blue tanks fire at the surviving red tanks, decreasing the number of surviving red tanks. On the right side, the surviving red tanks fire at the blue tanks, decreasing the number of surviving blue tanks. Causal loop 2 describes the cause and effect of engaging AGMs. On the left side, the casualties caused by the red tanks, IFVs, and AGMs decrease the number of surviving blue AGMs; the blue AGMs fire at the red tanks, IFVs, and AGMs and cause casualties. On the right side, the casualties caused by the blue tanks, IFVs, and AGMs decrease the number of surviving red AGMs; the red AGMs fire at the blue tanks, IFVs, and AGMs and cause casualties. Causal loop 3 describes the cause and effect of engaging IFVs. On the left side, the surviving red IFVs fire at the blue IFVs and AGMs, decreasing the number of surviving blue IFVs and AGMs. The casualties caused by the red tanks, IFVs, and AGMs decrease the number of surviving blue IFVs. On the right side, the blue IFVs fire at the red IFVs and AGMs, decreasing the number of surviving red IFVs and AGMs. The dotted lines show that the forces are reassigned to attack subordinate targets after their tasked targets are eliminated. The symbol "+" shows a positive correlation between the adjoining variables, and the symbol "-" shows a negative correlation between the adjoining variables. All of the above three causal loops have a positive cycle, and thus, will continue to worsen.

3.3 The quantitative model

After developing causal loop diagram, the stock and flow diagrams are created by identifying, formulating, and integrating the stocks, flows, and other variables in the system. The decision policies can be formulated to describe how a decision is made from the available information streams. The stock and flow diagrams of combined arms system dynamics model for modern land battle is constructed, as shown in Figures 2–4.

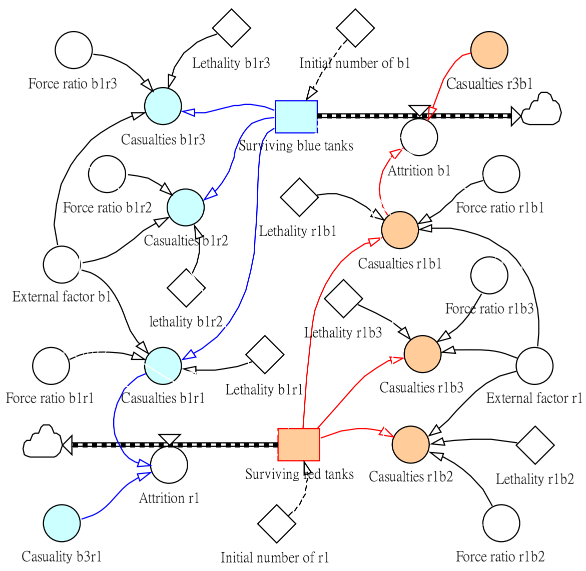

Fig. 2: The stock and flow diagram of engaging tanks.

In Fig. 2, on the right side, the red tanks fire at the blue tanks first, and, then, fire at the blue IFVs and AGMs once the blue tanks are eliminated. The casualties caused by the red tanks (Casualties r1b1) and AGMs (Casualties r3b1) decrease the number of surviving blue tanks. The casualties of blue IFVs and AGMs caused by the red tanks are connected to the other diagrams via nodes "Casualties r1b2" and "Casualties r1b3," respectively. On the left side, the blue tanks fire at the red tanks first, and, then, fire at the red IFVs and AGMs once the red tanks are eliminated. The casualties caused by the blue tanks (Casualties b1r1) and AGMs (Casualties b3r1) decrease the number of surviving red tanks. The casualties of blue IFVs and AGMs caused by the red tanks are connected to the other stock and flow diagrams via nodes "casualties b1r2" and "casualties b1r3," respectively.

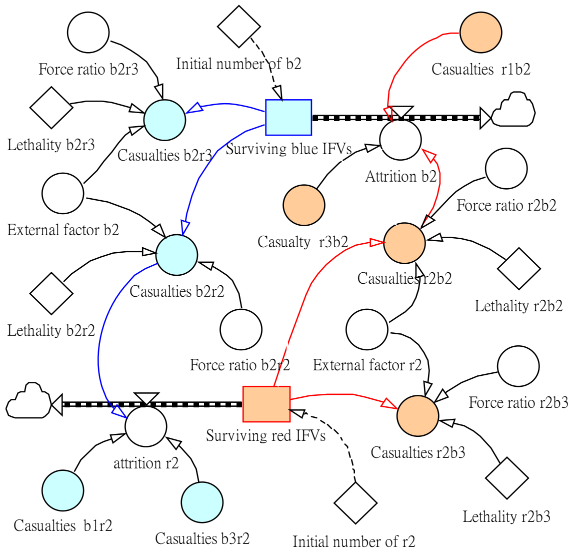

Fig. 3. The stock and flow diagram of the engaging IFVs.

In Fig. 3, on the right side, the red IFVs fire at the blue IFVs and AGMs with assigned fire distribution. The casualties caused by the red tanks (casualties r1b2), IFVs (casualties r2b2) and AGMs (casualties r3b2) decrease the number of the surviving blue IFVs. The casualties of the blue IFVs and AGMs caused by the red IFVs are connected to the other stock and flow diagrams via nodes "casualties r2b2" and "casualties r2b3," respectively. On the left side, the blue IFVs fire at the red IFVs and AGMs with assigned fire distribution. The casualties caused by the blue tanks (casualties b1r2), IFVs (casualties b2r2), and AGMs (casualties b3r2) decrease the number of the surviving red IFVs. The casualties of the red IFVs and AGMs caused by the blue IFVs are connected to the others stock and flow diagrams via nodes “casualties b2r2” and “casualties b2r3,” respectively.

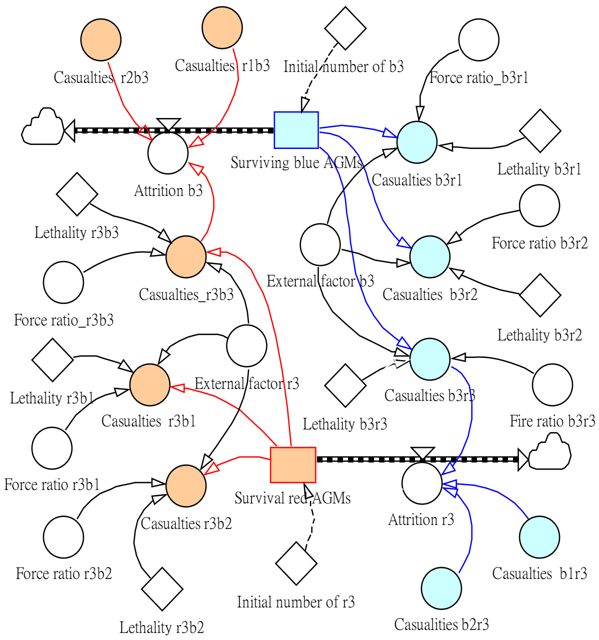

Fig. 4. The stock and flow diagram of the engaging IFVs.

In Fig. 4, on the right side, the red AGMs fire at the blue tanks first, and then fire at the blue IFVs and AGMs with reassigned fire distribution once the blue tanks are eliminated. The casualties caused by the red tanks (casualties r1b3), IFVs (casualties r2b3), and AGMs (casualties r3b3) decrease the number of the surviving blue AGMs. The casualties of the blue tanks and IFVs caused by the red AGMs are connected to the others stock and flow diagrams via nodes “casualties b3r1” and “casualties b3r2,” respectively. On the left side, the blue AGMs fire at the red tanks first, and then fire at the red IFVs and AGMs with reassigned fire distribution once the red tanks are eliminated. The casualties caused by the blue tanks (casualties b1r3), IFVs (casualties b1r2), and AGMs (casualties b1r3) decrease the number of the surviving red AGMs. The casualties of the red tanks and IFVs caused by the blue AGMs are connected to the other stock and flow diagrams via nodes “casualties b3r1” and “casualties b3r2,” respectively.

IV. MODEL VALIDATY

In general, different from analytic model, model validity in system dynamics includes structure validity and behavior validity. The main criteria for validation are structure tests, which are aimed at establishing confidence in the model structure. Behavior tests are also important after the structure tests have been conducted. The behavior adequacy of the model structure is tested through an analysis of the behavior exhibited by the structure [11, 14]. Both behavior tests and structure tests have to be verified by expert discussion in order to identify if the selection of variables and the causal relationship of qualitative and quantitative diagrams are meet the fact of real war. The validity of the model proposed here is shown as follows:

(a) Structure tests

First, the proposed model is developed based on Lanchester equations, which can be employed to represent the known and complex details of a system. Hence, the equations are reasonable. Next, the proposed model, including its structure, equations, links, and variable setting, is checked when using PowerSim software. All of the auxiliary variables and stocks are set when constructing the model. Finally, the sizes of the red and blue forces are set as zero, the results show that all values return to zero. And, set the sizes of the red and blue forces double and triple, the proposed model runs normally and no positive value and problem is observed.

(b) Behavior tests

All of the causal loops of the proposed model are positive feedback loops, the number of surviving weapons keep decreasing except for the combat reserve is committed to fight, and the surviving curves drop smoothly and monotonically. Besides, The proposed model allows both sides not only to concentrate their superior force to attack the tasked targets but also to reassign available force to attack surviving opposing force after the task targets have been eliminated. These tests confirm to the original concept of the proposed model and meet the fact of real battle.

V. SIMULATION AND DISSCUSSION

5.1 The assumptions and general scenario

Assume that the red force which is consists of 100 tanks, 70 IFVs and 60 AGMs has already landed at 5:20 a.m. on D-day and keeps on thrusting to the north. The blue force, carry 100 tanks, 70 IFVs and 60 AGMs, is assigned to counterthrust the red force. Both sides meet at 5:30 a.m. in hilly country and begin fighting immediately. The hilly area is rolling gentle and mixed, and, the weather is dry, overcast, and temperate. Both sides have the same morale condition, and there is no air support on both sides. Besides, both sides have enough manpower and ammunition. Mean time to shoot for all weapons is 3 minutes on either side because of the limitations of (target) acquisition and weapon cycle rate (shooting speed). The lethality of engaging weapons is set similar to real weapons, as shown in Table 2.

Table 2. The lethality of weapons

	Blue force			**Red force**		
	Tank	IFV	AGM	Tank	IFV	AGM
Blue tank				30.6	0.79	0.65
Blue IFV				0	0.43	0.45
Blue AGM				0.45	0.82	0.83
Red tank	0.29	0.75	0.62			
Red IFV	0	0.45	0.42			
Red AGM	0.42	0.8	0.82			

5.2 The comparison with MacKay's mixed forces model

In order to identify the model proposed here, two strategies are designed to compare with Mackay's mixed forces model. The results are given bellow:

Strategy 1

Suppose that both sides employ MacKay's mixed forces model. By simulating, the results give the number of all surviving weapons throughout the battle on both sides, as shown in Table 3 and Fig. 5.

Table 3. Surviving weapons of strategy 1

Time	**Blue force**			**Red force**		
	tank	IFV	AGM	tank	IFVs	AGM
5:30a.m.	100	70	60	100	70	60
5:33a.m.	89	48	44	88	48	43
5:36a.m.	80	30	32	77	33	31
5:39a.m.	71	17	23	67	22	22
5:42a.m.	63	7	17	57	15	15
5:45a.m.	55	0	12	49	10	11
5:48a.m.	48		9	41	6	8
5:51a.m.	43		6	34	4	5
5:54a.m.	38		5	27	2	3
5:57a.m.	34		4	22	1	2
6:00a.m.	31		3	17	0	1
6:03a.m.	29		3	12		0
6:06a.m.	28		2	7		
6:09a.m.	27		2	2		
6:12a.m.	27		2	0		

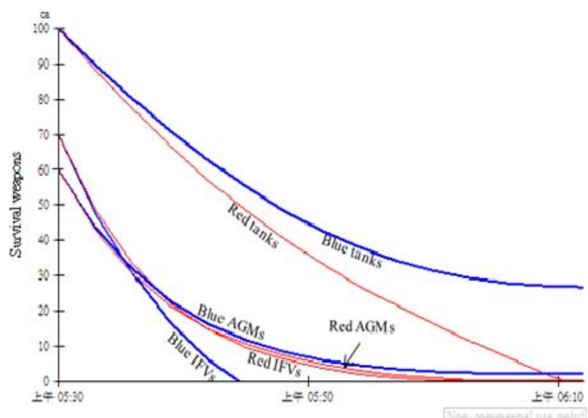

Fig. 5: Attrition curves of strategy 1.

Table 2 shows that target allocation B_i / B and R_α / R keep changing depending on the number of opposing surviving weapons. That is, the changing numbers of surviving weapons make both sides keep changing firepower depending on the proportion of the opposing force. In real battle, the main firepower is assigned to attack tasked target and fire distribution will not change until tasked targets vanish. Moreover, it is hard to calculate the number of residual forces throughout the battle because it is very difficult for either side to obtain the exact numbers for the opposing force over time. Besides, in Figure 7, the surviving curve of the red IFVs is steeper than that of the red tanks and ATMs. This shows that both sides disperse firepower to attack subordinate targets, resulting in heavy attrition but the tasked targets suffer less attack, including invalid attack.

Strategy 2

Assume that the blue force employs the model proposed here, while the red force adopts MacKay's model. By simulating, the results give the number of surviving weapons throughout the battle on both sides, as shown in Table 4. The surviving weapons' curves are shown in Fig. 6.

Table 4. Surviving weapons of strategy 2

Time	Blue force			Red force		
	tank	IFV	AGM	tank	IFVs	AGM
5:30a.m.	100	70	60	100	70	60
5:33a.m.	89	44	39	81	48	26
5:36a.m.	81	27	26	67	33	4
5:39a.m.	76	18	18	55	22	0
5:42a.m.	71	11	12	45	15	
5:45a.m.	66	7	8	35	10	
5:48a.m.	63	4	5	26	6	
5:51a.m.	60	3	4	17	4	
5:54a.m.	58	2	3	9	2	
5:57a.m.	57	1	2	0	1	
6:00a.m.	57	1	1		0	
6:03a.m.	57	1	0			

Table 4 shows that the blue force aggregates firepower on the red tanks and AGMs, resulting in heavy attrition. Relatively, the red force disperses firepower to attack the blue IFVs and ATMs, resulting in being eliminated first. Besides, as shown in Fig. 8, the slope of the surviving weapons curves of the red tanks and AGMs are relatively steeper, which implies that the blue force aggregates firepower to attack the red tanks and AGMs, resulting in heavy attrition. Once the red tanks and AGMs are eliminated, the surviving blue force is reassigned to attack the red IFVs, resulting in increasing attrition (The curves head downward rapidly). This shows that the red force assigns firepower divergently and ineffectively, but the blue force aggregates firepower effectively to attack tasked targets and win the war within a shorter period of time.

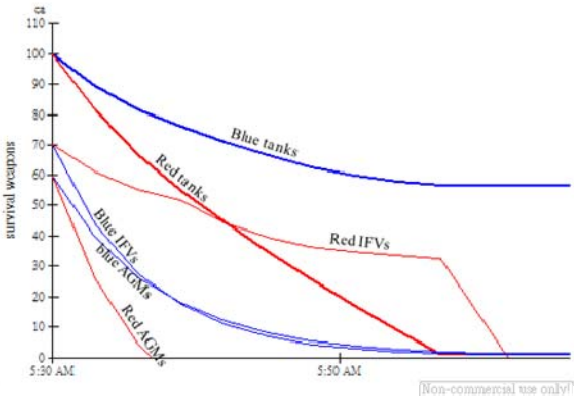

Fig. 6. Surviving weapons curve of strategy 2.

In this section, the results identify the problems of the latest Lanchester-type mixed forces model proposed by MacKay, as shown below:

(1) Both sides process several kinds of different weapons with different lethality, and thus, using the total number of blue (red) force to describe target allocation is meaningless.
(2) It is unreasonable to allocate firepower and keep changing it depending on the force ratio of the opposing force.
(3) Mackay's model compels tanks and AGMs to disperse firepower to attack subordinate targets and IFVs to waste firepower to attack invalid targets.

According to the results above, the model developed here is verified to be more reasonable and effective than MacKay's model. Besides, the advantages of the model developed here can be synthesized as follows:

(1) It makes the attacker and defender not only aggregate firepower to attack tasked targets but also to reassign the surviving force to attack residual opposing forces after tasked targets are eliminated.
(2) It can allocate firepower independently and

avoid invalid attack.

5.3 The use of tactics

Tactic is a science or art of maneuvering military forces in battle. It is the way to bring operational potential created by weapons into full play. In this section, five tactical scenarios are designed using the knowledge gained from experience to discuss some common tactics in meeting engagement. Assume that both sides employ the mode proposed here. The scenarios and results of using tactics are shown as follows:

Tactical scenario 1

Assume that both sides' commander have an insight into the battle situation before fighting and aggregate all available force to attack tasked targets (opposing tanks and AGMs) first, resulting in heavier attrition. Once the red tanks and AGMs are eliminated in turn, the surviving blue force is reassigned to attack the surviving red IFVs, resulting in increasing attrition. The result shows that the blue force wins the battle at 6:03 a.m. and 18 tanks survive, as shown in Fig. 7.

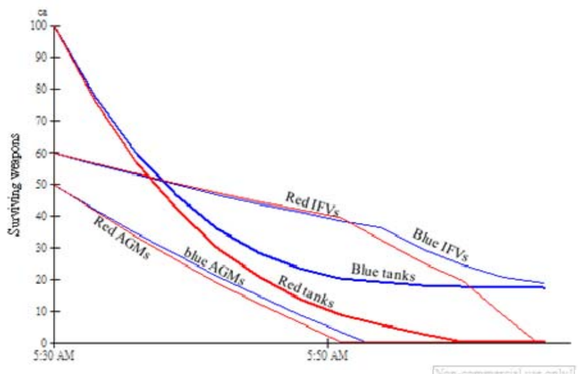

Fig 7. Surviving weapons curves of Tactical scenario 1.

Tactical scenario 2

Assume that the red commander has an insight into the battle situation and attempts to take advantage of terrain to win the battle. He would like to put additional AGMs into the battlefield before fighting. The result shows that the red force will win the battle if it reassigns at least 6 AGMs, as shown in Fig. 8.

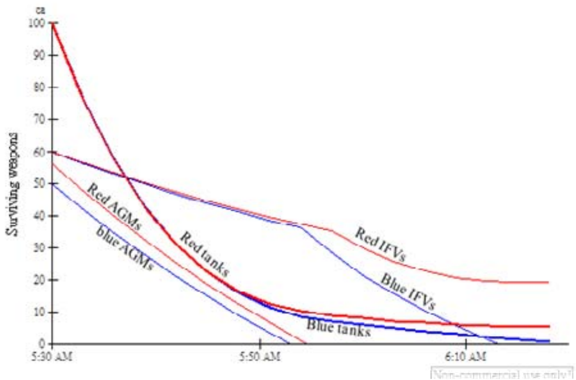

Fig. 8. The Surviving weapons' curves of Tactical scenario 1.

Tactical scenario 3

Assume that the blue commander consider the cost and effect of fighting. He puts reinforcement, 20 AGMS, into battlefield and decreases the initial number of tanks. The result shows that the blue force assigns 70 AGMs and decreases the number of tanks to 73, as shown in Fig. 9.

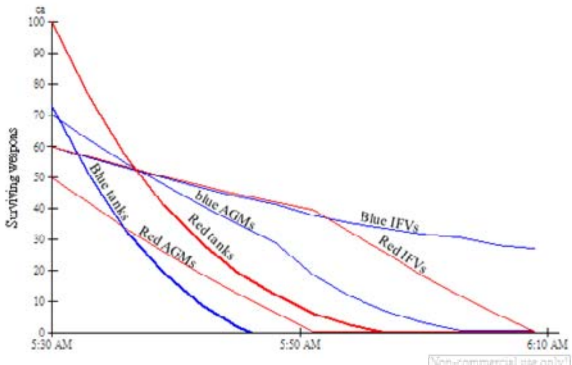

Fig. 9. Surviving weapons curves of Tactical scenario 3.

Tactical scenario 4

Assume that neither side knows the battle situation throughout the engagement. The landed red force retains 25 tanks as combat reserve and keeps making a thrust to the north. The red tank combat reserve is committed to fight if the initial main attack tanks reduce by one-fourth. The counterthrust blue force also retains 25 tanks as combat reserve and attempts to intercept the red force. The blue tank combat reserve is committed to fight if the initial main attack tanks reduce by one-fourth. The result shows that the blue force wins the battle at 6:12, as shown in

Fig. 10.

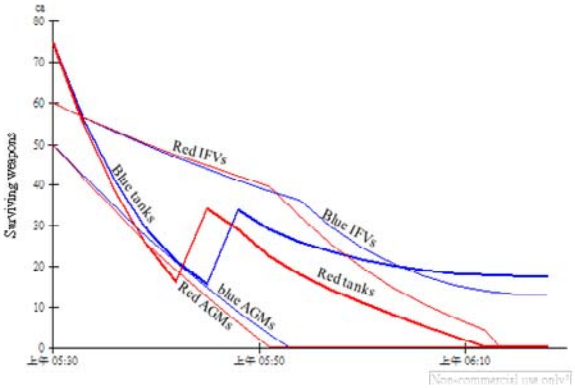

Fig. 10. Surviving weapons curves of Tactical scenario 4.

Tactical scenario 5

Assume that the red commander, a veteran of many battles, has already known the battle situation before fighting, and then, decides not to retain combat reserve. The blue force still retains 25 tanks as combat reserve. At the beginning, the red force aggregates superior tank force to attack the blue tank force. Then, the surviving red tank force is reassigned to attack blue IFVs and AGMs once the blue tanks are eliminated. The result shows that the red force wins the battle and 24 tanks survive, as shown in Fig. 11.

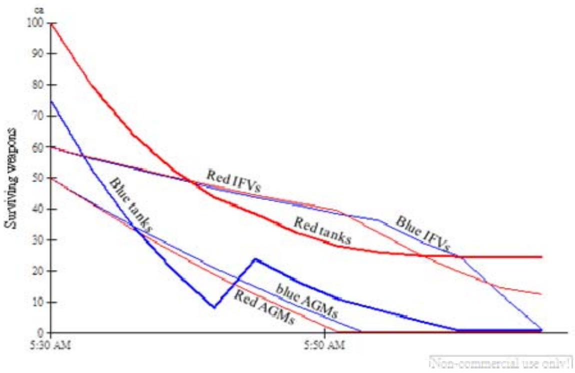

Fig. 11. Surviving weapons curves of Tactical scenario 5.

This section shows that different tactics enable inhomogeneous force processed different weapons to produce synergy. The results of simulation can be synthesized as follows:

(a) It is not necessary to maintain large combat reserve. If a commander has already known the battle situation, he can even put all available force into the battlefield to produce the synergy of aggregation.

(b) Using tactics can bring the operational potential produced by weapons into full play and make up the balance of force difference and even reverse the battle situation.

(c) Taking advantage of terrain and committing superior force can produce the effectiveness of an asymmetric operation.

VI. CONCLUTIONS

This study first combines Lanchester mixed forces equations with system dynamics to develop a model for modern combined arms land battle. Using simulation, the developed model is verified to be more reasonable and effective than the latest Lanchester-type mixed forces model and shows that using different tactics enable inhomogeneous force processed different weapons to produce synergy. In addition, the results show that it is possible to reserve inferior situation by managing adaptation and provide new thinking for a commander to manage adaptation by quantitative analysis approaches rather than subjective decision.

In the future, the model developed here can be expanded such that it becomes advantageous to use it when studying joint operations. It can also optimize tactics and force deployment in given scenarios.

REFERENCES

[1] Lanchester, F. W., Aircraft in Warfare: The Dawn of the Fourth Arm, Constable & Co., 1916.

[2] MacKay, N. J., "Lanchester models for mixed forces with Semi-dynamic modeling of heterogeneous land battle," Journal of The Operational Research Society, Vol.57, pp. 38–51, 2008.

[3] Coyle, R. G., "Model the lanchester law with system dynamics," Journal of The Operational Research Society, Vol. 32, pp.755-765, 1981.

[4] Benyamin, S. I., A system dynamics approach to aircraft survivability attrition analysis, Ph.D. dissertation, Virginia Polytechnic Institute and State University, 1984.

[5] Wolstenholme, E. F., "Defence operational

analysis using system dynamics," European Journal of Operational Research, Vol. 34, pp.10-18, 1988.
[6] Drew, D. R., "A System Dynamics Model for Managing Aircraft Survivability," The First International Conference of the System Dynamics Society, Chestnut Hill, MA USA, pp.778-791, 1983.
[7] Moussavi M and Santoso I, A system approach to management aircraft survivability, Ph.D. dissertation, Virginia Polytechnic Institute and State University, 1984.
[8] Moffat, J., "The system dynamics of future warfare," European Journal of Operational Research, Vol. 90, pp. 609-618, 1996.
[9] Michael, J. A. and Richard, F. D., "Model the lanchester law with system dynamics," JDMS., Vol.5, pp.1-20, 2008.
[10]Forrester, J W., Industrial Dynamics, Cambridge, MA: The MIT Press. Reprinted by Pegasus Communications, Waltham, MA., 1961.
[11] Sterman, J. D., Business dynamics - system thinking and modeling for a complex world, Massachusetts Institute of Technology Sloan School of Management, Irwin Mac Graw-Hill, 2000.
[12] Dupuy T. N., Numbers, prediction, and war: Using history to evaluate battle factors and predict the outcome of battles, Bobbs-Merrill Co., Indianapolis, pp.185 -225, 1979.
[13] Dupuy, T. N., Numbers, Predictions and War, Revised Ed., HERO Books, Fairfax, Virginia, pp.185-207, 1990.
[14]Forrester, J. W., "Tests for building confidence in system dynamics models," TIMS Studies in Management Sciences, Vol. 14, pp. 209-228.

APPENDIX

This appendix is made excerpts from "Numbers, prediction, and war: Using history to evaluate battle factors predicts the outcome of battles" and "Numbers, Predictions and War." These two books wrote by U.S. Ret. Col. Dupuy in 1979 after taking charge of the project of Quantified Judgment Method of Analysis for the U.S. Department of Defense and Quantified Judgment Method for the British Defence Operational Analysis Establishment. They are wildly employed to estimate operations. Because of this study discusses a meeting battle, the external factor adopted in this study are given as follows:

Table 1a. Weather factors

Weather	attacker	tank
Dry, sunshine, extreme heat	1.0	0.9
Dry, sunshine, temperate	1.0	1.0
Dry, sunshine, cold	0.9	0.9
Dry, overcast, extreme heat	1.0	1.0
Dry, overcast, temperate	1.0	1.0
Dry, overcast, cold	0.9	0.8
Wet, light, extreme heat	0.9	0.7
Wet, light, temperate	0.9	0.7
Wet, light, cold	0.9	0.7
Wet, heavy, extreme heat	0.6	0.6
Wet, heavy, temperate	0.7	0.4
Wet, heavy, cold	0.6	0.3

Table 2a. Terrain factors

Terrain	Mobility	Tank
Rugged, heavily wooded	0.4	0.3
Rugged, mixed	0.5	0.4
Rugged, bare	0.6	0.5
Rolling foothills, heavily wooded	0.6	0.6
Rolling foothills, mixed	0.8	0.7
Rolling foothills, bare	1.0	0.8
Rolling gentle, heavily wooded	0.65	0.65
Rolling gentle, mixed	0.85	0.75
Rolling gentle, bare	1.0	0.85
Flat, heavily wooded	0.7	0.7
Flat, mixed	0.9	0.8
Flat, bare, hard	1.05	1.0
Flat, desert	0.95	0.9
Rolling dunes	0.3	0.5
Swamp, jungle	0.3	0.3
Swamp, mixed or open	0.6	0.4
Urban	0.7	0.5